Robust Facility Location Problem for Hazardous Waste Transportation

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Abstract

We consider a robust facility location problem for hazardous materials (hazmat) transportation considering routing decisions of hazmat carriers. Given a network and a known set of nodes from which hazmat originate, we compute the locations of hazmat processing sites (*e.g.* incinerators) which will minimize total cost, in terms of fixed facility cost, transportation cost, and exposure risk. We assume that hazmat will be taken to the closest existing processing site. We present an exact full enumeration method, which is useful for small or medium-size problems. For larger problems, the use of a genetic algorithm is explored. Through numerical experiments, we discuss the impact of uncertainty and robust optimization in the hazmat combined location-routing problem.

1 Introduction

Hazardous wastes are generated by a large variety of commercial and industrial processes, on both small and large scale, dispersed throughout developed areas. They vary from the small amounts of waste generated by urban businesses such as dry cleaners and auto repair shops, to larger amounts produced by chemical plants and other heavy industries. For the most part, the producers of waste are responsible for arranging to dispose of it at appropriate processing or long-term storage facilities. The shippers of such waste typically

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make their own choices about which routes to follow to transport waste to the processing site. These decisions are therefore out of the control of planners, though route choices may be influenced by tolls or road bans.

In this paper, we formulate a mathematical optimization model that combines location and routing decisions for hazardous waste facilities under uncertainty. The literature on location and routing problems related to hazardous materials is extensive. An earlier survey of the area is Erkut and Neuman (1989). Alumur and Kara (2007) also include a fairly substantial literature review, though their work is not a survey as such. There are many different ways of approaching problems related to what are generally termed "obnoxious" facilities. Such facilities are necessary, but the facilities as well as transport going to and/or from them are undesirable, potentially dangerous or both. Because of this it will be undesirable to locate these facilities in certain areas. However, for a facility to be useful as well as to reduce the undesirable effects of transporting hazardous materials over long distances, it must be within a reasonable distance of the need it serves. As a result, locating such facilities represents a trade off between the desire to avoid risk and "obnoxiousness", which could result in the facility being located in some very remote area, and the desire for the facility to be cost effective, which would mean locating it in reasonable proximity to the demand it serves. For obnoxious facility location and routing models to address these conflicting objectives, they may optimize a weighted combination of cost and risk (as well as perhaps other objectives such as fairness), they may minimize risks subject to keeping cost below a given threshold, or minimize cost subject to keeping risk below some maximum acceptable level.

There are also a variety of approaches when both facility location and routing are concerned in the context of hazardous materials (hazmat). Some studies assume facility locations are given and solve a routing problem, and some solve a combined location and routing problem. The first study on combined location-routing decisions may be Zografos and Samara (1989), who propose a goal-programming approach to minimize multi-objectives of travel time, transportation risk, and disposal risk. ReVelle et al. (1991) choose sites and routes so as to minimize a convex combination of distance traveled and population exposure. List and Mirchandani (1991) consider equity for a combined location-routing problem, and Jacobs and Warmerdam (1994) use a linear programming based model for both the location of processing sites and the routing of hazardous waste for a single type of hazardous waste. Stowers and Palekar (1993) propose a combined model that minimizes the population exposure from transportation and long-term storage of hazardous wastes. Current and Ratick (1995) propose a multiobjective, mixed integer program considering minimization of cost and risk, and maximization of equity, and Giannikos (1998) presents a goal programming model considering minimization of operating cost and perceived risk, and equitable distribution of risk and disutility. Nema and Gupta (1999) create a model for the location of sites and the routing of waste with a composite objective function weighting risk and cost for multiple waste types. Cappanera et al. (2003) propose a discrete combined location-routing model that is referred as the Obnoxious Facility Location and Routing (OFLR) model and solved by a Lagrangean relaxation. Alumur and Kara (2007) use a multi-objective model (cost and exposure) for a deterministic problem where they find optimal sites for waste processing and disposal, and optimal routes between waste generation points, processing sites and disposal sites.

There are also related hazmat *network-design* problems that consider routing when the locations of origin-destination pairs are given. Kara and Verter (2004) assume that origin and destination points for hazardous material shipments are known, and address the problem of minimizing exposure risk by banning hazardous shipments from traveling on certain network arcs. Garrido (2008) and Marcotte et al. (2009) approach similar problems not by a road ban method but by road pricing, and Wang et al. (2012) extend the concept with dual-toll pricing policy. Erkut and Alp (2007) create a minimal road network (a tree or set of trees) for hazmat transport, then use a greedy algorithm to augment the network with links which will decrease cost and exposure risk. Erkut and Gzara (2008) provide a bi-level optimization approach for hazmat network design and propose a heuristic method. Bianco et al. (2009) consider a similar bi-level approach and Gzara (2012) provides a cutting-plane algorithm for hazmat network design. Bianco et al. (2013) overview the hazmat network design literature. Hazmat network design problems are often referred as *global* routing problems as opposed to *local* routing problems that determine a safe path between a single origin-destination pair.

Literature on local hazmat routing is abundant. We refer readers to a handbook chapter by Erkut et al. (2007) and references therein. A key to any local routing method is how to assess risk in a path. Various risk measures are introduced and corresponding optimization methods are devised. Erkut and Verter (1998) and Erkut and Ingolfsson (2005) discuss axioms that risk measures in hazmat routing are recommended to satisfy. For risk-averse routing, Bell (2006) considers mixed route strategies, and advanced risk measures such as value-at-risk and conditional value-at-risk are also considered in Kang et al. (2013) and Toumazis et al. (2013). A robust measure considering the worst-case is also introduced in Kwon et al. (2013).

This paper inherits the modeling components of the OFLR model of Cappanera et al. (2003). However, the proposed model of this paper has a unique modeling component that is not found in other combined location-routing models: independent behavior of hazmat carriers. While other combined models simply assign a path to each shipment, we assume that hazmat carriers are independent entities from the hazardous waste facility operators. When carriers and operators are the same, assigning a path to each shipment is possible. On the other hand, when the two groups are independent, such assignments are inappropriate or often impossible. Therefore we assume that the hazmat carriers select the shortest-path based on distances only. This modeling approach is common in the above-mentioned, more recent hazmat network design literature (Kara and Verter, 2004; Erkut and Alp, 2007; Erkut and Gzara, 2008; Marcotte et al., 2009; Bianco et al., 2009; Gzara, 2012). Therefore the proposed model in this paper has a similar model structure including bi-level mixed integer programming.

In the *non-hazmat* context, the location-routing problems are also well-studied. Earlier problems are reported in Franca and Luna (1982) and Laporte (1988), and comprehensive reviews are provided by Min et al. (1998) and Nagy and Salhi (2007). Stochastic location-routing problems are studied in Laporte et al. (1989) and in Albareda-Sambola et al. (2007) where the location decision and a priori routing decision are made in the first-stage and the routing recourse are determined in the second-stage. More recent studies on the combined location-routing problems include Shen (2007), Klibi et al. (2010), Bozkaya et al. (2010), and Toyoglu et al. (2012). We also refer the readers to Snyder (2006) for a review of facility location problems under uncertainty.

In hazmat location and routing problems, methods based on stochastic programming are less effective. This is because historical data are rarely sufficient to construct meaningful probability distributions for modeling parameters such as accident consequences. Although there are many accidents involving hazmat in the entire network, hazmat accidents are still rare events for each road segment or for each region. In addition, the consequences depend on various uncertain factors such as weather conditions, the number of people involved at the accident, the evacuation effectiveness, and the severity of the accident. Such uncertain factors make a probabilistic estimation of accident consequences difficult. This property makes a robust optimization approach more appropriate. To the best of our knowledge, this paper is the first attempt to solve a facility location and routing problem involving uncertainty by an robust optimization approach in *hazardous waste* management. A noxious facility location problem under uncertainty (Killmer et al., 2001) has been studied, but not combined with routing.

Robust optimization approaches to location problems, which are not combined with routing decisions, in the *non-hazmat* context include Averbakh and Berman (1997, 2000a,b), Carrizosa and Nickel (2003), Baron et al. (2011), and Gülpınar et al. (2013). However for the combined problems there seems to be no research using robust optimization reported in the literature. It may be because applying robust optimization methods to combined problems considering both demand uncertainty and transportation cost uncertainty requires treatment of *multiplicative* uncertain parameters, and an appropriate approach has only recently been proposed by Kwon et al. (2013). In addition, most robust optimization approaches consider only demand uncertainty. While demand uncertainty is the primary source of uncertainty in many location and transportation problems, transportation risk (transportation cost in general) is also a significant source of uncertainty in the hazmat context.

In most cases, hazmat facility location-routing problems are formulated as a 0-1 mixed integer program and solved by commercial optimization solvers like CPLEX or LINDO. One notable exception is the OFLR model by Cappanera et al. (2003) who solve the problem by a Lagrangean relaxation. In this paper, we provide a 0-1 mixed integer programming formulation that is solvable by CPLEX when the problem size is small. Sometimes we find that CPLEX fails to solve the problem even when the problem size is as small as 90-node. For such cases, we propose a genetic algorithm. While Lagrangean relaxation methods are popularly used for location problems and for the OFLR model, it seems inappropriate for the problem in this paper as the problem structure involves two additional levels: one to model carriers' behavior and the other to consider robustness.

In summary, the unique characteristics of the proposed model and the contributions of this paper are as follows:

- This research studies a hazardous waste facility location problems under demand and risk uncertainty.
- This research applies a robust optimization methodology for a combined location-routing problem.
- This research models independent route-choice behavior of hazmat carriers.
- This research provides a single-level mixed-integer program reformulation of the proposed location-routing model that is solvable by a commercial solver when the problem size is small. A genetic algorithm is proposed for large-scale problems.

For the present work we will assume that the origin points for hazardous waste are known, but the destination points (waste treatment or disposal sites) are not. We will then find the set of locations for waste processing sites which minimizes a linear combination of fixed facility cost and the risk posed by the shipment of hazardous wastes (*e.g.* due to spills). We will assume that the shipment of the waste will be done by third-party carriers who seek to minimize their own costs (that is, that they will take the shortest path to the nearest waste processing facility), and that they are not affected by tolls or road bans. (Or rather, if tolls or road bans do exist they are a *fait accompli* reflected in the existing arc costs of our network rather than something we need to calculate ourselves.)

2 Model

Assume that hazardous waste is generated at some known subset of the nodes of the network. Our goal is to choose, from some set of suitable network nodes, sites for facilities where this waste may be processed (treated, incinerated, etc.). If the cost of transporting the waste is borne by the shipper, and neither road bans nor tolls are in place, we will expect waste shipments to follow the lowest-cost route to any processing facility. Given this assumption, what is the choice of processing site locations which will minimize cost/exposure risk?

We will use the following notation:

- $\mathcal{N}$: set of nodes i
- $\mathcal{A}$: set of arcs (i, j)
- $\mathcal{G} \equiv (\mathcal{N}, \mathcal{A})$
- L_{ij}: length of arc (i, j)
- R_{ij} : a measure of the exposure risk due to one truck carrying hazardous waste through arc (i, j).
- $\mathcal{S}$: set of hazardous waste shipments
- $o(s)$: the node at which shipment $s \in \mathcal{S}$ originates
- $\mathcal{M}$: the set of candidate locations for facilities to handle waste. We will assume $o(s) \notin \mathcal{M}$ for all s.
- F_i: facility construction cost at node $i \in \mathcal{M}$
- N^s: the number of trucks required for shipment s
- y_i: 1 if a waste processing facility is located at node i, 0 otherwise
- x^s_{ij}: 1 if link (i, j) is used for shipment s, 0 otherwise.

2.1 Nominal Problem

For the nominal (deterministic) problem, we assume that N^s, the number of shipments originating from each site, is known exactly, as is R_{ij}, the exposure risk for each network arc. In this case we minimize facility construction cost plus exposure risk which is known with certainty for a given value of y. Note that as exposure risk and construction cost are not directly comparable we will have to make some sort of a trade-off between these two components of the objective function (that is, place a dollar amount on a unit of exposure risk). A higher dollar cost can be placed on a unit of exposure if the decision maker is risk averse, and a lower figure if he is less so. This results in the formulation:

$$\min_{y_i \in \{0,1\}} \left[w_1 \sum_{i \in \mathcal{M}} F_i y_i + w_2 \sum_{(i,j) \in \mathcal{A}} L_{ij} x_{ij}^s(y) + w_3 \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} N^s R_{ij} x_{ij}^s(y) \right] \tag{1}$$

where $x^s(y)$ defines a shortest path routing for shipment s given the set of possible disposal sites specified by the vector y, namely:

$$x^s(y) = \arg\min_{x^s \in X^s(y)} \sum_{(i,j) \in \mathcal{A}} L_{ij} x_{ij}^s \quad \forall s \in \mathcal{S} \tag{2}$$

and $X^s(y)$, the set of feasible flows, is defined by

$$\sum_{(i,k) \in \mathcal{A}} x_{ik}^s - \sum_{(k,i) \in \mathcal{A}} x_{ki}^s \begin{cases} = +1 & i = o(s) \\ \geq -y_i & i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{N}, s \in \mathcal{S} \tag{3}$$

$$x_{ij}^s \in \{0,1\} \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \tag{4}$$

In the objective function (1) of the upper-level problem, w_1, w_2, and w_3 represent the weight for each cost component. The first part represents the facility construction cost. The second part is the transportation cost. The weight factor w_2 may have two different meanings depending on the context. It may reflect the actual transportation cost that is required for hiring carriers, or may reflect the hazmat facility location decision maker's interest in reducing transportation cost so as to prevent the case when the facilities are located too far away. The weight factor w_2 may also be zero. The third part represents the risk component.

Constraints (3) specify that any node i which is an origination point for waste must have a positive net outflow ($x_{ij}^s = 1$). Nodes which are neither waste origin points nor waste processing sites must have a zero net outflow ($x_{ij}^s = 0$). Any node i which is eligible to be a waste processing site ($i \in \mathcal{M}$) may have a negative net outflow of waste if i is selected as a processing site (and therefore $y_i = 1$), otherwise it must have a zero net outflow.

For uncertain N^s and R_{ij} we may define a number of uncertain problems. For example, if the mean values of N^s and R_{ij} are known we may simply solve a problem equivalent to the above where we seek to minimize the total cost in the event that the uncertain parameters take their mean values. If the N^s and R_{ij} are uncertain but have known probability distributions, we may formulate a stochastic problem where the expected total cost is minimized.

2.2 Robust Problem with Budgeted Uncertainty Sets

In practice we may have limited information about uncertain parameters. Where probability distributions or even mean values are unknown, robust optimization is a useful technique. We define a bounding box where each uncertain parameter is constrained to fall within a specified range, and the total deviation from nominal values is limited by a budget of uncertainty. In this case, we define two separate budgets, one for the uncertainty in the number of trucks required for a given shipment and another for the uncertainty in exposure risk for a given arc. The robust problem therefore becomes:

$$\min_{y_i\in\{0,1\}} \left\{ w_1 \sum_{i\in\mathcal{M}} F_i y_i + w_2 \sum_{(i,j)\in\mathcal{A}} L_{ij} x^s_{ij}(y) + w_3 \max_{u\in\mathcal{U}, v\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} [N^s + K^s u^s][R_{ij} + Q_{ij} v_{ij}] x^s_{ij}(y) \right\}$$

subject to (2)-(4) above, which define the shortest feasible flow. Q_{ij} is the width of the uncertainty in exposure risk for arc (i,j) and K^s is the width of the uncertainty in the number of trucks required to transport shipment s. Note that N^s and R_{ij} now denote the minimum number of trucks and minimum exposure risk r, respectively. The uncertainty sets $\mathcal{U}$ and $\mathcal{V}$ with budgets of uncertainty Γ_u and Γ_v respectively are defined by

$$\mathcal{U} = \left\{ u : \sum_{s\in\mathcal{S}} u^s \le \Gamma_u, \quad 0 \le u^s \le 1 \right\}$$

$$\mathcal{V} = \left\{ v : \sum_{(i,j)\in\mathcal{A}} v_{ij} \le \Gamma_v, \quad 0 \le v_{ij} \le 1 \right\}$$

Note that we have two uncertainty sets, each with its own separate budget. As was shown in Kwon et al. (2013), it is not necessarily possible to model the problem accurately (or find the solution which minimizes risk) with a single budget. In addition, because the two uncertain parameters are for each shipment s and each arc (i,j), respectively, it is not obvious how we would build a single merged uncertainty set.

2.2.1 Linearization of the Lower-Level Problem

Our first step in solving the problem will be to convert it from a bi-level problem to a single level problem. We begin with the fact that, for any given y, the lower-level shortest path problem is unimodular, as noted by Kara and Verter (2004), and must therefore have an integral optimal solution. Therefore the problem is equivalent to that obtained by relaxing the binary constraints on the x^s_{ij}. The relaxed version of the lower-level problem is

$$\min_{x^s} \sum_{(i,j)\in\mathcal{A}} L_{ij} x^s_{ij}$$

subject to

$$(-\zeta^s_i) \qquad \sum_{(i,k)\in\mathcal{A}} x^s_{ik} - \sum_{(k,i)\in\mathcal{A}} x^s_{ki} \begin{cases} = +1 & i = o(s) \\ \geq -y_i & i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{N} \tag{5}$$

$$(\phi^s_{ij}) \qquad -x^s_{ij} \leq 0 \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \tag{6}$$

Note that we omit the constraint $x^s_{ij} \leq 1$. We can do this because although feasible solutions may exist with some $x^s_{ij} > 1$ (but satisfying all other constraints), no such solution will ever be optimal (for such a solution it is easy to show how to construct a lower-cost solution). Let us introduce dual variables $-\zeta^s_i$ and ϕ^s_{ij} for constraints (5) and (6), respectively. Please note that the '-' sign in $-\zeta^s_i$ is just to match with the common notational convention in the shortest-path related literature. The KKT conditions for the above problem are

$$L_{ij} - \zeta^s_i + \zeta^s_j - \phi^s_{ij} = 0 \qquad \forall (i,j) \in \mathcal{A} \tag{7}$$

$$\phi^s_{ij} x^s_{ij} = 0 \qquad \forall (i,j) \in \mathcal{A} \tag{8}$$

$$\Big(- \sum_{(i,k)\in\mathcal{A}} x^s_{ik} + \sum_{(k,i)\in\mathcal{A}} x^s_{ki} - y_i \Big) \zeta_i = 0 \quad \forall i = \mathcal{M} \tag{9}$$

$$\phi^s_{ij} \geq 0 \qquad \forall (i,j) \in \mathcal{A} \tag{10}$$

$$\zeta^s_i \geq 0 \qquad \forall i \in \mathcal{M} \tag{11}$$

A more detailed development of these KKT conditions is given in Appendix A. The complementarity conditions (8) can be linearized using a large number M as follows:

$$\phi^s_{ij} \leq M(1 - x^s_{ij}) \quad \forall (i,j) \in \mathcal{A}$$

Similarly the other complementarity conditions (9) can be linearized as:

$$\zeta^s_i \leq M\left[1 - \Big(\sum_{(i,k)\in\mathcal{A}} x^s_{ik} - \sum_{(k,i)\in\mathcal{A}} x^s_{ki} + y_i \Big)\right] \quad \forall i \in \mathcal{M}$$

Therefore the robust hazardous waste facility location problem becomes:

$$\min_{y_i \in \{0,1\}} \left\{ \sum_{i \in \mathcal{M}} F_i y_i + \max_{u \in \mathcal{U}, v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} x_{ij}^s [N^s + K^s u^s][R_{ij} + Q_{ij} v_{ij}] \right\}$$

subject to

$$\sum_{(i,k) \in \mathcal{A}} x_{ik}^s - \sum_{(k,i) \in \mathcal{A}} x_{ki}^s \begin{cases} = 1 & i = o(s) \\ \geq -y_i & i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} \quad \forall i \in \mathcal{N}$$

$$L_{ij} - \zeta_i^s + \zeta_j^s - \phi_{ij}^s = 0 \quad \forall (i,j) \in \mathcal{A}$$

$$\phi_{ij}^s \leq M(1 - x_{ij}^s) \quad \forall (i,j) \in \mathcal{A}$$

$$\zeta_i^s \leq M \left[1 - \left(\sum_{(i,k) \in \mathcal{A}} x_{ik}^s - \sum_{(k,i) \in \mathcal{A}} x_{ki}^s + y_i \right) \right] \quad \forall i \in \mathcal{M}$$

$$\phi_{ij}^s \geq 0 \quad \forall (i,j) \in \mathcal{A}$$

$$\zeta_i^s \geq 0 \quad \forall i \in \mathcal{M}$$

$$\zeta_i^s \text{ free} \quad \forall i \notin \mathcal{M}$$

$$x_{ij}^s \in \{0,1\} \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}$$

2.2.2 Linearization and Dualization of the Inner Maximization Problem

We proceed to linearize the inner maximization problem and then convert it into a minimization problem by replacing it with its dual, whereupon we will be left with an ordinary mixed-integer linear programming problem. To do so, first we note that the objective function of the inner maximization problem can be written as follows:

$$\begin{aligned}
&\max_{u \in \mathcal{U}, v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} x_{ij}^s [N^s + K^s u^s][R_{ij} + Q_{ij} v_{ij}] \\
=&\max_{u \in \mathcal{U}, v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} x_{ij}^s [N^s R_{ij} + N^s Q_{ij} v_{ij} + K^s R_{ij} u^s + K^s Q_{ij} u^s v_{ij}] \\
=&\left[\sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} x_{ij}^s N^s R_{ij} + \max_{u \in \mathcal{U}, v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} x_{ij}^s \left\{ N^s Q_{ij} v_{ij} + K^s R_{ij} u^s + K^s Q_{ij} u^s v_{ij} \right\} \right]
\end{aligned}$$

For any given x, the inner maximization problem is equivalent to the following:

$$\max_{u\in\mathcal{U},v\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} x_{ij}^s \Big\{ N^s Q_{ij} v_{ij} + K^s R_{ij} u^s + K^s Q_{ij} u^s v_{ij} \Big\}$$

subject to

$$\mathcal{U} = \left\{ u : \sum_{s\in\mathcal{S}} u^s \le \Gamma_u, \quad 0 \le u^s \le 1 \right\}$$

$$\mathcal{V} = \left\{ v : \sum_{(i,j)\in\mathcal{A}} v_{ij} \le \Gamma_v, \quad 0 \le v_{ij} \le 1 \right\}$$

which can be linearized as follows (Kwon et al., 2013):

$$\max_{u\in\mathcal{U},v\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} x_{ij}^s \Big\{ N^s Q_{ij} v_{ij} + K^s R_{ij} u^s + K^s Q_{ij} w_{ij}^s \Big\}$$

subject to

$$\begin{aligned}
u^s &\le 1 && (\rho^s) \\
v_{ij} &\le 1 && (\mu_{ij}) \\
-u^s + w_{ij}^s &\le 0 && (\eta_{ij}^s) \\
-v_{ij} + w_{ij}^s &\le 0 && (\pi_{ij}^s) \\
\sum_{s\in\mathcal{S}} u^s &\le \Gamma_u && (\theta_u) \\
\sum_{(i,j)\in\mathcal{A}} v_{ij} &\le \Gamma_v && (\theta_v) \\
u^s, v_{ij}, w_{ij}^s &\ge 0
\end{aligned}$$

The dual problem of the above problem is:

$$\min_{\theta_u,\theta_v,\rho,\mu,\eta,\pi} \left[\Gamma_u\theta_u + \Gamma_v\theta_v + \sum_{(i,j)\in\mathcal{A}} \mu_{ij} + \sum_{s\in\mathcal{S}} \rho^s \right]$$

subject to

$$\rho^s - \sum_{(i,j)\in\mathcal{A}} \eta_{ij}^s + \theta_u \ge \sum_{(i,j)\in\mathcal{A}} K^s R_{ij} x_{ij}^s \quad \forall s \in \mathcal{S}$$

$$\mu_{ij} - \sum_{s\in\mathcal{S}} \pi_{ij}^s + \theta_v \ge \sum_{s\in\mathcal{S}} N^s Q_{ij} x_{ij}^s \quad \forall (i,j) \in \mathcal{A}$$

$$\eta_{ij}^s + \pi_{ij}^s \geq K^s Q_{ij} x_{ij}^s \quad \forall s \in \mathcal{S}, (i,j) \in \mathcal{A}$$
$$\rho^s, \mu_{ij}, \eta_{ij}^s, \pi_{ij}^s, \theta_u, \theta_v \geq 0$$

2.2.3 Single-Level Robust Facility Location Problem

Replacing the inner maximization problem with its dual, we obtain a single-level optimization problem of the form:

$$\min_{y_i \in \{0,1\}} \left\{ w_1 \sum_{i \in \mathcal{M}} F_i y_i + w_2 \sum_{(i,j) \in \mathcal{A}} L_{ij} x_{ij}^s + w_3 \left(\sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} N^s R_{ij} x_{ij}^s + \Gamma_u \theta_u + \Gamma_v \theta_v + \sum_{(i,j) \in \mathcal{A}} \mu_{ij} + \sum_{s \in \mathcal{S}} \rho^s \right) \right\}$$

subject to

$$\sum_{(i,k) \in \mathcal{A}} x_{ik}^s - \sum_{(k,i) \in \mathcal{A}} x_{ki}^s \begin{cases} = +1 & i = o(s) \\ \geq -y_i & i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} \quad \forall i \in \mathcal{N}, s \in \mathcal{S}$$
$$L_{ij} - \zeta_i^s + \zeta_j^s - \phi_{ij}^s = 0 \quad \forall (i,j) \in \mathcal{A}$$
$$\phi_{ij}^s \leq M(1 - x_{ij}^s) \quad \forall (i,j) \in \mathcal{A}$$
$$\zeta_i^s \leq M \left[1 - \left(\sum_{(i,k) \in \mathcal{A}} x_{ik}^s - \sum_{(k,i) \in \mathcal{A}} x_{ki}^s + y_i \right) \right] \quad \forall i \in \mathcal{M}$$
$$\phi_{ij}^s \geq 0 \quad \forall (i,j) \in \mathcal{A}$$
$$\zeta_i^s \geq 0 \quad \forall i \in \mathcal{M}$$
$$\zeta_i^s \text{ free} \quad \forall i \notin \mathcal{M}$$
$$\rho^s - \sum_{(i,j) \in \mathcal{A}} \eta_{ij}^s + \theta_u \geq \sum_{(i,j) \in \mathcal{A}} K^s R_{ij} x_{ij}^s \quad \forall s \in \mathcal{S}$$
$$\mu_{ij} - \sum_{s \in \mathcal{S}} \pi_{ij}^s + \theta_v \geq \sum_{s \in \mathcal{S}} N^s Q_{ij} x_{ij}^s \quad \forall (i,j) \in \mathcal{A}$$
$$\eta_{ij}^s + \pi_{ij}^s \geq K^s Q_{ij} x_{ij}^s \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}$$
$$x_{ij}^s \in \{0,1\} \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}$$
$$\rho^s, \mu_{ij}, \eta_{ij}^s, \pi_{ij}^s, \theta_u, \theta_v \geq 0 \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}$$

This is a mixed integer linear program with all integers being binary. Small instances may be solved using CPLEX. Beyond a certain problem size, however, CPLEX requires too much memory. (For example, a problem with $|\mathcal{N}| = 1000$ $|\mathcal{A}| = 3000$ and $|\mathcal{S}| = 20$ requires 203,080 constraints (not including nonnegativity and binarity constraints) and 264,022 decision variables.) We must therefore use a heuristic to find good-quality solutions for larger problems.

CPLEX sometimes fails to solve even small examples (*e.g.* the 90-node example) for certain values of Γ_u and Γ_v. The branch and cut tree grows until memory is exhausted. In such cases it is possible, for sufficiently small $|\mathcal{M}|$, to solve them by complete enumeration. The number of feasible solutions is $2^{|\mathcal{M}|} - 1$. For any given value of y the corresponding x_{ij}^s values may be computed by solving $|\mathcal{S}| \times |\mathcal{M}|$ routing problems. Once this is done, both x and y are known so the formulation no longer has any integer variables but instead reduces to a linear program of the form

$$\max_{\mathcal{U}\times\mathcal{V}} \left[\sum_{s\in\mathcal{S}} \sum_{(i,j)\in\mathcal{A}} \left[u^s K^s R_{ij} + v_{ij} N^s Q_{ij} + w_{ij}^s K^s Q_{ij} \right] \right]$$

subject to

$$\begin{aligned}
\sum_{(i,j)\in\mathcal{A}} v_{ij} &\leq \Gamma_v \\
\sum_{s\in\mathcal{S}} u^s &\leq \Gamma_u \\
w_{ij}^s &\leq v_{ij} \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
w_{ij}^s &\leq u^s \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
u^s &\leq 1 \quad \forall s \in \mathcal{S} \\
v_{ij} &\leq 1 \quad (i,j) \in \mathcal{A} \\
u, v, w &\geq 0
\end{aligned}$$

The problem may then be solved by full enumeration using a simple algorithm:

Step 1: Solve the $|\mathcal{S}| \times |\mathcal{M}|$ routing problems to determine the optimal routes from the set of waste origin points $\{o(s) : s \in \mathcal{S}\}$ to the set of possible destinations $\mathcal{M}$, and cache the resulting paths in the set $\mathcal{H}$.

Step 2: For each possible value of y

[a.] For each shipment $s \in \mathcal{S}$, use the cached shortest paths from the set $\mathcal{H}$ to determine the shortest path to any facility (thereby determining the value of x).

[b.] Now that x and y are known, solve the above LP to determine the objective function value

Step 3: The optimal solution will of course be the value of y which minimizes the objective function of the LP

3 Genetic Algorithm

For large problems, neither using CPLEX to solve the MILP formulation nor complete enumeration will be practical. Many location problems are NP-hard and therefore require some sort of heuristic approach for large instances. Genetic algorithms have long been used for solving location problems. See Jaramillo et al. (2002) for some discussion of the use of genetic algorithms for location problems. In our case, the genetic algorithm is a natural choice because y, our vector of decision variables is easily encoded into a binary vector where essentially every possible vector corresponds to a feasible solution. (If we use as our genome the subset of y defined as $\{y_i : i \in \mathcal{M}\}$ then the only infeasible solution which may be coded for by the genome is the trivial case of the vector of zeros.) For example, if we have 10 candidate locations ($|\mathcal{M}| = 10$), and a feasible solution can be coded as

$$1\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 1$$

when locating 4 facilities are optimal and their locations are first, fourth, sixth, and tenth candidates; '1' means that the candidate is chosen, and '0' means not. In the genetic algorithm, we will consider variations, through mutation and crossover, of such a coded solution. The total number as well as the location of '1'-genes may change.

3.1 Basic Structure of the Algorithm

We will use a simple structure for the genetic algorithm, as follows:

Step 0: Create an initial population of n individuals by generating random binary strings.

Step 1: Members of the current population reproduce using a "crossover" method.

Step 2: Members undergo spontaneous "mutation" (each bit in the genome of an individual flips with given probability).

Step 3: The objective function value (fitness) is calculated for each member of the population.

Step 4: If the desired number of generations has elapsed, stop. The fittest individual in the current generation is the best solution found. Otherwise cull all but the n fittest individuals from the population and return to step 1.

Certain parameters such as the number of individuals in the population, the number of generations to run the algorithm and the probability of mutation may be adjusted so as to improve the likelihood of getting a good solution quickly.

3.2 Calculating the Objective Function Value

When using a genetic algorithm, we need to be able to calculate the fitness of a given individual. In this case, given the vector y (*i.e.* where waste processing facilities are located), calculate the objective function of the robust optimization problem

$$
\begin{aligned}
Z^R = w_1 \sum_{i \in \mathcal{M}} F_i y_i + w_2 \sum_{(i,j)\in\mathcal{A}} L_{ij} x_{ij}^s(y) + w_3 \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} x_{ij}^s(y) N^s R_{ij} \\
+ w_3 \max_{u\in\mathcal{U}, v\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} x_{ij}^s(y) \left[N^s Q_{ij} v_{ij} + K^s R_{ij} u^s + K^s Q_{ij} u^s v_{ij}\right]
\end{aligned}
$$

To solve this, we must first solve the $\mid \mathcal{S} \mid \times \mid \mathcal{M} \mid$ shortest-path problems to compute the cost of transporting waste from each origin point to each possible destination, as we did when doing full enumeration. Although this may be computationally expensive if $\mathcal{S}$, $\mathcal{M}$ and $\mathcal{N}$ are large, it need be done only once if we cache the results. This information may be used to quickly determine the $x_{ij}^s(y)$ values for any given y. Once this is done, the terms $\sum_{i\in\mathcal{M}} y_i F_i$, $\sum_{(i,j)\in\mathcal{A}} L_{ij} x_{ij}^s(y)$ and $\sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} x_{ij}^s(y) N^s R_{ij}$ reduce to constants and what remains is the quadratic problem

$$
\max_{u\in\mathcal{U}, v\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} x_{ij}^s(y) \left[N^s Q_{ij} v_{ij} + K^s R_{ij} u^s + K^s Q_{ij} u^s v_{ij}\right]
$$

Which although it is quadratic is much smaller than the corresponding instance of the mixed-integer linear problem from Section 2.2.3, and may be solved by CPLEX even when $|\mathcal{S}|$ and $|\mathcal{A}|$ are fairly large. (It could be linearized by the method used in Section 2.2.2, but in our experience this was not necessary; CPLEX solved even fairly large-size examples of this quadratic problem without difficulty.)

4 Numerical Experiments

We solved the robust and nominal problems for examples of size $|\mathcal{N}| = 50$, 90, 200 and 1000. The 50 and 90-node problem use subsets of the road network for the Albany, New York area (shown in Figure 1), and the 200 and 1000-node examples use subsets of the road network in San Joaquin County, California. The three smaller instances were solvable with CPLEX, but the 1000-node problem was too large for CPLEX and was solved by the genetic algorithm. We assumed $(w_1, w_2, w_3) = (1, 0, 1)$. In a computer running 64-bit Windows 7 with 3.10 GHz CPU and 6 GB RAM and MATLAB R2010b, the full enumeration took 2,771 seconds, while the genetic algorithm took 975 seconds until to run 50 generations for the 90-node Albany network with $|\mathcal{S}| = 10$ and $|\mathcal{M}| = 10$. However, the best solution found was found after only six generations. Of the total time, 6.84 seconds were used to compute and cache the shortest-paths.

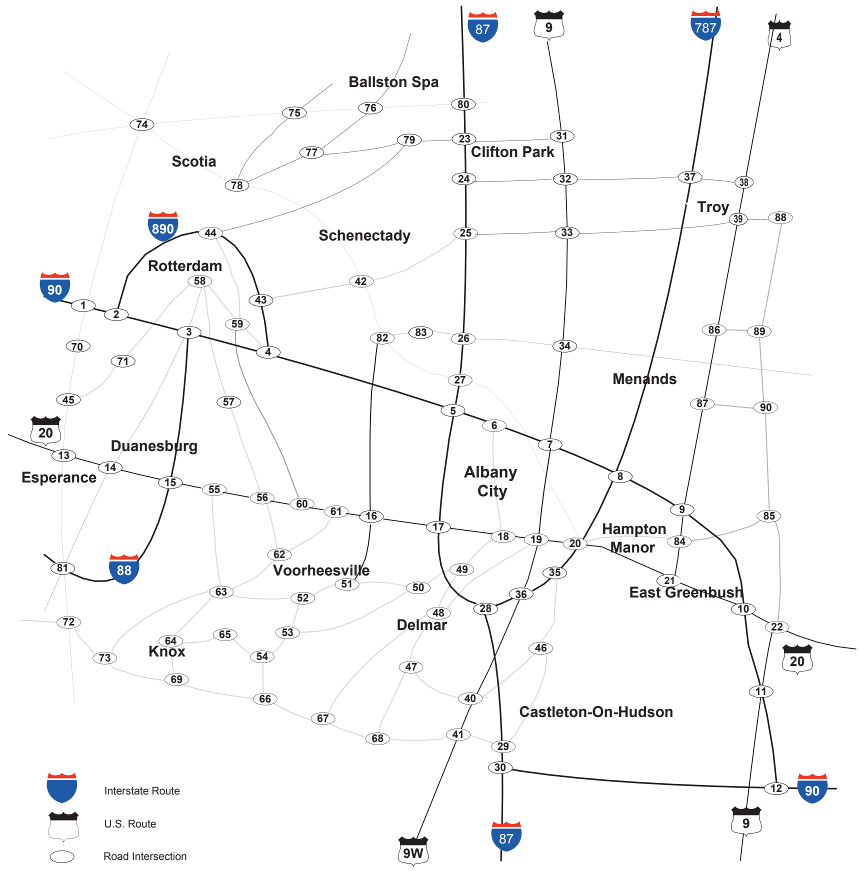


Figure 1: Albany, New York area road network. From Kwon et al. (2013)

4.1 Results

The robust and nominal problems were solved. The resulting solutions were then compared to see how well they performed over a sample of scenarios (sampled from uniform distributions) from the uncertainty set $\mathcal{U} \times \mathcal{V}$. The results for the comparison for the Albany, New York area problem are shown in Figure 2. (For this example the values $\Gamma_u = 4$ and $\Gamma_v = 18$ were used.) We see that the nominal and robust solutions perform almost identically in a simulation with 10,000 samples. Table 2 shows worst case costs for the

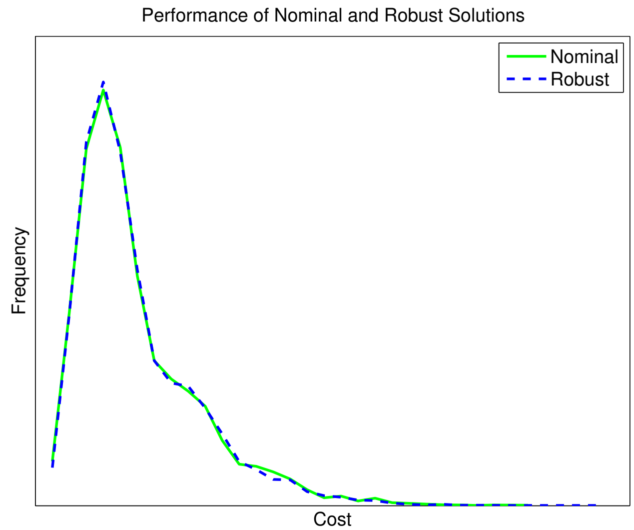


Figure 2: The performance of robust and nominal solutions over a random sample from the uncertainty set $\mathcal{U} \times \mathcal{V}$ for a 90-node problem (Albany, New York area). For this example the values $\Gamma_u = 4$ and $\Gamma_v = 18$ were used.

Table 1: Statistics for the Experiments

	Values for Figure 2		Values for Figure 5		Values for Figure 6	
	norminal	robust	norminal	robust	norminal	robust
mean	748,916	746,089	3,855,337	3,771,625	2,544,805	2,256,858
variance	5.886×10^{10}	5.539×10^{10}	1.800×10^{13}	1.705×10^{13}	5.445×10^{11}	5.371×10^{11}
min	384,200	393,990	384,200	393,990	754,090	689,393
max	2,304,795	2,514,635	33,351,098	30,742,916	5,263,779	5,002,351

nominal and robust solutions for a variety of values of Γ_u and Γ_v. The robust solution outperforms the nominal solution more often than not, but not always. Theoretically, the robust solution must always outperform the nominal solution (Table 3), but in simulations the sampled cases may not include the theoretical worst case. For example, see the case with $\Gamma_u = 3$ and $\Gamma_v = 10$ in Table 2. The results reflect the unlikely but high costs events of arc exposure risk. For example, with $\Gamma_u = 2$ and $\Gamma_v = 8$ the robust solution had an anomalously high worst-case cost. Some statistics from the numerical simulations are provided in Table 1. Another thing we notice here is the predictable nature of u-uncertainty as opposed to v-uncertainty. When $\Gamma_v = 0$ the robust solution never has poorer worst case performance than the nominal solution. This is because u-uncertainty always impacts both the nominal and robust solutions. This is not true for v-uncertainty; the nominal and robust solutions use different sets of arcs, so one solution may be impacted when the other is not.

	Γ_u										
Γ_v	0	1	2	3	4	5	6	7	8	9	10
0	212052	407108	565291	686676	788256	880175	965860	1034641	1090229	1122976	1138009
	212052	379760	529312	671441	773021	864940	950626	1019407	1074994	1107742	1122775
2	433387	998554	1148224	1427494	1637587	1747729	1949187	1937895	2102830	2566790	2635571
	431797	799659	1188006	1378990	1512452	1762649	1910574	1922661	2057870	2166926	2600824
4	461646	911462	1499699	1622988	1992766	1983807	2058966	2430664	2325766	2560046	2635571
	467021	914987	1607905	1620267	1685497	1993598	2022411	2415430	2310532	2544813	2600824
6	524773	1168519	1412895	1592700	2226439	1957824	2255120	2340063	2605443	3193143	2686605
	490511	1851976	1933752	1357636	2211206	1955102	2252399	2402015	2590210	2912727	2600824
8	613189	1168519	1471038	1682258	2176574	2228360	2752229	2432462	2826157	3131626	3016031
	537666	1759687	2356537	1692049	2186365	2238151	2412753	2490909	2810923	2746935	2981431
10	567716	1168519	1505822	1739956	2086025	2398714	3060647	2557722	2946249	3186506	3160289
	591397	1220618	1587703	1814018	1861926	2395993	3045414	2542489	2821647	3099892	3088648
12	549779	1264578	1473422	1895745	2155389	2555403	2783667	2896806	2888160	2939549	3455560
	595477	1389384	1556218	1908125	2111607	2290690	2768433	3006906	3172330	2888140	3440327
14	562431	1397862	1612719	1746281	2168990	2426234	2470639	2928803	3236065	3983351	3424293
	599602	1356219	1409038	1701229	2115530	2423512	2512782	2659616	3034531	3419803	3801104
16	637611	1231769	1575151	2206424	2391205	2422627	2906276	2922631	3064590	3086923	3491406
	667450	1283980	1558321	1983569	2411323	2419905	2891042	2723183	2886670	3334166	3360195
18	603181	1491333	1726945	2167496	2383912	2846380	2753561	2952703	3141548	3449898	3638043
	616185	1392735	1626118	1947476	2381191	2761449	2744453	3061771	3077647	3434665	3549253
20	620762	1431550	1600276	2070160	2318111	2658734	3636056	3324363	3463651	3774097	3648113
	623334	1248910	1791129	2292389	2567153	2613292	3072507	3112560	3698477	3293526	3510234
22	644507	1432757	1854425	2101004	2641668	2584526	3177004	3196477	3442348	3643193	3885633
	675422	1282786	1793114	2098283	2626435	2658589	2684206	3234020	3238746	3451706	4092347

Table 2: The worst case costs for nominal (above) and robust (below) solutions for varying values of Γ_u and Γ_v. In each case, the value is the worst case cost out of a 10,000-scenario sample of the uncertainty set.

Γ_v	Γ_u										
	0	1	2	3	4	5	6	7	8	9	10
0	0.21205	0.40711	0.56529	0.68668	0.78826	0.88017	0.96586	1.0346	1.0902	1.123	1.138
	0.21205	0.37976	0.52931	0.67144	0.77302	0.86494	0.95063	1.0194	1.075	1.1077	1.1228
2	0.46165	1.1685	1.5892	1.9266	2.0848	2.2356	2.4174	2.519	2.5878	2.6205	2.6356
	0.43505	1.2121	1.6079	1.8472	1.9929	2.2328	2.3827	2.4843	2.553	2.5858	2.6008
4	0.63108	1.7556	2.2697	2.6071	2.8794	3.0376	3.4341	3.5356	3.6044	3.6372	3.6522
	0.59663	1.4909	2.2582	2.6169	2.8892	3.0349	3.2921	3.3937	3.4625	3.4952	3.5102
6	0.76476	1.9351	2.5421	2.9554	3.6831	4.0205	4.2361	4.3377	4.4065	4.4392	4.4542
	0.71067	1.636	2.497	2.9619	3.3279	3.8551	4.0698	4.1713	4.2401	4.2729	4.2879
8	0.89114	2.0615	2.9842	3.7125	4.0861	4.6002	4.8725	5.0099	5.1647	5.1975	5.2125
	0.80731	1.7713	2.6632	3.5472	3.9207	4.4348	4.7071	4.8445	4.9994	5.0321	5.0471
10	0.97075	2.1411	3.0902	3.8185	4.37	4.8841	5.1668	5.4876	5.6424	5.6752	5.6902
	0.89486	1.8663	2.7571	3.6531	4.2046	4.7187	5.0014	5.3222	5.4771	5.5098	5.5248
12	1.0372	2.2075	3.1566	3.8849	4.4364	4.9506	5.5493	5.8216	5.9765	6.0738	6.0888
	0.97925	1.9357	2.8305	3.7177	4.2692	4.7852	5.3839	5.6562	5.8111	5.9084	5.9234
14	1.0764	2.2467	3.1958	3.9241	4.5336	5.0851	5.5992	5.9278	6.1755	6.3088	6.3239
	1.0545	1.9825	2.8798	3.7587	4.3682	4.9197	5.4338	5.7624	6.0101	6.1434	6.1585
16	1.1046	2.2749	3.224	3.9523	4.5618	5.1133	5.6274	5.961	6.3104	6.4781	6.4932
	1.0893	2.0109	2.9082	3.7869	4.3964	4.9479	5.462	5.7956	6.145	6.3128	6.3278
18	1.1233	2.2937	3.2428	3.9711	4.5806	5.1321	5.6462	6.0585	6.392	6.5885	6.6057
	1.1081	2.0318	2.9291	3.8057	4.4152	4.9667	5.4808	5.8931	6.2267	6.4231	6.4403
20	1.138	2.3083	3.2574	3.9858	4.5952	5.1467	5.6609	6.0735	6.4071	6.6036	6.6938
	1.1228	2.0496	2.9469	3.8204	4.4298	4.9814	5.4955	5.9082	6.2417	6.4382	6.5284
22	1.138	2.3083	3.2574	3.9858	4.5952	5.1467	5.6609	6.0735	6.4071	6.6036	6.6938
	1.1228	2.0625	3.1671	3.8204	4.4298	4.9814	5.4955	5.9082	6.2417	6.4382	6.5284

Table 3: The theoretical worst case costs (in millions) for nominal (above) and robust (below) solutions for varying values of Γ_u and Γ_v.

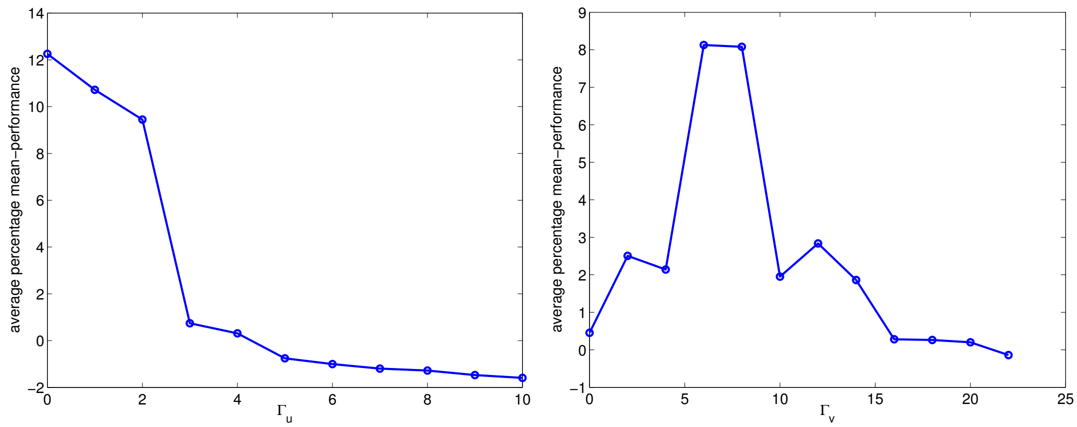
When we try the same experiment but with much wider uncertainty intervals for exposure risk, we see that again the performance graphs for the nominal and robust solutions are similar to each other (see Figure 5). However here the nominal problem has a higher worse case cost, 8.5% higher than that of the robust problem. Note also that the curves are shaped differently. This is because with smaller exposure risk uncertainty, traffic volume uncertainty dominated. When exposure risk uncertainty is made much larger, its effect dominates the resulting costs. Exposure risk uncertainty is an avoidable uncertainty because it affects arcs which may or may not be used by the solution. Traffic volume uncertainty is an unavoidable uncertainty because all hazardous waste must be shipped.

This is apparent in Figure 3, where we plot the percentage comparison between the nominal and robust solutions based on the data in Table 2. After evaluating 10,000 samples for each solution, we first compute the percentage differences of the cost function values of the nominal and robust solution. In Figure 3(a), we plot the average of such percentage differences for each Γ_u-value, and in Figure 3(b) for each Γ_v-value. We observe that as Γ_u becomes bigger and unavoidable uncertainty dominates, the robust solutions perform better on average (about 2% better when $\Gamma_u = 10$), and the trend is consistent. However, in Figure 3(b) we do not observe any consistent trend with increases of Γ_v. Similarly, in Figure 4, we compare the percentage differences of the (simulated) worst-case costs. As expected, for larger Γ_u and Γ_v, the robust solutions perform better, although we do not observe any significant trend in either case. The chaotic appearance of the graphs in Figure 4 may be explained by the fact that very bad performance is an extremely rare event (as illustrated by the very long tails in Figures 2 and 5), so even a fairly large sample (10,000 for each Γ_u, Γ_v combination) is insufficient to make the resulting graph smooth.

By contrast, the results of the comparison for a 200-node problem with larger Γ_u and Γ_v values are seen in Figure 6. The robust solution clearly outperforms the nominal solution. This is achieved by building additional facilities. The additional cost is more than compensated for by the resulting decrease in the large exposure risk resulting from high Γ values. Maps of the facilities selected and routes chosen for the nominal and robust problems are shown in Figures 7 and 8 respectively.

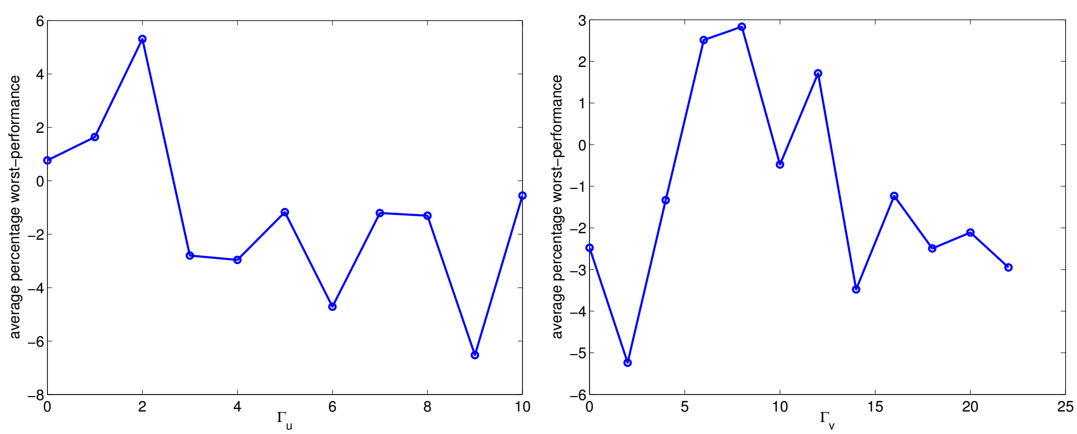
The performance of the genetic algorithm in solving a 1000-node problem is shown in Figure 9. The vertical axis shows (on a logarithmic scale) the objective function of the current best solution, plotted for a run of 100 generations.

The objective function increases in a smooth, predictable way as Γ_u or Γ_v increase (as shown in Figure 10). This should not be surprising, as increasing the budget of uncertainty can never decrease the objective function of the robust problem, which means that Z^R will be a monotonically increasing function of both Γ_u and Γ_v. It is interesting however



(a) Γ_u, Uncertainty in Traffic Volume (b) Γ_v, Uncertainty in Accident Risk

Figure 3: Percentage Comparison of the *Mean* Performances of the Nominal and Robust Solutions, from Table 2 with 10,000 samples



(a) Γ_u, Uncertainty in Traffic Volume (b) Γ_v, Uncertainty in Accident Risk

Figure 4: Percentage Comparison of the *Worst-Case* Performances of the Nominal and Robust Solutions, from Table 2 with 10,000 samples

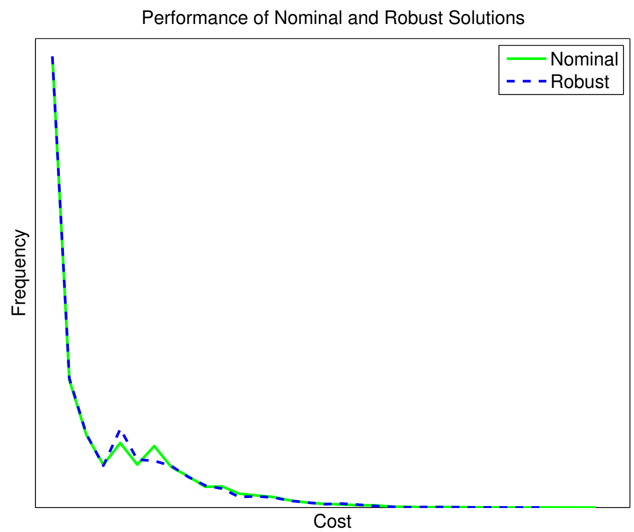


Figure 5: The performance of robust and nominal solutions over a random sample from the uncertainty set $\mathcal{U} \times \mathcal{V}$ for a 90-node problem (Albany, New York area). For this example the values $\Gamma_u = 4$ and $\Gamma_v = 18$ were used (the same as for the previous example), but the width of the exposure risk uncertainty was 20 times higher.

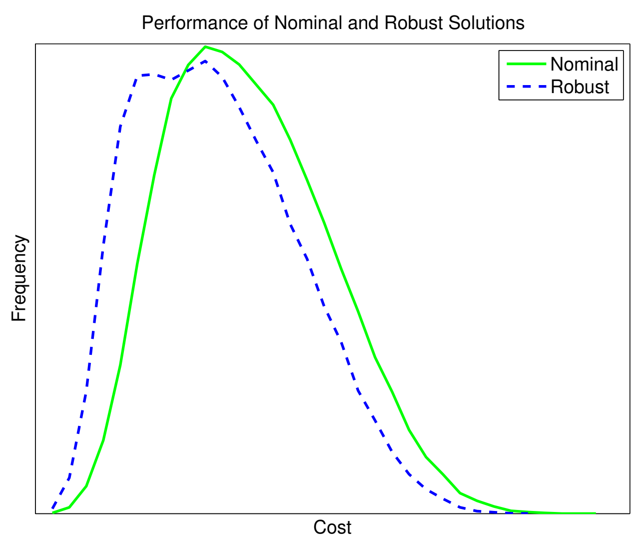


Figure 6: The performance of robust and nominal solutions over a random sample from the uncertainty set $\mathcal{U} \times \mathcal{V}$ for a 200-node problem. For this example the values $\Gamma_u = 5$ and $\Gamma_v = 300$ were used.

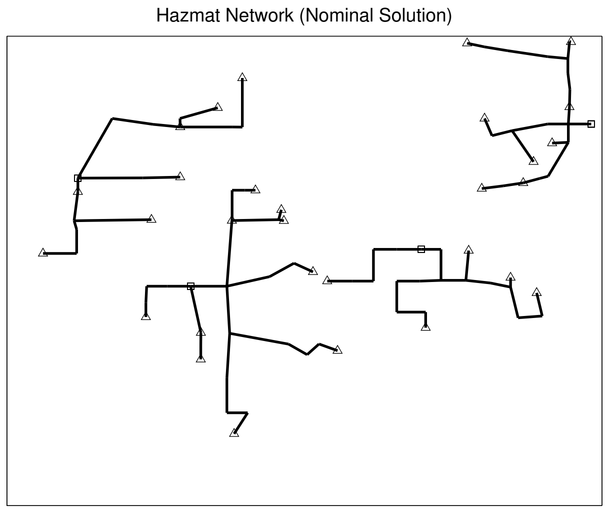


Figure 7: The squares are sites chosen for facilites. The triangles show the origin points of waste shipments. The lines indicate the resulting paths along which waste is shipped.

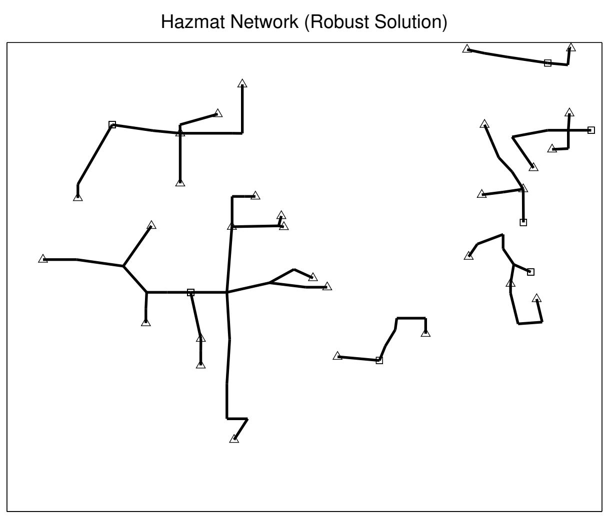


Figure 8: The squares are sites chosen for facilites. The triangles show the origin points of waste shipments. The lines indicate the resulting paths along which waste is shipped. Note how more facilities are built than for the nominal solution, which causes the number of arcs impacted to be reduced.

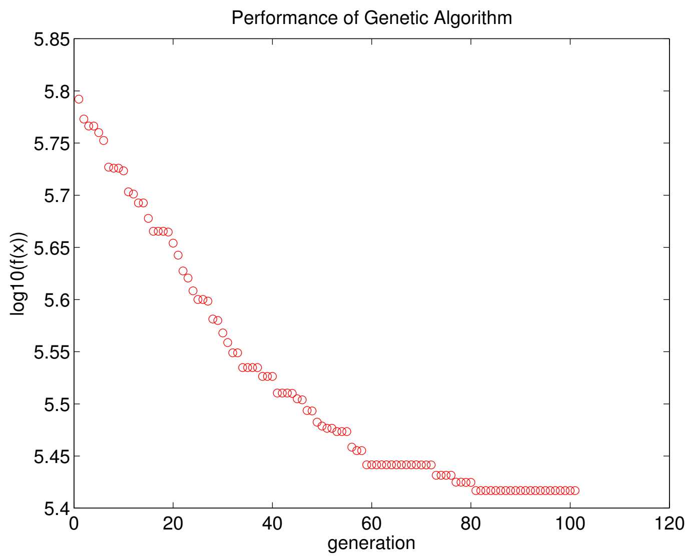


Figure 9: This figure shows the performance of the genetic algorithm on a 1000-node, 2254 arc problem. The algorithm converged on a solution after approximately 80 generations. The time to run 100 generations was 7 hours 36 minutes.

to note that the optimal solution varies in irregular and unpredictable way. Figure 11 along with Table 4 illustrate which feasible solution was optimal for a 90-node Albany, New York area problem when Γ_u and Γ_v varied.

5 Concluding Remarks

For low levels of uncertainty (small uncertainty interval widths), we see that robust and nominal solutions preform similarly. For higher levels of uncertainty, the robust solution generates superior results by spending more money (building more facilities, or building them in expensive but efficient locations). This may be seen as a result of risk averseness.

We also note the contrast between avoidable and unavoidable uncertainty. Here arc risk uncertainty was an avoidable type of uncertainty, and shipment volume uncertainty an unavoidable type. As before, we saw that in the presence of substantial avoidable uncertainty robust optimization generated results that were typically superior to the nominal solution. However, when most of the uncertainty was of the unavoidable type robust optimization did not prove as valuable.

There are a number of possible extensions to our model. A natural extension of the problem would be to have capacitated waste processing sites. However it is not clear

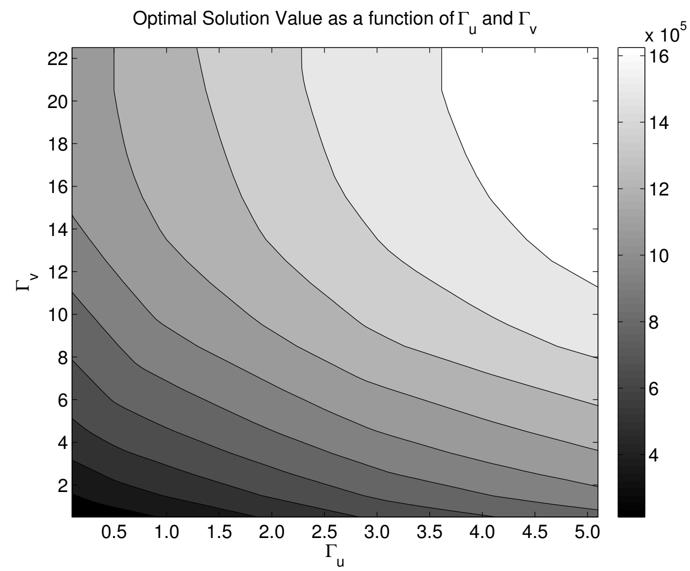


Figure 10: The optimal solution value increases in a smooth fashion as either Γ_u or Γ_v increases. The problem is the 90-node Albany, New York area problem.

solution key	indices of locations used in solution
A	2,10,41
B	46,51,78
C	2,10,68,78
D	2,10,68
E	2,10,51,78
F	2,10,51
G	2,40,46,51,78
H	10,51,78
I	2,10,63,78
J	2,46,51,78

Table 4: For each optimal solution in Figure 11, which nodes were used as hazardous waste facilities.

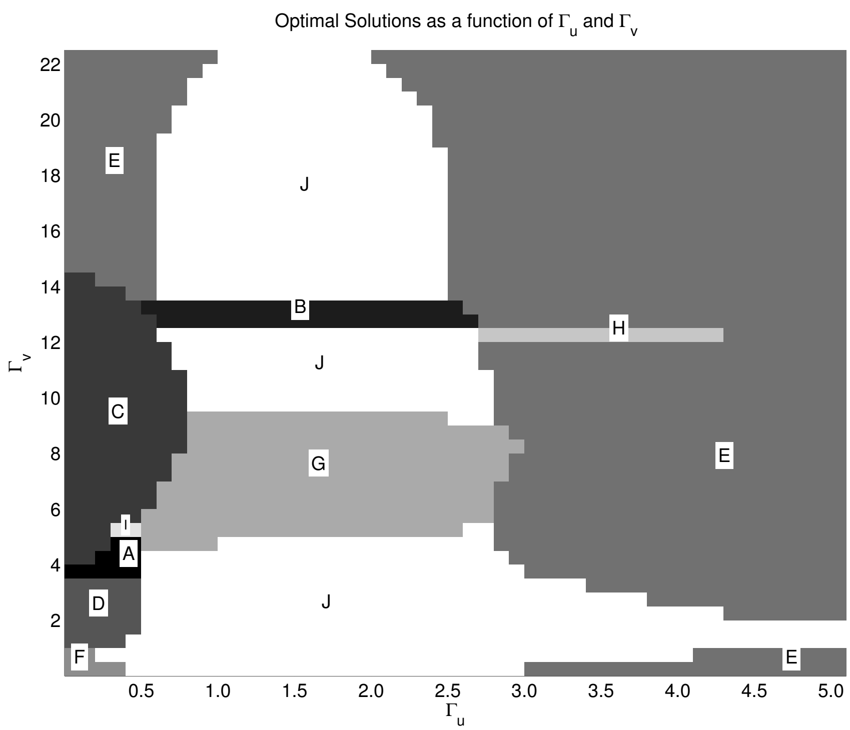


Figure 11: The optimal solution may change in irregular fashion as Γ_u and Γ_v vary. The nominal solution "F" is of course optimal when $\Gamma_u = \Gamma_v = 0$. The problem is the 90-node Albany, New York area problem. See Table 4 for the nodes contained in each solution A,B,. . .,J.

how such constraints could be integrated into our model given that we assume that each shipper makes his own choice of which facility to use so as to minimize his own cost.

Much of the existing literature deals with multiple waste types. If we assume that each waste type may be processed by exactly one type of facility, problems with multiple waste types are separable by type of facility and remains equivalent to the foregoing. If some waste types may be processed by multiple types of facility, it may be necessary to formulate a larger and more complex model with additional constraints to ensure that each waste type is processed by a compatible facility. A similar complicating generalization would be to have both processing and disposal sites as in the model of Alumur and Kara (2007).

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Appendix

A Karush Kuhn-Tucker Conditions for the Shortest Path Problem

In order to convert the two-level problem into a single level optimization problem, we want to be able to replace the inner minimization (shortest path) problem with an equivalent set of constraints. As we recall, the inner shortest path problem for shipment s was

$$\min_{x^s} \sum_{(i,j)\in\mathcal{A}} L_{ij} x_{ij}^s$$

subject to

$$\begin{aligned} &\sum_{(i,k)\in\mathcal{A}} x_{ik}^s - \sum_{(k,i)\in\mathcal{A}} x_{ki}^s \begin{cases} = +1 & i = o(s) \\ \geq -y_i & i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{N} \\ &-x_{ij}^s \leq 0 \quad \forall (i,j) \in \mathcal{A} \end{aligned}$$

which we put into the standard $g(x) \leq 0$, $h(x) = 0$ form:

$$\begin{aligned} -\sum_{(i,k)\in\mathcal{A}} x_{ik}^s + \sum_{(k,i)\in\mathcal{A}} x_{ki}^s - y_i &\leq 0 \quad \forall i \in \mathcal{M} \\ -x_{ij}^s &\leq 0 \quad \forall (i,j) \in \mathcal{A} \\ -\sum_{(o(s),k)\in\mathcal{A}} x_{o(s),k}^s + \sum_{(k,o(s))\in\mathcal{A}} x_{k,o(s)}^s + 1 &= 0 \\ -\sum_{(i,k)\in\mathcal{A}} x_{ik}^s + \sum_{(k,i)\in\mathcal{A}} x_{ki}^s &= 0 \quad \forall i \in \mathcal{N}, i \neq o(s), i \notin \mathcal{M} \end{aligned}$$

The corresponding KKT conditions are, first, stationarity

$$L_{ij} - \zeta_i^s + \zeta_j^s - \phi_{ij}^s = 0 \qquad \forall (i,j) \in \mathcal{A}$$

then complementary slackness

$$
\begin{aligned}
\phi_{ij}^{s} x_{ij}^{s} &= 0 \qquad \forall (i,j) \in \mathcal{A} \\
\Big(- \sum_{(i,k)\in\mathcal{A}} x_{ik}^{s} + \sum_{(k,i)\in\mathcal{A}} x_{ki}^{s} - y_i \Big) \zeta_i &= 0 \qquad \forall i = \mathcal{M}
\end{aligned}
$$

and finally dual feasibility

$$
\begin{aligned}
\phi_{ij}^{s} &\geq 0 \qquad \forall (i,j) \in \mathcal{A} \\
\zeta_i^{s} &\geq 0 \qquad \forall i \in \mathcal{M}
\end{aligned}
$$