

# Inventory Rebalancing through Pricing in Public Bike Sharing Systems

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## Abstract

This paper presents a new conceptual approach to improve the operational performance of public bike sharing systems using pricing schemes. Its methodological developments are accompanied by experimental analyses with bike demand data from Capital Bikeshare program of Washington, DC. An optimized price vector determines the incentive levels that can persuade system customers to take bicycles from, or park them at, neighboring stations so as to strategically minimize the number of imbalanced stations. This strategy intentionally makes some imbalanced stations even more imbalanced, creating hub stations. This reduces the need for trucks and dedicated staff to carry out inventory repositioning. For smaller networks, a bi-level optimization model is introduced to minimize the number of imbalanced stations optimally. The results are compared with a heuristic approach that adjusts route prices by segregating the stations into different categories based on their current inventory profile, projected future demand, and maximum and minimum inventory values calculated to fulfill certain desired service level requirements. We use a routing model for repositioning trucks to show that the proposed optimization model and the latter heuristic approach, called the iterative price adjustment scheme (IPAS), reduce the overall operating cost while partially or fully obviating the need for a manual repositioning operation.

**Keywords:** bike-sharing; shared-mobility; rebalancing; pricing; heuristics

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# 1 Introduction

Increasingly, Public Bike Sharing Systems (BSS) are being adopted by many major cities throughout the world. Bicycles are being touted as a way to achieve sustainable mobility in an urban setting while also helping to alleviate the last mile problem in urban transportation (Shaheen et al., 2010). As of March 12, 2017, some form of bike sharing system is operating in 1232 cities worldwide and another 382 such systems are in planning or under construction with a growing interest in more and more cities (Meddin and DeMaio, 2014). One of the major problems faced by these systems is the operational issue of repositioning of bicycles between different stations. Demand variability causes certain stations to become too full or too empty to effectively service new customers. This not only affects the desired service level but also incurs spurious operational costs. According to a report by New York City Department of City Planning (2009) based on different case studies, the total capital cost for a bike sharing system varies from \$3000/bike to \$4400/bike in different cities. When averaged across programs, the average yearly operating cost for a bike share program is around \$1,600/bicycle.

The operating cost consists mainly of system operations, administration, marketing and utility costs associated with hardwired stations. System operation forms the largest share of these costs and includes functions such as: maintenance of all equipment, rebalancing of bicycles, customer service operations, and website and IT support (The Pennsylvania Environmental Council, 2013). Clearly, the repositioning of bicycles from stations too full to stations too empty is a huge operational overhead. In fact, for Vélib system in Paris, the average cost of a single repositioning for a single bike is \$3 (DeMaio, 2009). A system-wide snapshot of Capital Bikeshare at 9:30 a.m. on May 15, 2014 shows that 88 out of 202 stations are *imbalanced* considering 90% service level (see Section 3.1).

The contribution of this work lies in the development of methods - both exact and inexact - and algorithms that bike sharing system managers can use to reduce the number of imbalanced stations by rebalancing their inventory through price incentives/disincentives. To do so, we will intentionally make some imbalanced stations even more imbalanced, making them function as *hubs*. If only a few highly imbalanced stations exist in the system, then bike redistribution can be handled with just a few regular short time truck trips, in every operational period (e.g., a 30-minute-long one). With the reduced number of imbalanced stations, the operation of truck redistribution becomes simpler and more efficient resulting in operating cost reduction. This observation is key to our idea of designing dynamic pricing policies. We seek to ensure that surplus bicycles are gathered predominantly at designated ‘surplus accumulation’ stations, and similarly, the deficiency/lack of bicycles predominantly occurs at ‘lack accumulation’ stations.

We understand that the practical implementation of the pricing policy can be challenging and must be discussed. Nowadays, many modern bike sharing stations are equipped with a computer terminal with a touch screen. When a bike user tries to checkout a bike, the user will be asked to

choose his or her destination. Based on the current state of the system, the user will be provided with alternate journey choices with information about the price to be charged for each choice at the time of return. A mobile webpage or an application can also be used to provide on the go information about the prices even before the bike user approaches a bike station.

We also assume that bike users exhibit a homogeneous sensitivity to the price and always seek to maximize their utility. These assumption are commonplace in the road pricing literature, where drivers are assumed to choose routes that minimizes a weighted sum of travel time and toll prices. By taking account of such responses, we can say at best that the system operator indirectly controls bike users. Such indirect control in the bi-level programming context is also very common in the Stackelberg game (or leader-follower game) setting for economics and policy studies. We also did not consider the demand elasticity of price. We assumed that the travel demand is fixed and the users will choose the lowest priced alternative to make the journey.

To determine the price incentives, we first formulate a bi-level optimization model in Section 3 and provide a single-level reformulation that may be useful for small networks. The inputs to the model include a set of stations, their initial inventory and capacity, their minimum and maximum inventory values calculated using a pre-processing step explained in section 3.1, a set of all origin destination pairs (OD pairs), a travel time matrix between OD pairs and a demand matrix based on historical data. The output of the model is a price vector between OD pairs that minimizes the number of imbalanced stations. In Section 4 we propose a heuristic algorithm, called the Iterative Price Adjustment Scheme (IPAS), and compare its performance with the single-level optimization model solved by a commercial solver. We conclude that we can successfully reduce the number of imbalanced stations, by giving travelers multiple journey choices and changing the cost of those journeys through pricing. We have also demonstrated that the cost of the same degree of manual rebalancing outweighs the price incentives offered.

The performance of IPAS is demonstrated by computational experiments in Section 5. Using the data from Capital Bikeshare in Washington, D.C., we show how our approaches manage to successfully minimize the number of imbalanced stations. The efficacy of our heuristic approaches vis-à-vis execution time, while bringing satisfactory improvement to the overall objective of minimizing the number of imbalanced stations is also shown. In section 5.2.2, we use a routing model to show how the smaller number of imbalanced stations achieved as a result of a pricing scheme translates into a simpler and more efficient static repositioning operation using trucks.

## 2 Literature Review

Bike sharing systems have recently garnered an increased interest from the research community due to their growing importance in sustainable urban transport systems. DeMaio (2009) and Shaheen et al. (2010) separately discuss the history, impacts, models of provision and the future of public BSS. Introducing what they call the Fourth generation of BSS, they identify improved

redistribution of bicycles as a key challenge facing BSS. Schuijbroek et al. (2017) have an excellent and comprehensive description of BSS literature. They divide up the BSS literature into four major streams including strategic design, demand analysis, service level analysis, and rebalancing operations. We, thus, refer readers to Schuijbroek et al. (2017) for general literature review, and limit this section to reviewing most relevant literature on bike sharing systems, in particular, rebalancing operations.

Rebalancing operations are a big part of operating costs of a bike sharing system (The Pennsylvania Environmental Council, 2013). Most such systems are run in collaboration with city governments and depend financially on public funds and corporate sponsorship. Because these systems are often cash strapped, it is not possible to indiscriminately add extra bicycles or docks to the system. Furthermore, the demand depends on many factors and is hardly predictable. This necessitates some kind of rebalancing of the system. Generally, bike sharing systems employ two methods to redistribute the bicycles: truck-based manual redistribution and pricing-based rebalancing.

Most bike sharing systems have a fleet of trucks that move around and pick and drop bicycles. Vélib has 20 trucks (Benchimol et al., 2011) operating 24 hours to carry out manual rebalancing. Trucks and crew required to operate these have huge associated costs. Paul DeMaio of MetroBike, LLC, mentions a conversation with Luud Schimmelpennick, a pioneer of bike sharing concept, in DeMaio (2009). He reports that according to Schimmelpennick the cost for distribution of a single bike for JCDecaux is \$3 and that any scheme that offers incentives to customers would increase the redistribution efficiency at a fraction of the current cost. Since some kind of manual balancing is always required, most of rebalancing literature is focused on optimal truck routing. Benchimol et al. (2011) introduces several approximation algorithms for static rebalancing of bicycles at the end of the day when there is not much bike movement.

Several papers have recently studied truck-based manual bicycle redistribution. Raviv et al. (2013) introduced several formulations for static rebalancing problem but their objective function minimizes the expected user dissatisfaction rather than minimizing the total travel distance. Fricker and Gast (2016) study a simple model with symmetry where all the bicycle stations have the same parameters. The authors establish that even in this perfect scenario, the probability of a station being full or empty is  $2/(K+1)$  where  $K$  is the capacity of each station, and then, propose to improve this situation through incentives and regulation. Contardo et al. (2012) introduce a dynamic public bike sharing balancing problem (DPBSBP) to rebalance a BSS during daytime which constitutes peak hours. They solve the DPBSBP problem using Dantzig-Wolfe decomposition and Benders decomposition to derive lower bounds and fast feasible solutions. Caggiani and Ottomanelli (2012) construct a modular Decision Support System (DSS) for dynamic bike redistribution. Shu et al. (2013) discuss under-utilization of bike sharing systems in Chinese cities and propose a deterministic model to optimally deploy bicycles and docking capacity at different stations. They also evaluate the value of redistribution and its impact on the number of trips supported by the system. They

conclude that for systems with more than 30,000 bicycles, frequent periodic redistribution does not add much value, and that a small number of daily redistributions are recommended.

There is a recent trend in BBS literature to introduce a scheme of incentives to get users to move bicycles away from the crowded stations and into the less occupied stations. Vélib operates a V+ scheme to induce users to avoid certain stations and prefer others. Users get 15 minutes of added travel time if they place the bicycles at one of the hundred uphill stations (Fricker and Gast, 2016). The incentives can be in the form of extra added time, as is the case with Vélib, or some cash discounts. The literature on user incentive schemes is not as plentiful as that on rebalancing through trucks (Fricker and Gast, 2016). Fricker and Gast (2016) present a two-choice model in which each user is provided with two station choices at the time of a rental and is given an incentive to choose the station with the lower load as a destination. They show that even if a fraction of the users make the intended choice, the number of imbalanced stations comes down dramatically. Waserhole et al. (2012) solve an optimization model called Vehicle Sharing System Pricing Problem for setting the trip prices through a Markov Decision Process framework based on Continuous-Time Markov Chain. Finally, they present a Fluid Approximation approach and build a mathematical programming model for the fluid approximation of the Stochastic VSS Pricing Problem with continuous prices. A simulation model with different demand intensity, gravitation factors and tides is also implemented to check the performance of fluid approximation heuristic.

Pfrommer et al. (2014) introduced a tailored algorithm for dynamic route planning for multiple trucks for redistribution of bicycles and then devised a system of price incentives computed based on Model Predictive Control (MPC) to draw users away from full or empty stations. They attempted to increase the service level of the system by adding repositioning trucks and increasing the incentive payouts. They observed that with the increase in number of trucks and incentive payouts, diminishing gains to service level are reported. They also concluded that a system of price incentives is more effective on weekends as compared to weekdays when work related commuting takes place.

The problem of inventory imbalance and subsequent need for redistribution also exists in car sharing systems. While in bike sharing a user can make a trip on any origin destination pair, most car sharing systems require users to return their vehicles back to the origin to avoid inventory imbalance. Effective relocation policies are essential for allowing one way trips and making car sharing systems more viable and user friendly.

Barth et al. (2004) use price incentives in a shared-use electric vehicle system called UCR Intellishare to encourage user-based vehicle relocation. They provide a system of price incentives to users for trip splitting and trip merging. The results of their simulation model suggest a 42% percent decrease in overall number of relocations if incentives for trip joining and trip splitting are offered at the time of rental.

Kek et al. (2009) devise a three-phase Optimization-Trend-Simulation (OTS) decision support

system for vehicle relocation problem in car sharing systems. In the first phase, the *optimizer* provides the lowest cost resource allocation giving optimized staff strength, staff activities and relocations. In phase two, a *trend filter* uses a series of heuristics to filter the optimum results from phase one giving a set of recommended operation parameters. Finally a *simulator* evaluates the effectiveness of these parameters using three system performance measures related to system's inventory situation, including the number of relocations. The simulation results suggest that using the suggested parameters from OTS can reduce the number of relocations by 37-41 %.

Correia and Antunes (2012) recommend relocation operations at the end of each day in one way car sharing systems. They present three MIP models corresponding to three trip selection schemes. They conclude that profits could be achieved by pre selecting the trips or by rejecting the undesirable trips that result in system imbalance. They also suggest a scheme of price rate discrimination to incentivize or disincentivize certain trips.

Correia et al. (2014) determine the added value of information and user flexibility for a one way car sharing system. Their approach is similar to what we propose. They allow users the flexibility to choose neighboring stations other than the intended origin and destination pair to address the stock imbalance problem. They show that in the scenario where flexible users have information on station status beforehand, the total daily profit and satisfied demand increase considerably.

Jorge et al. (2013) present a mathematical model to optimize the vehicle relocation operations using drivers with the objective of maximizing the profit of one way car sharing systems. They also build a simulation model to test the two real time relocation policies. Their policies divide the stations into suppliers and demanders based on their current inventory situation and expected demand. They incentivize the trips to demander stations by artificially reducing the travel time. The relocation policies when applied to a full network in Lisbon increased the profitability and coverage of the system.

In our paper, we aim to establish a system of cash discounts and penalties on user fee to modify user decisions. The operators have real time status data on all stations and based on this data a price vector can be calculated for all journeys. This information can be provided to users at Point of Sale or through mobile applications. As described by Waserhole et al. (2012), price incentives can be offered in discrete jumps with certain small increments, or they can be continuous within a range with a defined maximum and minimum value. In this paper, pricing policies are dynamically changed in real time depending upon the current state of the system and the expected future demand. They can, however, be static, i.e., independent of the system's current state, and set in advance.

### 3 Model Formulation

The decision-making process for bike sharing systems is bi-level, as shown in Figure 1. Decisions regarding the location and size of stations as well as pricing are made by the operator running the

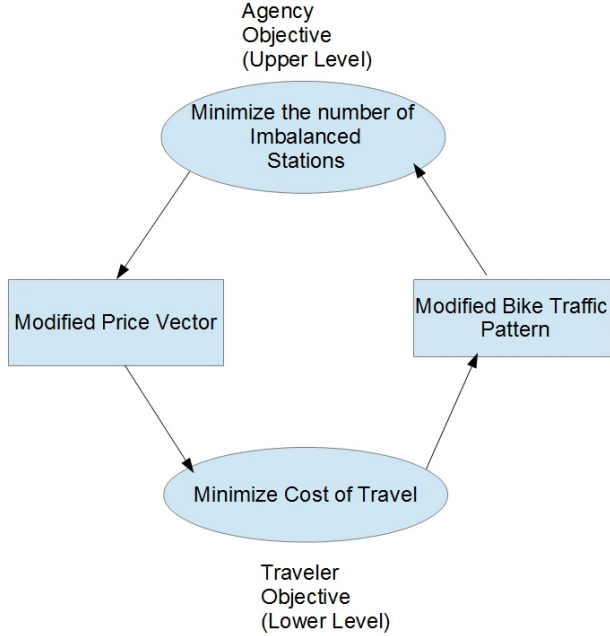


Figure 1: Bilevel Decision Making Process

system, while lower level journey decisions are made by the customers using the bicycles. In our model, the upper level (operator) objective is to minimize the total number of *imbalanced* stations. The lower level customer objective is to make a journey between two points at the minimum possible cost. The underlying assumption is that travelers will always take the minimum cost route. This section works to develop a detailed formulation of the problem that exploits the idea of strategic customer incentives. A few explanations, leading to the definition of *imbalanced* stations, are first in order.

### 3.1 Service Level Requirements

As described earlier, bike sharing systems are subject to two demands. On one hand, there exists the demand for bicycles while on the other hand, there is the demand for empty docks, i.e., for parking the bicycles at the end of the journey. A bike procured from one station is eventually parked back at the same station or any other station in the system. Every time a station is full or empty, a service opportunity is wasted. Most bike sharing systems keep track of the station inventory level. Some systems measure the number of instances (e.g. Capital Bike share in Washington, DC) while others measure the fraction of time (e.g. Vélib in Paris) that the stations are empty or full. Operators do it for efficient rebalancing operations and to determine the need for expansion or reduction in the number of docks available at a particular station.

Schuijbroek et al. (2017) define a measurable Type-2 service level: the fraction of demand satisfied directly should be larger than  $\beta_i^-$  for pickups and larger than  $\beta_i^+$  for returns, assuming no

back-orders. We use the same definition:

$$\frac{E[\text{Satisfied bike pickup demands}]}{E[\text{Total bike pickup demands}]} \geq \beta_i^-,$$

$$\frac{E[\text{Satisfied bike return demands}]}{E[\text{Total bike return demands}]} \geq \beta_i^+.$$

Schuijbroek et al. (2017) then go on to establish a method to evaluate the values of minimum and maximum inventory for each station in the system, respectively designated as  $s_i^{\min}$  and  $s_i^{\max}$  for given  $\beta_i^-$  and  $\beta_i^+$ . They model the inventory  $s_i$  at station  $i$  as an  $M/M/1/K$  queuing process with customers in the queue for bicycles or docks representing the inventory. We use their system of equations to evaluate the values of  $s_i^{\min}$  and  $s_i^{\max}$ . For given  $\beta_i^-$  and  $\beta_i^+$ , starting inventory  $s_i^0$  should ideally be rebalanced so that:

$$s_i^{\min} \leq s_i \leq s_i^{\max}.$$

If a station does not satisfy the above inequality, the station is termed *imbalanced*. Implicit in this definition of an *imbalanced* station is the idea of service level requirements. Only those stations that cannot fulfill the future demand for bikes and docks with a  $\beta_i^-, \beta_i^+$  service level are considered imbalanced. It bears repeating that a station is lack-imbalanced when  $s_i \leq s_i^{\min}$  and surplus-imbalanced when  $s_i \geq s_i^{\max}$ . It must be noted that  $s_i^{\min}$  is always greater than or equal to 0 and  $s_i^{\max}$  is always less than or equal to the maximum capacity of station  $i$ , i.e., the number of docks installed in station  $i$ . So a station can be imbalanced even when it is not completely full or empty. The following parameters are required to calculate  $s_i^{\min}$  and  $s_i^{\max}$  for station  $i$ :  $\beta_i^-, \beta_i^+$ , number of docks installed, the rate of arrival of users to pick up bicycles, and the rate of arrival of users to return bicycles.

### 3.2 A Bi-Level Formulation

This section presents the mathematical formulation of the bi-level problem. The first level represents the price change vector to minimize the number of *imbalanced* stations while the lower level corresponds to a minimum cost network flow problem which determines the route choices made by the travelers. Let  $\mathcal{S}$  be the set of stations in the system. Let us assume that for a single journey,  $r$  is the origin station and  $s$  is the destination station where  $r, s \in \mathcal{S}$ . Let us also assume that  $(r, s)$  is the OD pair for a single one way trip and  $\mathcal{W}$  is the set of all possible OD pairs, i.e.,  $(r, s) \in \mathcal{W}$ . If the number of stations in the network is  $n$  then the number of OD pairs is  $n^2$ . The distance  $d$  between two stations is the distance along the shortest bike route and not the euclidean distance. In an urban setting, each station has a number of *neighborhood* stations. We assume that two stations less than 600 meters apart are *neighborhood* stations. On average, this accounts for less than 6 minutes of walking. For every OD pair, both origin and destination have a number of



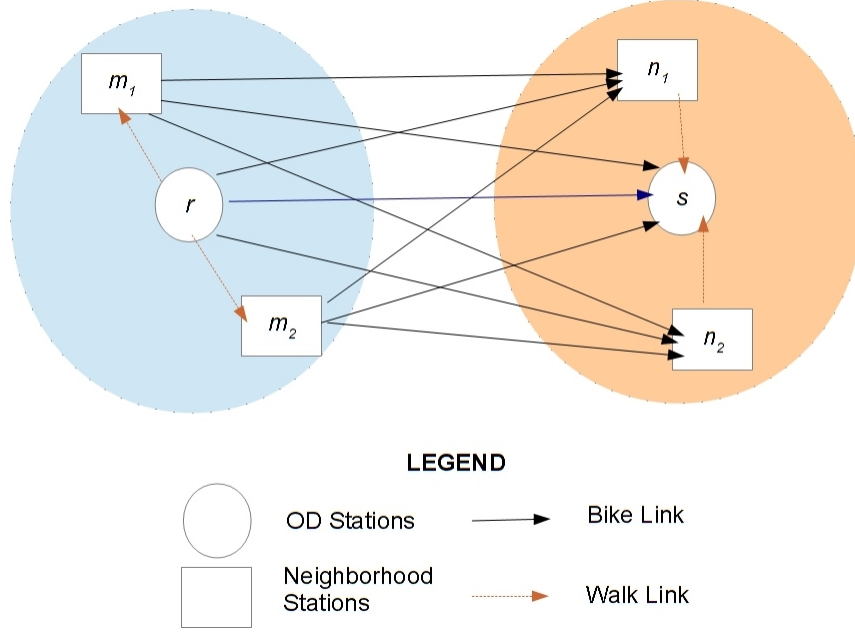


Figure 2: Network structure for every OD station pair

neighborhood stations as illustrated in Figure 2. The colored circles around the stations represent the neighborhood radius.

Let us designate a full, directed network  $G(\mathcal{N}, \mathcal{A}, \mathcal{P})$  where  $\mathcal{N}$  denotes the set of nodes (or stations) in the network,  $\mathcal{A}$  denotes the set of arcs and  $\mathcal{P}$  denotes the set of paths. In this network a direct arc between any two nodes is also the shortest path between them. For every single OD pair in set  $\mathcal{W}$ , we construct a directed network  $G^{rs}(\mathcal{N}^{rs}, \mathcal{A}^{rs}, \mathcal{P}^{rs})$ . In this smaller network  $\mathcal{N}^{rs}$  is the set of nodes that includes the origin station, the destination station and their respective neighborhood stations.  $\mathcal{A}^{rs}$  is the set of directional arcs for every OD pair  $(r, s)$  and  $\mathcal{P}^{rs}$  is the set of multiple paths from origin station to destination station. Observe that  $\mathcal{N}^{rs} \subset \mathcal{N}$  but  $\mathcal{A}^{rs} \not\subset \mathcal{A}$  and  $\mathcal{P}^{rs} \not\subset \mathcal{P}$ . As shown in Figure 2, a traveller intending to go from origin  $r$  to destination  $s$  can now take any one of the many paths available to him. If  $\text{No}_r$  and  $\text{No}_s$  are the numbers of neighborhood points of origin and destination, respectively, then the total number of alternate paths available is  $(\text{No}_r + 1) \times (\text{No}_s + 1)$ . Each path can consist of one, two or three arcs. For example, path  $r \rightarrow s$  consists of one arc, while path  $r \rightarrow m_2 \rightarrow n_1 \rightarrow s$  consists of three arcs. A traveller taking the latter path walks from origin  $r$  to its neighborhood point  $m_2$ , rents a bicycle and bikes to destination's neighborhood point  $n_1$ , then parks the bicycle at an empty dock and walks to the destination  $s$ . In Figure 2, *bike links* are represented by black arrows, while red arrows represent *walk links*. Note that every path contains only one bike link.

For a given OD pair  $(r, s)$ , let  $C_{mn}^{rs}$  be the cost matrix for all links  $(m, n)$  where  $(m, n) \in \mathcal{A}^{rs}$  and  $m, n \in \mathcal{S}$ . The cost  $C_{mn}^{rs}$  of traversing a link consists of various subcosts. These include the cost of walking, cost of biking, and the price of renting a bicycle to travel on a bike link. The

rental price is determined by the operator. Let these subcosts be denoted by  $w_{mn}^{rs}, v_{mn}^{rs}$  and  $p_{mn}$ , respectively. The costs  $w_{mn}^{rs}, v_{mn}^{rs}$  are calculated as

$$w_{mn}^{rs} = \nu_3 t_{mn}^{rs} \quad \text{and} \quad v_{mn}^{rs} = \nu_2 t_{mn}^{rs}.$$

In the above expressions  $t_{mn}^{rs}$  is the time of travel between two stations  $m$  and  $n$  using a bicycle.  $\nu_2$  and  $\nu_3$  are the coefficients that convert distance between stations to the cost depending upon biking and walking travel times, respectively. As stated earlier, the price  $p_{mn}$  is defined by the operator. The price associated with a single given link  $(m, n)$  in multiple OD pair networks is the same. For example, the value of  $p_{19,3}$  is the same for all  $(19, 3)$  links in all the feasible OD pair networks. Hence, all  $p_{mn}$  values in an OD pair network form a price vector associated with the OD pair  $(m, n)$  and defined by the operator.

Also, for every OD pair  $(r, s)$ , let  $(m, n) \in \mathcal{A}_B^{rs}$  be the links where a bicycle is used to traverse and hence the cost of travel consists of  $p_{mn}$  and  $v_{mn}^{rs}$ . Similarly, Let  $(m, n) \in \mathcal{A}_W^{rs}$  be the links where a traveller walks and hence the cost of travel consists of  $w_{mn}^{rs}$  alone. The price vector  $p_{mn}$  further consists of a fixed component and a variable component,

$$p_{mn} = p_{mn}^0 + q_{mn}, \quad (1)$$

where  $p_{mn}^0$  is the fixed base price set by the operator and depending on the time of the journey  $(m, n)$  while  $q_{mn}$  is the variable component capturing a penalty or incentive within a fixed range  $[q_{mn}^{\min}, q_{mn}^{\max}]$ , determined again by the operator, with

$$q_{mn}^{\min} \leq q_{mn} \leq q_{mn}^{\max}. \quad (2)$$

Note that all the costs mentioned above except  $q_{mn}$  are fixed costs depending only on travel time. By modifying the price change vector  $q_{mn}$ , the operator can modify the cost matrix and influence the traveller's decision about which path to take. Here we introduce a binary variable  $x_{mn}^{rs}$  which is equal to 1 if link  $(m, n)$  is used to travel between OD pair  $(r, s)$ , and 0 otherwise. Since for every OD pair, one or more links  $(m, n)$  can be used to travel between origin  $r$  and destination  $s$ , one or more variables  $x_{mn}^{rs}$  can take on the value of 1. The outcome  $x_{mn}^{rs}$  of the lower level program which depends on traveller choices is used at the upper level to calculate the bicycle inventory  $s_i$  for each station  $i$  at each instant. This information feeds into the upper level objective function to minimize the number of imbalanced stations. To calculate the inventory, we only require links where a bicycle is used. Let parametric vector  $\delta_{mn}^{rs}$  have value 1 for a bike link and 0 for a walk link. Then the product  $\delta_{mn}^{rs} x_{mn}^{rs}$  is 1 only if a link  $(m, n)$  used to travel between an OD pair is a bike link and 0 if it is not a bike link. Observe that for every OD pair combination,  $\delta_{mn}^{rs} x_{mn}^{rs}$  is 1 for only one link  $(m, n)$ . Table 1 details the mathematical notation used in the model.

Table 1: Mathematical Notation

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$\mathcal{S}$	Set of stations
$\mathcal{W}$	Set of Origin-Destination (OD) pairs
$\mathcal{A}$	Set of arcs in the directed network for every OD pair
$\mathcal{P}$	Set of Possible Paths for each OD pair
$s_i^{\max}$	Maximum number of permissible bicycles at a station $i$ , beyond which the station is considered imbalanced
$s_i^{\min}$	Minimum number of permissible bicycles at a station $i$ , beyond which the station is considered imbalanced
$s_i^0$	Starting level of bike inventory at station $i$ at any time $t$
$s_i$	Current level of bike inventory at station $i$ at a later time $t + \delta t$
$s_i^T$	Truncated level of bike inventory at station $i$ at a later time $t + \delta t$
$\delta_{mn}^{rs}$	1 for a bike link and 0 for a walking link
$\alpha_{mn}^{rs}$	1 for a walking link and 0 for a bike link
$y_i$	1 for a surplus station where surplus station is a station where $s_i > s_i^{\max}$
$z_i$	1 for a lack station where lack station is a station where $s_i < s_i^{\min}$
$q_{mn}$	Price change (Incentive or penalty) for traversing link $(m, n)$
$x_{mn}^{rs}$	1 when link $(m, n)$ is used to travel between OD pair $(r, s)$
$d^{rs}$	Demand for OD pair $(r, s)$ in current time period
$\text{No}_r$	The number of neighborhood points of origin
$\text{No}_s$	The number of neighborhood points of destination

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## The Upper Level Pricing Problem of the System Operator

Using the notation defined in Table 1, we formulate an upper level optimization problem to determine the price change vector  $q_{mn}$  as follows:

$$(P) \quad \min_{x,y,z,q} \sum_i y_i + \sum_i z_i \quad (3)$$

subject to

$$s_i - s_i^{\max} \leq My_i \quad \forall i \in \mathcal{S}, \quad (4)$$

$$s_i^{\min} - s_i \leq Mz_i \quad \forall i \in \mathcal{S}, \quad (5)$$

$$s_i = s_i^0 - \sum_{(r,s) \in \mathcal{W}} \sum_{n \in \mathcal{N}^{rs}} x_{in}^{rs} \delta_{in}^{rs} d^{rs} + \sum_{(r,s) \in \mathcal{W}} \sum_{n \in \mathcal{N}^{rs}} x_{ni}^{rs} \delta_{ni}^{rs} d^{rs} \quad \forall i \in \mathcal{S}, \quad (6)$$

$$q_{mn}^{\min} \leq q_{mn} \leq q_{mn}^{\max} \quad \forall (m, n) \in \mathcal{A}, \quad (7)$$

where values of variables  $x_{mn}^{rs}$  depend on the minimum cost route choice of bike users and is determined by a lower level problem to be introduced later. It is possible in constraint (6) for the inventory  $s_i$  to go beyond capacity  $c_i$  or fall below zero as we do not explicitly consider any bounds on the inventory. However, we use a variable  $s_i^T$  to represent the truncated value of inventory if  $s_i$  goes beyond capacity or below zero. Following inequalities are used as constraints in our model to represent the relationship between the real final inventory and the truncated inventory. The auxiliary binary variables  $a_i$  and  $b_i$  take a value of 1 when  $s_i$  is below zero or above capacity, respectively. If one of these binary variables  $a_i$  or  $b_i$  is equal to 1, The corresponding truncated inventory value  $s_i^T$  is equal to 0 or  $c_i$ , respectively. If  $s_i$  is between 0 and  $c_i$ , i.e.,  $a_i$  and  $b_i$  are both 0, the truncated counterpart is simply equal to  $s_i$ . The value  $s_i - s_i^T$  summed over all  $i$  is the portion of demand that remains unsatisfied.

$$s_i \leq c_i + Mb_i \quad \forall i \in \mathcal{S}, \quad (8)$$

$$s_i \leq M(1 - a_i) \quad \forall i \in \mathcal{S}, \quad (9)$$

$$s_i \geq c_i - M(1 - b_i) \quad \forall i \in \mathcal{S}, \quad (10)$$

$$s_i \geq -Ma_i \quad \forall i \in \mathcal{S}, \quad (11)$$

$$a_i + b_i \leq 1 \quad \forall i \in \mathcal{S}, \quad (12)$$

$$s_i^T \leq s_i + Ma_i + Mb_i \quad \forall i \in \mathcal{S}, \quad (13)$$

$$s_i^T \geq s_i - M(a_i + b_i) \quad \forall i \in \mathcal{S}, \quad (14)$$

$$s_i^T \geq M(1 - a_i) \quad \forall i \in \mathcal{S}, \quad (15)$$

$$s_i^T \geq c_i - M(1 - b_i) \quad \forall i \in \mathcal{S}, \quad (16)$$

$$0 \leq s_i^T \leq c_i \quad \forall i \in \mathcal{S}, \quad (17)$$

Using this approach, we avoid the possible infeasibility in our model by not considering any explicit bounds on the final inventory  $s_i$ . At the same time, however, we make sure that any deviations from the realistic range of  $[0, c_i]$  are accounted for. Later in our computational experiments, we report the unsatisfied demand for comparison purposes between our pricing approaches and the situation without pricing.

### The Lower Level Routing Problem of the Bike Users

In the lower level problem bike users who want to travel from origin  $r$  to destination  $s$  use minimum cost paths so the objective is,

$$\min \sum_{(r,s) \in \mathcal{W}} \sum_{(m,n) \in \mathcal{A}^{rs}} c_{mn}^{rs} x_{mn}^{rs} \quad (18)$$

subject to

$$\sum_{(m,n) \in \mathcal{A}^{rs}} x_{mn}^{rs} - \sum_{(n,m) \in \mathcal{A}^{rs}} x_{nm}^{rs} = e_m^{rs} \quad \forall m \in \mathcal{N}^{rs}, (r,s) \in \mathcal{W}, \quad (19)$$

$$x_{mn}^{rs} \in \{0, 1\} \quad \forall (m,n) \in \mathcal{A}^{rs}, (r,s) \in \mathcal{W}, \quad (20)$$

where

$$c_{mn}^{rs} = \begin{cases} p_{mn}^0 + q_{mn} + \nu_2 v_{mn}^{rs} & \forall (m,n) \in \mathcal{A}_B^{rs} \\ \nu_3 w_{mn}^{rs} & \forall (m,n) \in \mathcal{A}_W^{rs}. \end{cases} \quad (21)$$

In (19),  $e_m^{rs}$  takes the value 1 (respectively, -1), if node  $m$  is the origin (respectively, destination) of the trip, and 0 otherwise.

### 3.3 A Single Level Reformulation

In the lower level problem the integrality requirement for variables  $x_{mn}^{rs}$  can be replaced by the constraints  $x_{mn}^{rs} \geq 0$ . This is so because the lower level program is a minimum cost network flow problem: its right hand side can only be integer and the coefficient matrix in the left hand side forms a totally unimodular matrix. Now we can represent the lower problem by its optimality conditions or Karush-Kuhn-Tucker (KKT) conditions of its LP relaxation. Since the lower-level problem is a linear optimization problem, we can replace it by following KKT optimality conditions:

$$\delta_{mn}^{rs} (p_{mn}^0 + q_{mn} + \nu_2 v_{mn}^{rs}) + \nu_3 \alpha_{mn}^{rs} w_{mn}^{rs} - \lambda_m^{rs} + \lambda_n^{rs} - \mu_{mn}^{rs} = 0 \quad \forall (m,n) \in \mathcal{A}^{rs}, (r,s) \in \mathcal{W}, \quad (22)$$

$$\sum_{(m,n) \in \mathcal{A}^{rs}} x_{mn}^{rs} - \sum_{(n,m) \in \mathcal{A}^{rs}} x_{nm}^{rs} = e_m^{rs} \quad \forall m \in \mathcal{N}^{rs}, (r,s) \in \mathcal{W}, \quad (23)$$

$$-x_{mn}^{rs} \leq 0 \quad \forall (m,n) \in \mathcal{A}^{rs}, (r,s) \in \mathcal{W}, \quad (24)$$

$$\mu_{mn}^{rs} \geq 0 \quad \forall (m,n) \in \mathcal{A}^{rs}, (r,s) \in \mathcal{W}, \quad (25)$$

$$\lambda_m^{rs} \text{ free} \quad \forall (m, n) \in \mathcal{A}^{rs}, (r, s) \in \mathcal{W}, \quad (26)$$

$$-\mu_{mn}^{rs} x_{mn}^{rs} = 0 \quad \forall (m, n) \in \mathcal{A}^{rs}, (r, s) \in \mathcal{W}. \quad (27)$$

The complementary slackness conditions (27) are non-convex, and we can linearize them taking advantage of binary nature of  $x_{mn}^{rs}$ . Suppose  $M$  is a very large number then the linearized constraint would be:

$$\mu_{mn}^{rs} \leq M(1 - x_{mn}^{rs}) \quad \forall (m, n) \in \mathcal{A}^{rs}, (r, s) \in \mathcal{W}. \quad (28)$$

We state the final formulation of our optimization model here:

$$(P) \quad \min_{x, y, z, q} \quad \sum_i y_i + \sum_i z_i \quad (29)$$

subject to

$$s_i - s_i^{\max} \leq M y_i \quad \forall i \in \mathcal{S}, \quad (30)$$

$$s_i^{\min} - s_i \leq M z_i \quad \forall i \in \mathcal{S}, \quad (31)$$

$$s_i = s_i^0 - \sum_{(r,s) \in \mathcal{W}} \sum_{n \in \mathcal{N}^{rs}} x_{in}^{rs} \delta_{in}^{rs} d^{rs} + \sum_{(r,s) \in \mathcal{W}} \sum_{n \in \mathcal{N}^{rs}} x_{ni}^{rs} \delta_{ni}^{rs} d^{rs} \quad \forall i \in \mathcal{S}, \quad (32)$$

$$q_{mn}^{\min} \leq q_{mn} \leq q_{mn}^{\max} \quad \forall (m, n) \in \mathcal{A}, \quad (33)$$

$$\delta_{mn}^{rs} (p_{mn}^0 + q_{mn} + \nu_2 v_{mn}^{rs}) + \nu_3 \alpha_{mn}^{rs} w_{mn}^{rs} - \lambda_m^{rs} + \lambda_n^{rs} - \mu_{mn}^{rs} = 0 \quad \forall (m, n) \in \mathcal{A}^{rs}, (r, s) \in \mathcal{W}, \quad (34)$$

$$\sum_{(m,n) \in \mathcal{A}^{rs}} x_{mn}^{rs} - \sum_{(n,m) \in \mathcal{A}^{rs}} x_{nm}^{rs} = e_m^{rs}, \quad \forall m \in \mathcal{N}^{rs}, (r, s) \in \mathcal{W}, \quad (35)$$

$$-x_{mn}^{rs} \leq 0 \quad \forall (m, n) \in \mathcal{A}^{rs}, (r, s) \in \mathcal{W}, \quad (36)$$

$$\mu_{mn}^{rs} \geq 0 \quad \forall (m, n) \in \mathcal{A}^{rs}, (r, s) \in \mathcal{W}, \quad (37)$$

$$\lambda_m^{rs} \text{ free} \quad \forall (m, n) \in \mathcal{A}^{rs}, (r, s) \in \mathcal{W}, \quad (38)$$

$$\mu_{mn}^{rs} \leq M(1 - x_{mn}^{rs}) \quad \forall (m, n) \in \mathcal{A}^{rs}, (r, s) \in \mathcal{W}. \quad (39)$$

In the final model represented by Equations (29) to (39) alongside Equations (8) to (17), for every OD pair  $(r, s)$ , the number of decision variables is calculated as  $2\text{No}_r \text{No}_s + 3(\text{No}_r + \text{No}_s)$ . For a network of 200 stations, this amounts to approximately 1 million variables and 1 million constraints. Therefore, while this formulation can be useful for a comparative evaluation of other methods (with small-scale problems), more scalable solutions are desirable for practical purposes.

Before we describe the computational experiments, however, it is important to point out that our choice of the objective function is an obvious departure from the functions that try to minimize the system-wide number of unsatisfied demands. Rather than trying to minimize the system-wide service level, we look at the service level for every single station and try to achieve a certain desirable value for it for as many stations as possible. Also, the alternative objective functions may provide us

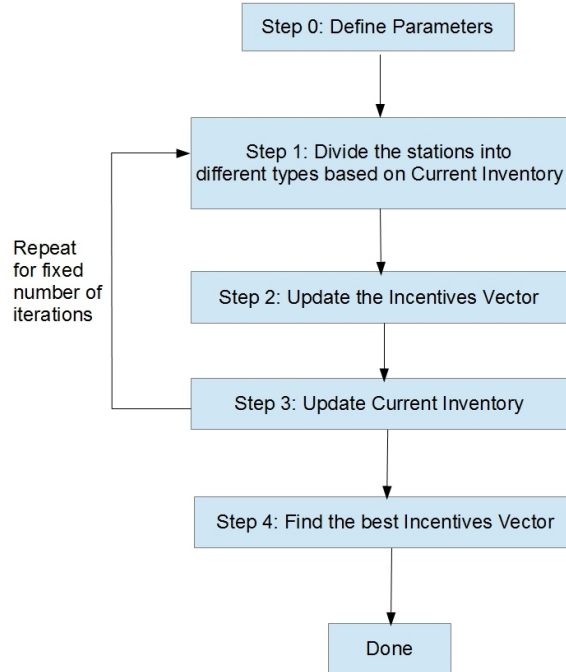


Figure 3: Flow chart of the Iterative Price Adjustment Scheme (IPAS)

a price vector but they will naturally redistribute the unsatisfied demand between different stations in equal measure. This outcome is unfavorable to our basic idea of trying to create *hub* stations and concentrating the unsatisfied demand on those stations, thus making the subsequent manual truck repositioning operation much easier and simpler.

## 4 Iterative Price Adjustment Scheme

This section presents a novel heuristic method to solve model (P). Keeping the upper level problem intact, we work with the lower level problem: instead of performing the exact minimum cost optimization to determine the optimum price change vector  $q_{mn}$ , we determine  $q_{mn}$  heuristically. The overall objective of minimizing the number of imbalanced stations remains the same. This section presents an iterative heuristic method called IPAS that produces a price change vector using discrete increments and decrements on the price between different station categories.

The proposed iterative price adjustment scheme (IPAS) relies on a simple decision making process that classifies the bike stations into different categories based on their starting inventory level  $s_i^0$ , the maximum and minimum inventory values, and the demand in the next time period. Based on these factors we divide the stations into six different types: 1) Balanced Stations with Bikes Needed, 2) Balanced Stations with Docks Needed, 3) Imbalanced stations with Bikes Needed, 4) Imbalanced stations with Docks Needed, 5) HUB Stations with Bikes Needed (also called Lack accumulation stations), and 6) HUB Stations with Docks Needed (also called Surplus accumulation

stations). We will identify stations that are “slightly” imbalanced at the current time and try to make those stations balanced. We will also identify stations that are “highly” imbalanced and try to make them even more imbalanced for the sake of preserving the balance at other neighboring stations.

As the primary objective of the pricing problem lies in identifying the accumulation stations, while simultaneously reducing the number of such stations, we will first develop a heuristic to identify such stations where accumulation happens naturally. Further accumulation may be possible and likely whenever a station is expected to experience an increase in the imbalance at the current pricing scheme. Here, we will look at the possibility of transferring the surplus inventory from high surplus stations to the neighboring stations which lack bicycles. If a station is expected to have a significant surplus and there are no stations in its neighborhood where lack accumulation is expected, then such a station is a suitable candidate for a surplus accumulation hub. For this kind of station, to help further accumulation, we can decrease prices for bike returns to this station and simultaneously increase regular prices on bike checkouts from the station. For a likely lack accumulation station, we can do the opposite: increase regular prices on bike returns and offer discounts for checkouts. Based on such selection of accumulation stations and price changes, we may evaluate the objective function in (P) and iteratively adjust prices. Figure 3 shows a flow chart for the proposed iterative price adjustment scheme.

### **Initialization: Define the Algorithm Parameters**

First of all, we define some parameters that will be used in the proposed scheme. The basic parameters are as follows: The parameters  $s_i^{\max}$ ,  $s_i^{\min}$ ,  $s_i^0$ , and  $s_i$  as defined in Table 1;  $k$  is an integer number representing the running iteration of the algorithm; and  $q_{mn}^k$  is the Price change (Incentive or Penalty) vector for traversing link  $(m, n)$  during iteration number  $k$  of the algorithm where  $q_{mn}$  for a given link  $(m, n)$  is the same for all OD pairs. The starting price change vector called  $q_{mn}^k$  for  $k = 0$  can generally have all values equal to 0.

Parameter  $\theta_i^{\text{in}}$  describes the maximum number of bicycles that can possibly come into a certain station  $i$ , which is the sum of the number of bicycles coming in from all the other stations of the network into the station  $i$  and the number of bicycles coming in from all the other stations of the network into the neighborhood stations of  $i$ . We assume that if big enough incentives were offered, all the traffic coming into neighborhood points of  $i$  will be redirected to  $i$  and users will take rest of the trip walking.

Parameter  $\theta_i^{\text{out}}$  describes the maximum number of bicycles that can possibly go out of a certain station  $i$ , which is the sum of the number of bicycles going to all the other stations of the network from the station  $i$  and the number of bicycles going to all the other stations of the network from the neighborhood stations of  $i$ . We assume that if big enough incentives were offered, all the traffic heading from neighborhood points of  $i$  will be redirected through  $i$  and users will walk to  $i$  and



take a bike forward to their destinations.

The *rank ratio*, denoted by  $\rho$ , is the parameter used to rank different stations based on their current inventory status,

$$\rho_i = \frac{(s_i - s_i^{\max})^2 + (s_i - s_i^{\min})^2}{(s_i^{\max} - s_i^{\min})^2}. \quad (40)$$

The value of  $\rho$  varies from 0.5 to  $\infty$  with lesser values implying a more balanced station and larger values implying a more imbalanced station. A station with value of  $\rho \geq 1$  implies that either of the two inequalities  $s_i \leq s_i^{\max}$  and  $s_i \geq s_i^{\min}$  are unsatisfied. The metric  $\rho$  looks at the current inventory and min-max range of a station to define the “distance” of the current inventory of the station from its min max extremes. If for a station  $i$ ,  $s_i^{\min} = 0$ ,  $s_i^{\max} = 10$ , and  $s_i = 5$ , then its  $\rho$  value will be 0.5, which is the minimum possible. Any deviation from this middle value will increase  $\rho$  and the station will be considered less balanced. We could very well have constructed some other measures but the underlying logic would stay the same. The purpose of the rank ratio is to sort stations that fall into one of the six categories listed below.

#### 4.1 Step 1: Define Station Types

The stations fall in only one of the six subsets of stations based on the following definitions:

**HUB Stations with Bikes Needed (HSBN)** have very low bike inventory, so that we would not have enough bicycles to make this station balanced again, even if the incoming bike inventory from all the neighboring stations were to be redirected; hence they become good candidates for Lack Accumulation Stations. Instead of making such a station balanced by gaining bicycles, we make it even more imbalanced by losing bicycles further. Such a station must satisfy the following inequality:

$$\pi s_i^{\min} - s_i^0 \leq \theta_i^{\text{in}}.$$

**HUB Stations with Docks Needed (HSDN)** have very high bike inventory so that it is very difficult to lose enough bicycles to make the station balanced again, even if all the outgoing bicycles from its neighborhood stations were to be redirected through it. Hence these stations become good candidates for Surplus Accumulation Station. So instead of making it balanced by losing bicycles, we make it even more imbalanced by accumulating more bicycles. Such station must satisfy the following inequality

$$s_i^0 - \pi s_i^{\max} \leq \theta_i^{\text{out}}.$$

where  $\pi$  is a constant with value ranging from 0 to 1. A higher value of  $\pi$  means less HUB

stations and vice versa.

**Imbalanced stations with Bikes Needed (ISBN)** are stations which cannot satisfy user demand at  $\beta_i^-$  service level with their current bike inventory, although they might not be completely vacant. Such station must satisfy the following two inequalities:

$$\begin{aligned} s_i &< s_i^{\min}, \\ \pi s_i^{\min} - s_i^0 &\leq \theta_i^{\text{in}}. \end{aligned}$$

The first inequality defines an imbalanced Station with Bikes Needed while the second makes sure that the station is not an HSBN.

**Imbalanced stations with Docks Needed (ISDN)** are stations which cannot satisfy user demand at  $\beta_i^+$  service level with their current inventory, although they might not be filled to capacity. Such stations must satisfy the following two inequalities:

$$\begin{aligned} s_i &> s_i^{\max}, \\ s_i^0 - \pi s_i^{\max} &\leq \theta_i^{\text{out}}. \end{aligned}$$

The first inequality defines an imbalanced Station with Docks Needed while the second makes sure that the station is not an HSDN.

**Balanced Stations with Bikes Needed (BSBN)** are stations which satisfy the inequality  $s_i \geq s_i^{\min}$ , and hence, are balanced by definition but their current inventory is relatively closer to  $s_i^{\min}$  than  $s_i^{\max}$ . Such stations must satisfy the following two inequalities:

$$\begin{aligned} s_i &\geq s_i^{\min}, \\ \pi s_i^{\min} - s_i^0 &\leq \theta_i^{\text{in}}. \end{aligned}$$

**Balanced Stations with Docks Needed (BSDN)** are stations which satisfy the inequality  $s_i \leq s_i^{\max}$ , and hence, are balanced by definition but their current inventory is relatively closer to  $s_i^{\max}$  than  $s_i^{\min}$ . Such stations must satisfy the following two inequalities:

$$\begin{aligned} s_i &\leq s_i^{\max}, \\ s_i^0 - \pi s_i^{\max} &\leq \theta_i^{\text{out}}. \end{aligned}$$

We denote each subset of stations by  $\mathcal{S}_{\text{HSDN}}$ ,  $\mathcal{S}_{\text{HSBN}}$ ,  $\mathcal{S}_{\text{ISBN}}$ ,  $\mathcal{S}_{\text{ISDN}}$ ,  $\mathcal{S}_{\text{BSBN}}$ , and  $\mathcal{S}_{\text{BSDN}}$  respectively. Within each subset, we order stations from the most balanced to the least balanced. We denote the number of stations in each subset by  $|\mathcal{S}_{\text{HSDN}}|$ ,  $|\mathcal{S}_{\text{HSBN}}|$ ,  $|\mathcal{S}_{\text{ISBN}}|$ ,  $|\mathcal{S}_{\text{ISDN}}|$ ,  $|\mathcal{S}_{\text{BSBN}}|$ , and  $|\mathcal{S}_{\text{BSDN}}|$  respectively.

Table 2: Calculating values of  $\Delta q_{mn}$ , where  $j_m$  and  $j_n$  are the order indices starting from zero within the set  $\mathcal{S}_{\text{ISBN}}$  of stations  $m$  and  $n$ , respectively—for example, if station  $m$  is the most balanced station in the set  $\mathcal{S}_{\text{ISBN}}$ , then  $j_m=0$ —and similarly,  $k_m$  and  $k_n$  are the order indices starting from zero within the set  $\mathcal{S}_{\text{ISDN}}$ . We let  $\Delta q_{mn} = 0$  for all  $m \in \mathcal{S}_{\text{BSBN}}$  and  $n \in \mathcal{S}_{\text{BSDN}}$ .

Type of Station $m$	Type of Station $n$			
	HSBN	HSDN	ISBN	ISDN
HSBN	0	$-\Delta i_s$	$-\Delta i_l \sin \frac{\pi j_n}{ \mathcal{S}_{\text{ISBN}} }$	$\Delta i_s \sin \frac{\pi k_n}{ \mathcal{S}_{\text{ISDN}} }$
HSDN	$\Delta i_s$	0	$-\Delta i_s \sin \frac{\pi j_n}{ \mathcal{S}_{\text{ISBN}} }$	$\Delta i_l \sin \frac{\pi k_n}{ \mathcal{S}_{\text{ISDN}} }$
ISBN	$\Delta i_l \sin \frac{\pi j_m}{ \mathcal{S}_{\text{ISBN}} }$	$\Delta i_s \sin \frac{\pi j_m}{ \mathcal{S}_{\text{ISBN}} }$	$\frac{\Delta i_s}{2} \left( -\cos \frac{\pi j_n}{ \mathcal{S}_{\text{ISBN}} } + \cos \frac{\pi j_m}{ \mathcal{S}_{\text{ISBN}} } \right)$	$\Delta i_l \sin \frac{\pi(j_n+k_m)}{ \mathcal{S}_{\text{ISBN}} + \mathcal{S}_{\text{ISDN}} }$
ISDN	$-\Delta i_s \sin \frac{\pi k_m}{ \mathcal{S}_{\text{ISDN}} }$	$-\Delta i_l \sin \frac{\pi k_m}{ \mathcal{S}_{\text{ISDN}} }$	$-\Delta i_l \sin \frac{\pi(j_n+k_m)}{ \mathcal{S}_{\text{ISBN}} + \mathcal{S}_{\text{ISDN}} }$	$\frac{\Delta i_s}{2} \left( -\cos \frac{\pi k_n}{ \mathcal{S}_{\text{ISDN}} } + \cos \frac{\pi k_m}{ \mathcal{S}_{\text{ISDN}} } \right)$

## 4.2 Step 2: Update the Incentives Vector

After the network has been divided into six exclusive sets of stations, it is time to update the price change vector based on the following equation:

$$q_{mn}^k = q_{mn}^{k-1} + \Delta q_{mn},$$

where  $\Delta q_{mn}$  is the price update that we will calculate as follows.

For each link  $(m, n)$ , the price change is updated in discrete jumps. We define  $\Delta i_s$  and  $\Delta i_l$  as small and large discrete jumps. Their values are constant numbers. We use \$0.017 and \$0.1 for experimental purposes. As shown in Table 2 the discrete jumps can thus vary from  $-\$0.017$  to  $\$0.017$  when  $\delta_s$  is used or from  $-\$0.1$  to  $\$0.1$  when  $\delta_l$  is used. The trigonometric distribution functions used to calculate  $\Delta q_{mn}$  are plotted in Figure 4. We have not included stations of type BSBN and BSDN in the table because the price change for travel between and to the *balanced* stations is zero. The stations belonging to each of the described categories are sorted according to their rank ratio values and stored as a sorted array. The stations in the beginning of the array are more balanced (or less imbalanced) and those at the end are less balanced (or more imbalanced). When deciding the prices between stations of each category, we take into account the position of each station in their respective arrays.

We use trigonometric functions because their shape allows us to have smoothly increasing or decreasing price changes based on the position of a station in an array. For instance, the value of  $\Delta q_{mn}$  between stations of HSBN and ISBN types follows the distribution in figure 4a with stations at both ends of the  $\mathcal{S}_{\text{ISBN}}$  vector getting smaller decrements while those in the middle getting the maximum decrements. This makes sure that our pricing favors the movement of bicycles from stations in the subset  $\mathcal{S}_{\text{HSDN}}$  to the middle stations of the subset  $\mathcal{S}_{\text{ISBN}}$ . The reason for favoring middle stations is that the stations at the start are substantially close to being balanced and hence do not need large changes in price while the stations at the end are critically imbalanced and it is difficult to make them balanced again using price changes. The middle stations in the array are moderately imbalanced and hence are more prone to respond to price changes. Similarly, the value of  $\Delta q_{mn}$  between stations of ISBN and ISDN types follows the distribution in Figure 4d. The price increment is maximum for stations in the middle making it difficult for bicycles to move from stations in subset  $\mathcal{S}_{\text{ISBN}}$  to stations in subset  $\mathcal{S}_{\text{ISDN}}$ , so on and so forth. The logic behind using trigonometric functions is their ability to give us two sided minima. Admittedly, we could have used a simple triangle function for simpler situations in 4a and 4b but trigonometric functions are better suited for constructing the more complex functions in 4c, 4d and 4e.

### 4.3 Step 3: Calculate the Number of Imbalanced stations

We use (P) to calculate the number of imbalanced stations given the price change vector  $q_{mn}$ . We store this value for every iteration. (P) is also used to calculate the updated values of  $s_i$ . After this, Step 1 is repeated with updated  $s_i$  and stations are again categorized into different types based on Step 1. This process is repeated until the designated number of iterations.

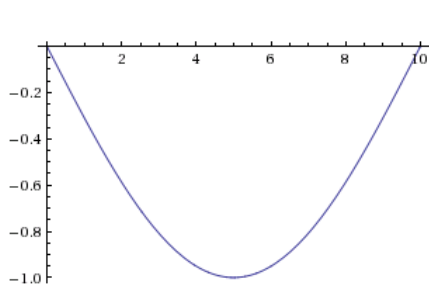
### 4.4 Step 4: Find the Best Price Change Vector

The number of imbalanced stations calculated in Step 3 are compared to the minimum value. If the current price change vector improves on the objective function, the vector and the objective value are stored. After this, Step 0 is repeated with updated  $s_i$  and the values of some of the parameters are calculated based on new  $s_i$ . Stations are again categorized into different types based on Step 1. This process is repeated until the designated number of iterations. After the iterations run out, this scheme gives the price change vector with minimum number of imbalanced stations as its output.

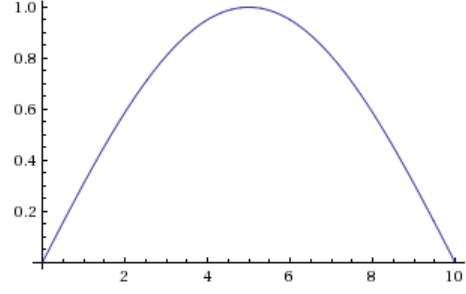
## 5 Numerical Experiments

The following publicly available data sets were retrieved from the Capital BikeShare website:

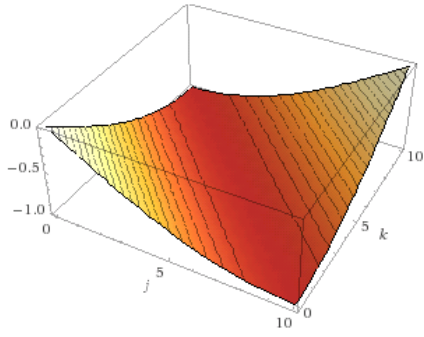
1. Journey Data: The data of 0.85 million bike rides during the three months period starting from July 1st to September 30th, 2013 (See Table 3).



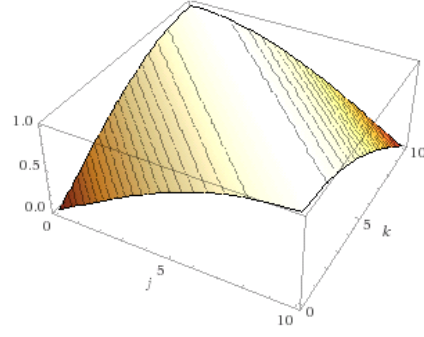
(a)  $-\sin \frac{\pi j}{|\mathcal{S}_{\text{ISBN}}|}$  for  $j \in [0, |\mathcal{S}_{\text{ISBN}}| - 1]$



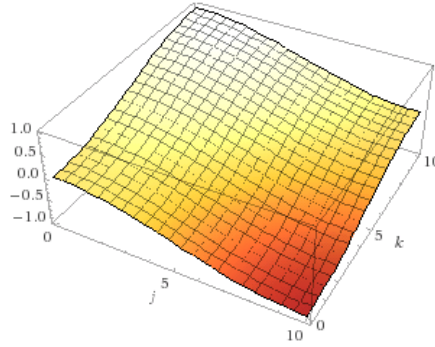
(b)  $\sin \frac{\pi j}{|\mathcal{S}_{\text{ISBN}}|}$  for  $j \in [0, |\mathcal{S}_{\text{ISBN}}| - 1]$



(c)  $-\sin \frac{\pi(j+k)}{|\mathcal{S}_{\text{ISBN}}|+|\mathcal{S}_{\text{ISDN}}|}$  for  $j \in [0, |\mathcal{S}_{\text{ISBN}}| - 1]$  and  $k \in [0, |\mathcal{S}_{\text{ISDN}}| - 1]$



(d)  $\sin \frac{\pi(j+k)}{|\mathcal{S}_{\text{ISBN}}|+|\mathcal{S}_{\text{ISDN}}|}$  for  $j \in [0, |\mathcal{S}_{\text{ISBN}}| - 1]$  and  $k \in [0, |\mathcal{S}_{\text{ISDN}}| - 1]$



(e)  $(-\cos \frac{\pi j}{|\mathcal{S}_{\text{ISBN}}|} + \cos \frac{\pi k}{|\mathcal{S}_{\text{ISBN}}|-1})$  for  $j, k \in [0, |\mathcal{S}_{\text{ISBN}}| - 1]$

Figure 4: Graphs of Different  $\Delta q_{mn}$  functions for  $|\mathcal{S}_{\text{ISBN}}| = 10$  and  $|\mathcal{S}_{\text{ISDN}}| = 10$

Table 3: Journey Data Sample

Start Time	Station ID	End Time	Station ID	Bicycle ID
7/1/2013 12:02:00 a.m.	31250	7/1/2013 12:21:00 a.m.	31506	W20231
9/4/2013 4:27:00 p.m.	31247	9/4/2013 5:11:00 p.m.	31248	W01484
8/9/2013 3:54:00 p.m.	31253	8/9/2013 4:01:00 p.m.	31229	W20198
9/23/2013 5:34:00 p.m.	31004	9/23/2013 5:52:00 p.m.	31007	W20523

Table 4: Station Status Data Sample

Station ID	Longitude	Latitude	No of Bicycles	No of Empty Docks
31000	-77.0512	38.8561	4	7
31605	-77.0023	38.8851	1	10
31609	-77.0213	38.8767	7	12
31403	-77.0198	38.9566	8	7

2. Station Data: The data of longitude and latitude of all stations, their ID numbers and their current inventory status retrieved from publicly available xml file (See Table 4).

Journey data is used to calculate the demand vector  $d^{rs}$ . Capital Bike Share is a relatively newer and smaller system. The number of journeys between each OD pair in a short enough time period is rather small. So we divide each day into four equal intervals of six hours. The first three intervals from 6:00 a.m. to 11:59 p.m. are used for price based repositioning while in the fourth interval, from 12:00 a.m. to 6:00 a.m. the next day, the trucks carry out static repositioning. We use the demand data from 20th September 2013. We assume that the demand on this day in all three intervals is known at the beginning of the intervals. At the beginning of each of the first three intervals, we use the Optimal model or the IPAS scheme to calculate a price vector. The journey data is also used to determine service level requirements and values of  $s_i^{\max}$  and  $s_i^{\min}$  at the beginning of the three intervals. We use  $\beta_i^-$  and  $\beta_i^+$  values of 0.95 for our calculations. In our computational experiments we run the optimization model or the IPAS scheme three times at the beginning of each of the three intervals. It is possible to further reduce the length of the interval and run the models more often during the day. We intentionally avoided this finer level of temporal granularity for two reasons. Firstly, running the model every hour gives us a very sparse demand matrix which contravenes our objective of creating Hub stations by modifying the demand matrix. Secondly, we observe that at station level and at system level, the inventory flow exhibits a similar trend within each of the time intervals. Simply, the demand in each hour largely follows a trend similar to that of the over-arching six hour interval. The first interval corresponds to the morning commute, the second corresponds to the commute back home and the third corresponds to the post

Table 5: Inventory Status for Different Network Sizes.

Network Name	No of Stations	No of Total Docks	No of Available Bikes	No of Empty Docks	System-wide Demand for Bicycles	System-wide Demand for Docks
MEDIUM	60	1606	1240	366	907	907
LARGE	202	4498	1012	3486	2531	2531

work activities in the evening and at night.

At the Capital Bikeshare website, the station data with sample in Table 4 is available in the form of an XML file and the current status of the stations in terms of inventory, check-ins and checkouts is continuously updated on to the server. This data is used to calculate the position of each station in the network. The longitude and latitude are used to calculate distances and time of travel between all OD pairs. The distance calculated is not euclidean and represents actual biking distance. We also get the number of bicycles, number of empty docks and capacity of each station using this data. The starting inventory  $s_i^0$  used in our calculations is the starting inventory of the network.

The starting inventory profile MEDIUM1 (M1) represents the system status at 6 a.m. on 20th September, 2013 for the 60 station network. In the subsequent intervals, the starting inventory is recursively updated. For second interval, the starting inventory is equal to the final inventory at the end of the first interval. For the fourth interval beginning at 12 a.m., the starting inventory (for truck based repositioning) is the same as the ending inventory for the third interval. The distance matrix is also used to determine the neighborhood stations of each station. We use 600 meters as a conservative measure with stations less than 600 meters apart becoming each other’s neighbors.

We build the data sets for two sets of problems of different network size. A medium sized network with 60 stations and a full sized network with 202 stations in it. It should be taken into account that the number of imbalanced stations cannot always be 0. In fact, our observations show that during high demand periods, like the one we have used for experimentation, the total inventory available in the system is much smaller than demand for the next time period.

In Table 5 we present the inventory situation for each of our data sets. As evident, there is always a measure of imbalance present in the system.

## 5.1 Experiments with Optimization Model and Heuristic Approaches

Before presenting the detailed experimental results, we must point out that in addition to the issue of time it takes to solve even the moderately sized instance of the Optimization model (P), we face another problem that is very instructive for any system trying to optimally determine the price

vector. For every OD pair  $(r, s)$ , the optimal price vector prescribed by the optimization model (P) is prone to give us path costs that can be approximately equal to each other. As an example, take the path costs for all the possible paths between node 2 as origin and node 10 as destination as shown in figure 5. The costs given in the third column are the summation of the nominal costs of the path and the price on the bike links suggested by the optimization model. The optimization model (P) chose path 2-19-10 as the optimal path with the minimum cost of 2.453 units. Another path, 2-10 also has approximately the same overall cost.

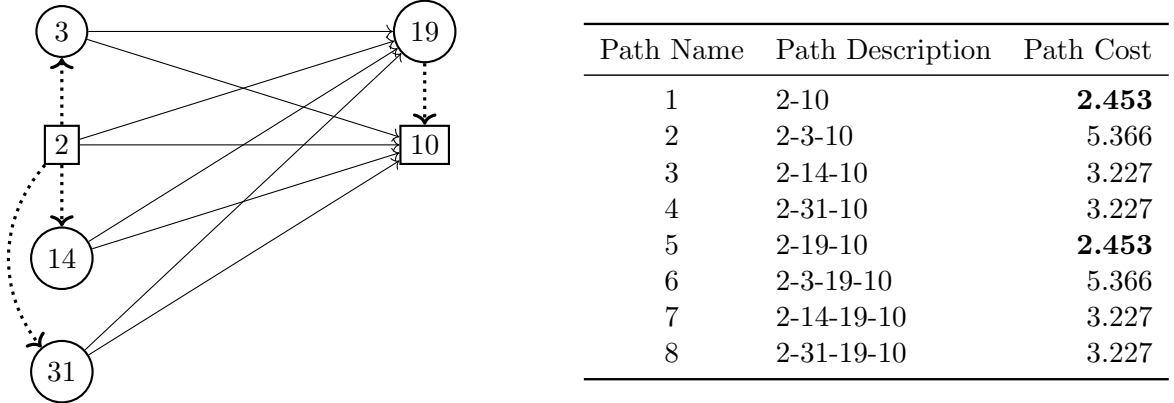


Figure 5: Possible paths between an OD pair  $(r = 2, s = 10)$

To summarize, it is obvious that in some cases, the price vector prescribed by the optimization model is not differentiated enough and can lead to undesirable choices by the decision makers. Since lower level decision makers are not explicitly aware of the upper level objective of minimizing the number of imbalanced stations, given ambiguous choices, they are likely to make decisions unfavorable to the upper level agency. Our original model does not take into account the undifferentiated nature of the pricing vector it recommends but we believe it is an issue worth exploring.

To take the effect of this fuzziness into account, we carry out some post processing steps after getting the pricing vector from our model (P). We construct a new model called  $(\bar{P})$ , a slightly modified version of the original model (P), and incorporate the price vector recommended by P, designated as  $q_{mn}^{opt}$ , into the new model as a constraint. We state the formulation of the model  $(\bar{P})$  here:

$$(\bar{P}) \quad \max_{x,y,z,q} \quad \sum_i y_i + \sum_i z_i \quad (41)$$

s.t.

$$(32), (34) \text{ to } (39), (8) \text{ to } (17),$$

$$s_i^{\max} - s_i \leq M(1 - y_i) \quad \forall i \in \mathcal{S}, \quad (42)$$

$$s_i - s_i^{\min} \leq M(1 - z_i) \quad \forall i \in \mathcal{S}, \quad (43)$$



$$q_{mn} = q_{mn}^{\text{opt}} \quad \forall (m, n) \in \mathcal{A}, \quad (44)$$

In the new model, the objective function is maximized, and constraints (30), (31) and (33) are replaced by constraints (42), (43) and (44). As evident, (42), (43) are modified to correspond to the changed objective function. Constraint (44) assigns the optimal price vector values from (P) to the price variables in the new model ( $\bar{P}$ ). The rest of the constraints in ( $\bar{P}$ ) are exactly the same as the model (P). The new model ( $\bar{P}$ ) does not try to determine a pricing vector. It uses the optimal vector suggested by (P) to come up with an alternate set of lower level decisions ( $x$ ). These decisions correspond to people making choices, in response to the predetermined pricing vector, that are unfavorable to the upper level objective (represented by maximization).

Using ( $\bar{P}$ ), we are interested in seeing the worst case scenario and answer the following question: What happens if we apply the optimal pricing vector in reality, but rather than minimizing the number of imbalanced stations (people making decisions favorable to the agency), we maximize the number of imbalanced stations (people making decisions unfavorable to the agency)? What would be the worst case result of applying the suggested price vector? The objective function value of ( $\bar{P}$ ) serves as a strict upper bound on the number of imbalanced stations if the optimal price vector recommended by model (P) is to be implemented. The second row in table 6 reports the objective function value for the new model. The values in first and second rows provide us with a range in which the objective function may lie for the given optimal price vector. For instance, if we use  $q_{mn}^{\text{opt}}$  in reality, the number of imbalanced stations for T1 could be in the range (26-44), 26 in the best case scenario and 44 in the worst case scenario. On the other hand, and very crucially, the IPAS heuristic generates the price vector based on iterative increments and decrements and hence the price vector thus generated is well differentiated.

The Objective Function values for optimization model (P), its maximization counterpart ( $\bar{P}$ ), IPAS scheme and the situation without pricing for different network sizes are tabulated in Table 6. All the numerical experiments were done on a machine with 2.30 GHz CPU clock speed, 8 GB RAM and 64-bit Windows 8 operating system. The model (P) was solved using the Java API of CPLEX V12.4 while the coding for the IPAS heuristic was also done in Java. The optimality gap for the optimization model (P) was set at 4.0%. For IPAS, as mentioned earlier, the heuristic was run for a designated number of iterations. We use 5000 iterations for our experiment. Mostly, our best result is found within the first 1000 iterations. We chose the number of iterations as our stopping criteria because of various reasons. Generally, the algorithms are stopped when the improvement between consecutive iterations, often parameterized by a small number  $\epsilon$ , is very small. The objective that we are comparing across iterations is the number of imbalanced stations which is an integer number with a very small range. It is not possible to look at two consecutive iterations and decide to stop the algorithm because no improvement was achieved. For most consecutive iterations of IPAS, there is indeed no improvement in the objective function. Also the objective function value goes up and down through the iterations. It does not linearly decrease. Hence, we run a large number

Table 6: Value of Objective Function for different starting inventories with (P), ( $\bar{P}$ ), and Iterative Price Adjustment Scheme (IPAS) for the MEDIUM and LARGE networks

Network	Method	Unsatisfied Demand	Number of Imbalanced Stations		
			T1	T2	T3
MEDIUM	(P)	355/83	26	21	15
	( $\bar{P}$ )	350/91	44	38	37
	IPAS	294/19	37	30	25
	N	252/48	47	46	38
LARGE	(P)	-	-	-	-
	( $\bar{P}$ )	-	-	-	-
	IPAS	390/972	110	89	84
	N	203/813	156	132	116

of iterations and report the iteration, and the corresponding pricing vector, that gave us the best solution first. If the optimal iteration is very close to the allowable number of iterations, we can simply choose to increase the number of iterations.

We find out that for the medium sized network, a single run of the optimization model is solved by CPLEX in approximately two hours. However, CPLEX fails to solve the full network model consisting of 202 stations in a reasonable amount of time. We see a huge improvement in the objective function when price change vectors are generated through optimization model and IPAS heuristic as compared with situation without pricing. Although the heuristic developed in this paper may potentially underperform model (P) in terms of objective function, the price vector generated by IPAS is well differentiated, and we may reasonably claim that IPAS can potentially outperform the optimization model (P) in terms of the objective function in scenarios close to the worst case for (P). On the other hand, time is a critical factor in calculating and updating a real time price vector. A single run of IPAS scheme only takes 40 seconds and 120 seconds for the 60 station and 202 station networks, respectively.

We also limit the maximum absolute value of price changes available to the operator. Table 7 shows the objective function values for different ranges of incentive and penalty values and different  $\nu_3$  values. Smaller values of  $\nu_3$  underestimate the cost of walking and hence alternative paths become more suitable for travellers. We have used  $\nu_3=1/50$  in our earlier calculations.

As evident, larger the price changes an operator is willing to introduce, the more balanced a network becomes. We see that as the price change range increases the total price change offered increases dramatically while offering much less in terms of objective function improvement. It is always better to offer smaller price changes first and gradually increase them if doing so provides considerable advantage.

Table 7: Value of Objective Function  $f(q_{mn})$  generated by IPAS for different  $q_{mn}$  ranges and  $\nu_3$  values for a LARGE network when neighborhood radius is 600m

	Range of $q_{mn}$					
	$[-5, 5]$	$[-4, 4]$	$[-3, 3]$	$[-2.5, 2.5]$	$[-2, 2]$	$[-1.5, 1.5]$
$\nu_3=1/25$	91	91	91	95	95	94
$\nu_3=1/50$	83	85	84	88	90	95
$\nu_3=1/75$	80	82	78	85	82	85

## 5.2 Models for Cost Comparison

To prove the efficacy of our model (P) and the IPAS scheme in delivering cost advantages, we discuss two approaches in this paper. The first approach is a naive method based on a simple costing heuristic for repositioning of bicycles. The second approach, which we discuss in relative detail, is a routing model that we borrow from Raviv et al. (2013).

### 5.2.1 Naive Method

In Table 8, we use the naive method for cost comparison between manual and price-induced repositioning schemes. The values in column A represent the total *positive* price change which is the amount of incentives doled out to customers. The values in column B are the total *negative* price change which is the total penalties levied on the travellers to stop them from making undesirable journey choices. Naturally, due to the price change vector, the travellers automatically move certain bikes between stations to make them balanced. The sum of A and B values is the total cost to the system for moving a certain number of bikes automatically. Column C indicates the cost of moving the same number of bikes using manual trucks and crew. Simply, column C is the cost that is saved as a result of pricing scheme. To find the cost values in column C, we use the average \$3.5, from DeMaio (2009), as a conservative measure for cost of a single manual repositioning and multiply it with the total number of bicycles moved as a direct result of our pricing scheme. As long as  $A+B \leq C$ , price induced repositioning is more viable than manual repositioning.

Note that if the range of incentives increases, the number of imbalanced stations comes down. Also, the number of bikes ‘automatically’ moved as a direct result of the pricing scheme increases and hence we see greater values in column A, B and C. Naturally, if the range of price change is larger, greater cost is saved.

### 5.2.2 Arc-Indexed Formulation for Repositioning Vehicle Routing

To evaluate the impact of pricing on repositioning costs we use, with minor modifications, the arc-indexed formulation presented in Raviv et al. (2013). The objective function of the formulation

Table 8: Using the Naive Method for Comparison of Cost(in Dollars) to the System for Price Induced Repositioning (P) and Manual Repositioning (No pricing), A=Total Incentives given, B= Total Penalties levied, A+B= Cost of Price Based Repositioning, C= Cost of Manual Repositioning

Range of $q_{mn}$	$\nu_3=1/25$				$\nu_3=1/50$				$\nu_3=1/75$			
	A	B	A+B	C	A	B	A+B	C	A	B	A+B	C
$[-5, 5]$	1311	-1955	-644	744	913	-1426	-513	1183	690	-1102	-412	1148
$[-4, 4]$	1251	-1742	-491	605	894	-1361	-467	1134	742	-1095	-337	1235
$[-3, 3]$	1039	-1337	-298	399	726	-1078	-352	952	604	-919	-315	1123
$[-2.5, 2.5]$	983	-1162	-181	224	668	-973	-305	896	537	-804	-267	1039
$[-2, 2]$	820	-954	-134	168	520	-829	-309	822	480	-697	-217	945
$[-1.5, 1.5]$	590	-724	-134	63	476	-689	-213	592	376	-577	-201	801

has two terms. The first term is the sum of discrete convex penalty functions which are linearized in an exact manner through a set of constraints. The second term represents the total operating costs for the repositioning operation. The objective function is given by the following expression:

$$\min \sum_{i \in N} f_i(s_i) + \alpha \sum_{i \in N_0} \sum_{j \in N_0} \sum_{v \in V} t_{ij} x_{ijv} \quad (45)$$

The penalty function  $f_i(s_i)$  is calculated in a pre-processing step for every inventory level from 0 to maximum capacity  $i$  for each station. In our modified model, however, we modify the first term of the objective function. Since we have already calculated  $s_i^{\min}$  and  $s_i^{\max}$  in section 3.1, we already have the inventory range that will ensure a 95% service level for a station taking into account the future demand. Given this range, we do not need to calculate the convex penalty function values for all inventory levels considering shortage costs and service costs as Raviv et al. (2013) do. Instead, We assume the penalty to be 0 if the inventory  $s_i$  lies between  $s_i^{\min}$  and  $s_i^{\max}$  and we use a simple linear function to calculate the penalties for all possible deviations from this range as shown in figure 6.

$$f_i(s_i) = 0 \quad \forall i \in N_0, \quad \forall s_i \in [s_i^{\min}, s_i^{\max}], \quad (46)$$

$$f_i(s_i) = \gamma(s_i^{\min} - s_i) \quad \forall i \in N_0, \forall s_i | s_i < s_i^{\min}, \quad (47)$$

$$f_i(s_i) = \gamma(s_i - s_i^{\max}) \quad \forall i \in N_0, \forall s_i | s_i > s_i^{\max}. \quad (48)$$

Where  $\gamma$  is the cost of unit deviation. Using the above expressions, we calculate the values of  $f_i(s_i)$  for all  $i$  and all  $s_i = \dots, -2, -1, 0, \dots, c_i, c_{i+1}, \dots$  in a preprocessing step. Next, we calculate parameters  $a_{iu}$  and  $b_{iu}$ , where  $u$  is the index over which  $s_i$  is defined, using the following equations:

$$b_{iu} \equiv f_i(u + 1) - f_i(u), \quad (49)$$

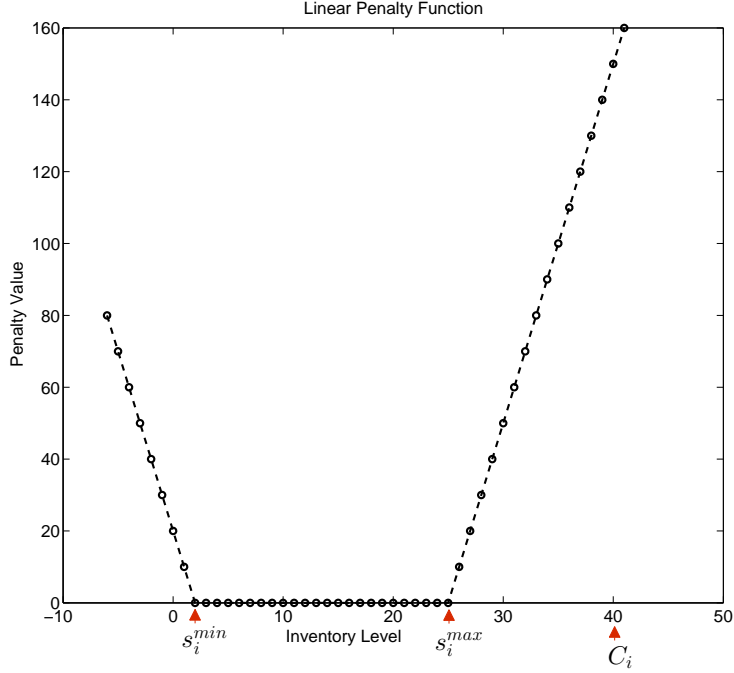


Figure 6: Linear Objective Function as a function of Inventory

$$a_{iu} \equiv f_i(u) - b_{iu}u. \quad (50)$$

Finally, if  $g_i$  is the variable representing the penalty incurred at station  $i$ , then we can replace the first set of terms in the objective function given in (52) by the respective linear terms and the constraint:

$$g_i \leq a_{iu} + b_{iu}s_i \quad \forall i \in N_0, \forall s_i = \dots, -2, -1, 0, \dots, c_i, c_{i+1}, \dots \quad (51)$$

The modified objective function is then given by:

$$\min \sum_{i \in N} g_i + \alpha \sum_{i \in N_0} \sum_{j \in N_0} \sum_{v \in V} t_{ij} x_{ijv} \quad (52)$$

The static or truck-based repositioning operation discussed in the model starts at 12 a.m. and continues for a specific period of time (We use 9000sec and 18000sec). The purpose of repositioning operation is to improve the starting conditions of the next day. This is done through adjusting the initial inventory levels before the start of the next day (at 6am) using the truck-based static repositioning. The dynamic pricing scheme, which precedes static repositioning, in turn adjusts the inventory level at 12 a.m. before the start of static repositioning. To compare the two situations i.e. with and without pricing, we compare the performance of the static repositioning model vis-à-vis its objective function given the different starting inventory situations at 12 a.m.

Using the arc-indexed formulation presented here, We compare the objective function of the routing problem with four different 12 a.m. inventory situations: one reached with optimal pricing considering the best case scenario (P), second reached with optimal pricing considering the worst case scenario ( $\bar{P}$ ), third reached without pricing, and the fourth reached as a result of IPAS pricing. We use Java API of CPLEX to solve the routing problem. We use setting of 4.0% for optimality gap and set the solver time to be 1 day. A sample of results is tabulated in table 9. We actually solve a large number of instances of the Routing Model. We solve the 60 station network with different settings for number of vehicles, the total repositioning time available, the total travelling time available, the capacity of the repositioning vehicle(s), the time it takes to load/unload a single bicycle, the value of constant  $\alpha$  in the objective function, and the type of pricing scheme employed.

We find considerable improvement in the overall objective function value when we use the final (12 a.m.) inventory situation which was achieved as a result of a pricing scheme. We also find improvement in other critical to performance factors, like the number of stations visited by the repositioning vehicle, the travel time used for repositioning and the total time used in the repositioning operation. As is evident in table 9, IPAS outperforms the without pricing situation by a considerable margin. This is especially true when full repositioning is allowed to happen, allowing for sufficient time for all the stations to become balanced, as is the case for data in table (9). Not only were smaller numbers of stations visited ( upto 55% less), the vehicle had to move smaller number of bikes. The travel time and the overall time used for respositioning were also smaller by a margin of upto 16% and 30%, respectively. We were not able to solve the problem with acceptable optimality gap within the set time limit for the FULL network of 202 stations. This is consistent with the findings by Raviv et al. (2013).

Furthermore, we also compare the improvements provided by the pricing schemes for different values of the parameters as explained earlier. We postulate that the value of the objective function and in turn the improvement due to the pricing scheme should depend on the following factors:

- The size of the repositioning vehicle: We postulate that larger the capacity of the repositioning trucks used, smaller (better) will be the objective function.
- The value of loading/unloading time for a single bicycle: If loading/unloading of individual bicycles is faster, there is more time for vehicles to travel to different stations and objective function value is smaller.

We test our results for different values of the mentioned parameters for all four pricing arrangements, i.e. Optimal Pricing in best case scenario (P), Optimal pricing in worst case scenario ( $\bar{P}$ ), IPAS Pricing, and Without Pricing situations. The results are plotted in Figure 7.

As evident from Figure 7, as the capacity of the repositioning vehicle increases, the objective function value decreases. This is expected because of two reasons. Firstly, a larger vehicle can carry more bikes at a time so it can afford to choose a route that minimizes the penalty function.

Table 9: No of Vehicles = 1, No of Stations = 60 Total Time = 9000 sec, Travel Time = 9000 sec, Pricing Scheme: (P) = Optimal Pricing Best case scenario, ( $\bar{P}$ ) = Optimal Pricing Worst case scenario (with number of imbalanced stations maximized), N = No Pricing, IPAS = IPAS Pricing, L/U Time = Loading/Unloading time per vehicle, T1 = Travel Time Used, T2 = Total Time Used

Vehicle Capacity	L/U Time	Pricing Scheme	No. of Stations Visited	Bikes Added/Removed	T1/T2	Objective Function	Average no. of Bikes moved per Station	Maximum Bikes carried	Optimality gap (%)
20	20	(P)	16	30/50	4594.0/6194.0	76	5.000	20.0	3.3%
		( $\bar{P}$ )	32	80/99	6276.0/9856.0	93	5.594	20.0	4.0%
		N	39	81/101	6966.0/10606.0	120	4.667	20.0	4.0%
		IPAS	25	62/77	5927.0/8807.0	89	5.560	20.0	4.0%
20	60	(P)	18	32/49	4695.0/9555.0	77	4.500	17.0	4.0%
		( $\bar{P}$ )	34	72/92	6225.0/16065.0	92	4.824	20.0	3.6%
		N	39	76/96	6373.0/16693.0	124	4.410	20.0	3.8%
		IPAS	28	51/70	5792.0/13232.0	88	4.321	20.0	3.7%
40	20	(P)	16	33/62	4278.0/6178.0	43	5.938	29.0	3.7%
		( $\bar{P}$ )	31	60/87	5608.0/8548.0	56	4.742	31.0	3.7%
		N	37	79/120	6215.0/10255.0	62	5.378	39.9	3.4%
		IPAS	26	58/90	5321.0/8320.0	53	5.692	31.9	1.8%
40	60	(P)	16	28/52	4298.0/9098.0	43	5.000	29.0	3.6%
		( $\bar{P}$ )	31	61/88	5770.0/14710.0	58	4.806	31.0	3.9%
		N	37	60/100	5968.0/15568.0	70	4.324	40.0	2.0%
		IPAS	28	48/81	5420.0/13220.0	54	4.607	32.0	3.8%

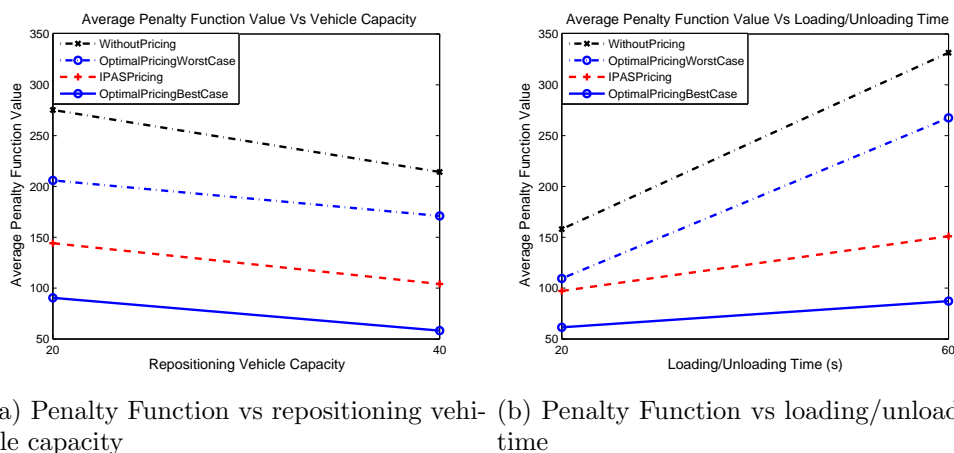


Figure 7: This figure shows how the Penalty Function varies with different parameters. Penalty Function is smaller if vehicles of larger capacity are used. If the overall carrying capacity is the same, larger number of small vehicles are better than smaller number of large vehicles. Similarly, repositioning is more effective when the loading/unloading operation is faster.

Secondly, a larger capacity vehicle can travel between different *hub* stations and easily carry a larger inventory between surplus and lack accumulation hubs. Similarly, smaller loading/unloading time gives us better value of the objective function. Since total time is limited and is divided between travel time and loading/unloading time, smaller value of loading/unloading time leaves more time for the vehicle to travel and choose a longer route thus minimizing the objective function. In all these instances however, the objective function and other critical to performance factors are always better for instances with (P) or IPAS pricing as compared to those with no pricing.

## 6 Conclusion

At the start of this paper, we set out to prove the efficacy of a pricing scheme for partially or fully rebalancing a BSS. This paper explores pricing and incentive schemes as a way to rebalance the network of a public bike sharing system. As already stated, the objective is to minimize the number of imbalanced stations to fully or partially obviate the need for a manual repositioning operation using trucks and crew. We develop a bi-level optimization model that works well for relatively smaller networks to deliver the optimal pricing scheme. We develop heuristics to solve the issue of time while still improving the objective function. We set out to develop a more intuitive heuristic approach based on the classification of different stations using the data available. This heuristic called iterative price adjustment scheme (IPAS) delivers much better computing time. For the full network, the IPAS takes only 120 seconds. The value of objective function is also markedly better than the situation without pricing. We conclude that this time is small enough to make it feasible for BSS operators to update their price vector in real time. The cost of offering incentives is also much smaller than the cost reduction from smaller number of imbalanced stations, smaller crew and trucking fleet. We use a routing model to show that the overall cost of repositioning, consisting of a linear penalty function penalizing the deviation from the min-max range for starting inventory at the beginning of the next day and an operating cost term, is markedly lesser when we employ a pricing scheme. The repositioning operation itself is easier to carry out and faster.

The demand vector we use is based on actual rides data after the fact. One obvious improvement is to model the demand more accurately using demand forecasting taking into account various factors that affect the demand for bicycles. The values of coefficients for deriving cost of travel have also been roughly determined. We assume the value of time to be the same for all customers which is obviously not the case. The values of these coefficients can be better estimated using insights into customer behavior. We also assumed that bike users are homogeneous and have the same level of sensitivity to price changes. In further research, we can consider heterogeneous probabilistic behavior or bounded rationality of bike users. As an extension of this work, we can also create a model in which the price based and dynamic manual repositioning happen side by side through out the day, complementing each other. We conclude that a real time dynamic pricing scheme cannot only solve the problem of system wide imbalance in BSS but also cut down on operating costs



of controlling that imbalance. As a practical solution to the repositioning problem, based on our findings, we recommend to have a dynamic pricing scheme during the day time complementing a static repositioning at the end of the day when the bike system is closed to the riders.

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# Appendix

## A Arc-Indexed Routing Formulation

### A.1 Notation

The model presented in Equations (53) to (73) in this Appendix is the basic arc-indexed formulation. We refer the reader to the paper by Raviv et al. (2013) for the detailed treatment of the problem.

We borrow the said paper's notation, summarized in table 10 in the Appendix.

## A.2 Mathematical Model

We use a linear penalty function  $f_i(s_i)$  rather than the convex penalty function used in Raviv et al. (2013). The model we present is the basic formulation from Raviv et al. (2013) with slight modifications. The formulation has the same constraints, with minor modifications, for inventory conservation, flow conservation, station capacity, repositioning vehicle capacity and upper bounds on loading, unloading and total vehicle travel times among others. We also used the valid inequalities presented in the paper to reduce the solution time for the problem.

$$(P2) \min \sum_{i \in N} g_i + \alpha \sum_{i \in N_0} \sum_{j \in N_0} \sum_{v \in V} t_{ij} x_{ijv} \quad (53)$$

subject to

$$s_i - s_i^{\max} \leq M y_i \quad \forall i \in N_0, \quad (54)$$

$$s_i^{\min} - s_i \leq M z_i \quad \forall i \in N_0, \quad (55)$$

$$y_i + z_i = p_i \quad \forall i \in N_0, \quad (56)$$

$$g_i \leq a_{iu} + b_{iu} s_i \quad \forall i \in N_0, \forall s_i = \dots, -2, -1, 0, \dots, c_i, c_{i+1}, \dots \quad (57)$$

$$s_i = s_i^0 - \sum_{v \in V} (y_{iv}^L - y_{iv}^U) \quad \forall i \in N_0, \quad (58)$$

$$y_{iv}^L - y_{iv}^U = \sum_{j \in N_0, j \neq i} y_{ijv} - \sum_{j \in N_0, j \neq i} y_{jiv} \quad \forall i \in N_0, \forall v \in V, \quad (59)$$

$$y_{ijv} \leq k_v x_{ijv} \quad \forall i, j \in N_0, i \neq j, \forall v \in V, \quad (60)$$

$$\sum_{j \in N_0, j \neq i} x_{ijv} = \sum_{j \in N_0, j \neq i} x_{jiv} \quad \forall i \in N_0, \forall v \in V, \quad (61)$$

$$\sum_{j \in N_0, j \neq i} x_{ijv} \leq 1 \quad \forall i \in N, \forall v \in V, \quad (62)$$

$$\sum_{v \in V} y_{iv}^L \leq \max(0, s_i^0) \quad \forall i \in N_0, \quad (63)$$

$$\sum_{v \in V} y_{iv}^U \leq \max(0, c_i - s_i^0) \quad \forall i \in N_0, \quad (64)$$

$$\sum_{i \in N_0} (y_{iv}^L - y_{iv}^U) = 0 \quad \forall v \in V, \quad (65)$$

$$\sum_{i \in N} (L y_{iv}^L + U y_{iv}^U) + \sum_{i \in N} (L y_{i0v} + U y_{i0v}) + \sum_{i, j \in N_0, i \neq j} t_{ij} x_{ijv} \leq T \quad \forall v \in V, \quad (66)$$

Table 10: Mathematical Notation for Arc-Indexed Static Repositioning Problem

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$N$	Set of stations indexed by $i = 1, \dots, N$
$N_0$	Set of nodes, including the stations at the depot (denoted by $i = 0, i = 0, \dots, N$ )
$s_i^0$	Number of bicycles at node $i$ before the repositioning operation starts
$s_i^{\min}$	Minimum number of permissible bicycles at a station $i$ , beyond which the station is considered imbalanced
$s_i^{\max}$	Maximum number of permissible bicycles at a station $i$ , beyond which the station is considered imbalanced
$c_i$	Number of lockers installed at station $i \in N_0$ , referred to as the station's capacity
$k_v$	Capacity (number of bicycles) of vehicle $v \in V$
$f_i(s_i)$	A convex penalty function for station $i \in N$ , the function is defined over the integers $s_i = \dots, -1, 0, \dots, c_i, c_i+1, \dots$
$t_{ij}$	Traveling time from station $i$ to station $j$
$\alpha$	Wight factor (in the objective function) of the operating costs relative to the penalty costs
$T$	Repositioning time, i.e., time allotted to the repositioning operations including travel time, loading time and unloading time
$T_t^v$	Travel time, i.e., time allotted for repositining travel for a particular vehicle $v \in V$
$L$	Time required to remove a bicycle from a station and load it onto the vehicle
$U$	Time required to unload a bicycle from the vehicle and hook it to a locker in a station
$M$	An upper bound on the number of arcs in a vehicle's tour whose length is at most $T$ time units, where the vehicle visits each station at most once
$x_{ijv}$	Binary variable which equals one if vehicle $v$ travels directly from node $i$ to node $j$ , and zero otherwise
$y_{ijv}$	Number of bicycles carried on vehicle $v$ when it travels from node $i$ to node $j$ . $y_{ijv}$ is zero if the vehicle $v$ does not travel directly from $i$ to $j$
$y_{iv}^L$	Number of bicycles loaded onto vehicle $v$ at node $i$
$y_{iv}^U$	Number of bicycles unloaded from vehicle $v$ at node $i$
$q_{iv}$	Auxiliary variables used for sub-tour elimination constraints
$s_i$	Inventory level at node $i$ at the end of the repositioning operation

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$$\sum_{i,j \in N_0: i \neq j} t_{ij} x_{ijv} \leq T_t^v \quad \forall v \in V, \quad (67)$$

$$q_{jv} \geq q_{iv} + 1 - M(1 - x_{ijv}) \quad \forall i \in N_0, \forall j \in N, i \neq j, \forall v \in V, \quad (68)$$

$$x_{ijv} \in \{0, 1\} \quad \forall i, j \in N_0 : i \neq j, \forall v \in V, \quad (69)$$

$$y_{iv}^L \geq 0, y_{iv}^U \geq 0, \quad \text{integer}, \forall i \in N_0, \forall v \in V, \quad (70)$$

$$y_{ijv} \geq 0, \quad \forall i, j \in N_0 : i \neq j, \forall v \in V, \quad (71)$$

$$s_i \geq 0, \quad \forall i \in N_0, \quad (72)$$

$$q_{iv} \geq 0, \quad \forall i \in N_0, \forall v \in V. \quad (73)$$