Generalized Route Planning Model for Hazardous Material Transportation with VaR and Equity Considerations

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Abstract

In an earlier paper, Kang, Batta, and Kwon (2011) introduced the Value-at-Risk (VaR) framework and applied it to the case of routing a single hazmat trip. In this paper, we develop upon this work in two important ways. First, we show how to apply the VaR concept to a more realistic multi-trip multi-hazmat type framework which aims at determining routes that minimize the global VaR value while satisfying equity constraints. Second, we show how to embed the solution algorithm for the single hazmat trip problem into a Lagrangian relaxation framework to obtain an efficient solution method for this general case. We test our computational experience based on a real-life hazmat routing scenario in the Albany district of New York State. Our results indicate that one can achieve a high degree of risk dispersion while controlling the VaR value within the desired confidence level.

Keywords: Value-at-Risk(VaR), multi-trip shipment, hazmat transportation, social risk mitigation, risk equity, dissimilar path

1 Introduction

Hazardous material transportation planning has been well-recognized as a Low-Probability-High-Consequence (LPHC) problem. A study of available data on hazmat transportation accident statistics in North America reveals that accident probabilities are extremely small, usually estimated at 10^{-6} per trip per mile traveled (Harwood, Viner, and Russell 1993). However, the low likelihood of hazmat accidents results in an insufficient or inaccurate set of historical records to predict the accident probabilities of future hazmat accidents. On the other hand, due to their disastrous consequences, prediction of accident probabilities of hazmat accidents is critical to reduction of societal risk. To illustrate this point further we note that there were only 167,680 hazmat transportation incidents for the ten year period from 2000 to 2009, which resulted in a total of just 133 fatalities and 2,784 injuries, while damage caused was $637,270,767 (U.S. Department of Transportation 2010) due to the evacuation and cleanup efforts needed after a hazmat accident. Hazmat transport planning has attracted the attention of numerous OR/MS researchers. Many models have been created, with objectives to minimize the expected risk (Alp 1995, Jin and Batta 1997), maximum risk (Erkut and Ingolfsson 2000), and the mean-variance of the risk (Erkut and Ingolfsson 2000). There are some other models that consider balancing hazmat risk and transportation cost (Verter and Kara 2008, Marcotte, Mercier, Savard, and Verter 2009). A common feature of all of these approaches is that they focus on a single criterion and rely on hazmat accident statistics being available and accurate.

To overcome the drawback associated with the reliance on the availability and the accuracy of hazmat accident statistics, in a recent paper we have introduced the Value-at-Risk (VaR) concept as it applies to hazmat routing and risk assessment (Kang, Batta, and Kwon 2011). Our previous paper focused on the introduction of the VaR concept and its application to a single trip case. In this paper, we expand on our earlier work in two significant directions. First, we demonstrate its application to the multi-trip, multi-hazmat situation while considering equity as a constraint. Second, we use Lagrangean relaxation coupled with the single trip solution algorithm to develop an efficient algorithm for the general case. Computational testing is conducted on a real-life routing scenario in the Albany district of New York State. Our principal finding is that by modest increases in overall VaR value one can achieve quite stringent levels of risk equity.

The remainder of this paper is organized as follows. The next section reviews some directly related literature. In Section 3, we present the mathematical formulation of the generalized routing problems for hazmat transport based on the hazmat VaR model. Sections 4 and 5 develop the corresponding solution framework on a large-sized transportation network. Section 6 contains our computational experience, based upon data from a real-life hazmat routing scenario in the Albany district of New York State. This experience illustrates the efficiency and effectiveness of our model and summarize our computational results. Finally, Section 7 provides our concluding remarks and offers suggestions for future research.

2 Literature Review

In this section we briefly review the relevant literature in the area of hazmat routing, equity modeling and dissimilar path generation.

2.1 Hamzat Routing

The research on hazardous material transportation problems basically focuses on two main issues: risk assessment and effective routing so as to mitigate risk. Much work has been done in risk modeling with a focus on the risk probability distribution over given areas — by taking into account the risk related to the transported substance (Abkowitz, Eiger, and Srinivasan 1984) or by map algebra techniques from Geographic Information Systems (Zhang, Hodgson, and Erkut 2000); impact of the commodities being carried – how to define the impact area (Batta and Chiu 1988), what levels they belong to and what consequences they will bring (Kara and Verter 2004, Martí, Velarde, and Duarte 2009); transport modality – by truck, train or freight (Abkowitz et al. 1984); and environmental conditions – the accident probabilities known (Patel and Horowitz 1994) or unknown (Bell 2007); the risk being varied (Erkut and Ingolfsson 2000).

Some prevailing studies emphasize the risk parameters to be measured during the transportation, including minimizing population exposure (ReVelle, Cohon, and Shobrys 1991), expected risk (Alp 1995), maximal risk (Erkut and Ingolfsson 2000), probability (Saccomanno and Chan 1985) or conditional probability (Sivakumar, Batta, and Karwan 1993), mean-variance of risk (Erkut and Ingolfsson 2000), and risk disutility (Erkut and Ingolfsson 2000). Recently researchers have focussed on other considerations like transportation cost or risk equity dispersion (Gopalan, Kolluri, Batta, and Karwan 1990b). Some bi-level hazmat transportation models have been developed to study the trade-off between the two conflicting objectives of minimizing transportation cost and risk of the hazmat transport (Kara and Verter 2004, Verter and Kara 2008, Marcotte et al. 2009). Another recent direction of study is on building a model which has less reliance on the accuracy of historical accident data. An example of this is the VaR model developed in Kang et al. (2011).

The other main issue is route planning of hazmat shipments, which involves a selection among the alternative paths between O-D pairs. Carriers usually focus on the routing problem of single-commodity single O-D pair shipments individually – i.e., select a route between a given O-D pair for a given hazmat type – which is called local route planning (Bianco, Caramia, and Giordani 2009). However, the main concern of a government authority is to control the total risk over the population and the equity distribution of this risk over population zones, which can only achieved by simultaneously considering all of the local route planning problems, i.e. through global route planning. Examples of this approach for a single O-D pair and multiple shipments of a single hazmat type are papers by Gopalan et al. (1990b) and Lindner-Dutton, Batta, and Karwan (1991).

2.2 Equity Modeling

The concept of risk equity is well defined in Keeney (1980). Keeney expresses equity as the magnitude of the largest difference in the level of risk among a fixed set of individuals. Holding the total risk constant, he focuses on comparing different distributions of that constant total risk across individuals. Several models have been proposed for addressing equity in the context of hazmat transport. Gopalan, Batta, and Karwan (1990a) develop a model for a single hazmat trip in which the objective is to minimize risk subject o a set of constraints that ensure that the difference in risk borne by population zones is less than a set threshold (equity specification). This model was later generalized by Gopalan et al. (1990b) to the case of multiple hazmat trips of a single O-D pair. For this situation, Gopalan et al. (1990b) were able to show empirically that high level of equity can be achieved by fairly modest increase in risk level.

Another method of enforcing equity is to limit the risk associated with a population zone or link. Current and Ratick (1995) take such an approach by minimizing the maximum risk for a zone. Carotenuto, Giordani, and Ricciardelli (2007) focus on minimizing the total risk while constraining the risk on each traversed link. Both Current and Ratick (1995) and Carotenuto et al. (2007) consider a single hazmat trip. Our model is much more general than those proposed above. We allow multiple O-D pairs and multiple trips for each O-D pair. Our method of modeling equity is the same as that used in Gopalan et al. (1990a) and Gopalan et al. (1990b). The other difference is that we use minimization of VaR as our objective. However, our approach can also be used with any other risk measures, including traditional risk as used in the majority of the literature.

2.3 Dissimilar Path Generation

The concept of dissimilar paths has been put forward in several contexts other than that of hazmat shipments. Kuby, Xu, and Xie (1997) apply dissimilar paths to reduce the search space for path-based models in a large, capacitated, multi-commodity network flow model. Lombard and Church (1993) suggest generating a number of topologically dissimilar paths to avoid the repeated attempt of infeasible or undesirable paths, in the case of restricted layout problems like the corridor location application.

In the hazmat transport problem, the generation of spatially dissimilar paths is necessary to spread the risk equitably all over the network whenever several hazmat shipments take place from a given origin to a given destination, and to provide more meaningful alternatives when the “best” path choice is not allowed in varied environments like bad weather conditions. There have been many methods to generate k shortest paths, like Yen (1971). However, Yen’s k-shortest path algorithm makes route choices based on transportation distance which results in the spatial similarity among the generated paths. This situation is rather undesirable in hazmat route choices, where the objective is to reduce transportation risk more than transportation cost and highly overlapped road segments would severely increase the consequences of traffic accidents. To overcome this problem, a “p-dispersion” model is proposed by Kuby (1987) to maximize the minimum dissimilarity of the paths on a general network. Erkut, Ülküsal, and Yenicerioğlu (1994) described and made an empirical comparison of ten heuristics for the discrete p-dispersion problem. Akgün, Erkut, and Batta (2000) present an improvement by selecting a subset from a large set of candidate paths using a dispersion model which maximizes the minimum dissimilarity in the selected subset, and compare their computational result with three other methods, the Iterative Penalty Method (IPM) (Johnson, Joy, Clarke, and Jacobi 1992), the Gateway Shortest Path Method (GSP) (Lombard and

Church 1993) and Minimax method (Kuby et al. 1997). Duarte and Marti (2007) proposed and solved a similar maximum diversity problem by employing a constructive semi-greedy algorithm with a tabu search method.

Though the above methods have clearly defined the path dissimilarity, their work are restricted to single criterion, the edge length, and only consider the edges in the definition of dissimilarity. Dell'Olmo, Gentili, and Scozzari (2005) developed the dissimilarity path problem from a multi-objective perspective and proposed a multi-criteria shortest path algorithm (MSPA) to generate a set of non-dominated paths according to the multi-objective concept with respect to both length and risk. Thyagarajan, Batta, Karwan, and Szczerba (2005) consider both spatial and temporal information to solve the problem of determining dissimilar paths for military aircraft during mission ingress. Later on, Martí et al. (2009) developed a modified Greedy Randomized Adaptive Search Procedure (GRASP) (See Resende and Ribeiro (2003)) heuristic for a bi-objective path dissimilarity problem by making the trade-off between the two conflicting objectives – minimizing the average length of the paths while maximizing the dissimilarity among the paths.

In our model, we apply the dissimilar path concept to reduce the search space of the generalized hazmat VaR model. The generation of the p disperse paths can be generated by any of the above methods. The main difference of our model lies in the generation of the candidate set — the k minimal VaR paths, which cannot be converted into the k shortest path problem.

3 Formulation

Consider a transport network $G = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ represents the node set, and $\mathcal{A}$ represents the link set. Define I as the O-D pair set within this network and H as the set of hazmat types. Suppose for each O-D pair $i \in I$, there is a set of candidate paths $\mathcal{P}^i$. Each shipment $s \in S$ is defined as a trip on an O-D pair $i \in I$ with a type of hazmat $h \in H$, where S is the set of shipments. Given a risk confidence level $\alpha \in (0, 1)$, the objective of our problem is to find a set of paths to minimize the total VaR over the set of shipments S while incorporating VaR equity. To write the VaR equity constraint, we assume that the geographical region encompassed by the transportation network G can be divided into a set of mutually disjoint zones K, with each zone denoted as Z_k. We define a zone pair ordering as an ordered pair (Z_a, Z_b) of zones. The risk equity constraint is to maintain the difference in total VaR between each zone pair within a set threshold μ. Table 1 presents the mathematical notation used in this paper.

Let $x_{\alpha i}^{j(h)}$ be an integer decision variable, representing the volume of hazmat type $h \in H$ on the jth path of the O-D pair $i \in I$ given confidence level α, where $j \in \mathcal{P}^i$. A formulation for the generalized hazmat VaR routing problem is as follows:

$$(P) \qquad \min_x \sum_{h\in H}\sum_{i\in I}\sum_{j\in\mathcal{P}^i} \beta_{\alpha i}^{j(h)} x_{\alpha i}^{j(h)} \tag{1}$$

subject to

$$\sum_{j\in\mathcal{P}^i} x_{\alpha i}^{j(h)} = n_{hi}, \quad \forall \quad i \in I, \quad \forall \quad h \in H \tag{2}$$

$$\sum_{h\in H}\sum_{i\in I}\sum_{j\in\mathcal{P}^i} (\hat{\beta}_{\alpha a}^{h}(i,j) - \hat{\beta}_{\alpha b}^{h}(i,j)) x_{\alpha i}^{j(h)} \le \mu, \quad \forall \quad a, b \in K \tag{3}$$

$$x_{\alpha i}^{j(h)} \quad \text{integer}, \quad \forall \quad j \in \mathcal{P}^i \quad \forall \quad i \in I \quad \forall h \in H \tag{4}$$

where $\beta_{\alpha i}^{j(h)}$ denotes the maximum cutoff risk brought by hazmat $h \in H$ along path $j \in \mathcal{P}^i$ of the O-D pair $i \in I$ under confidence level α, and $\hat{\beta}_{\alpha k}^{h}(i,j)$ represents the maximum cutoff risk to zone Z_k brought by $h \in H$ along $j \in \mathcal{P}^i$ given α for all zones $k \in K$.

The objective of the generalized hazmat VaR routing problem is to minimize the cumulative VaR values of all hazmat shipments in the road network G given α. The constraint set (2) constrains the number of shipments on O-D pair $i \in I$ carrying hazmat type $h \in H$. The constraint set (3) represents the risk equity constraints. Here $\hat{\beta}_{\alpha k}^{h}(i,j)$ represents the cumulative VaR value to a zone Z_k given α. We note that ordered zone pair (Z_a, Z_b) is different from (Z_b, Z_a), and we have no *a priori* knowledge about which zone pair sustains the maximum risk difference for traveling on an arbitrary path. Therefore, it is necessary to compare the VaR dissimilarity for each zone pair ordering (Z_a, Z_b) rather than a random selection of a pair of zones. Finally constraint set (4) restricts $x_{\alpha i}^{j(h)}$, the number of shipments with hazmat type $h \in H$ on each path $j \in \hat{P}_{\alpha}^{i}$ of an O-D pair $i \in I$, as integer.

The generalized hazmat VaR routing model is difficult to solve due to the risk equity constraint (3). If this constraint is absent, then the problem can be decomposed into separate routing problems

Table 1: Mathematical Notation Table

Notation	Explanation
Sets	
$G(\mathcal{N},\mathcal{A})$	a graph of road network
$\mathcal{N}$	set of nodes, $\|\mathcal{N}\| = n$
$\mathcal{A}$	set of links, $\|\mathcal{A}\| = m$
I	Set of O-D (Origin-Destination) pairs
S	Set of shipments
H	Set of hazmat types
K	Set of geographical zones in G
$\mathcal{P}$	Set of available paths for a shipment
$\mathcal{P}^i$	Set of available paths for O-D pair $i \in I$
$\hat{M}_\alpha$	Set of minimal VaR paths for a shipment given confidence level α, $\hat{M}_\alpha \subseteq \mathcal{P}$
$\hat{M}^i_\alpha$	Set of minimal VaR paths for O-D pair $i \in I$ under confidence level α, $\hat{M}^i_\alpha \subseteq \mathcal{P}^i$, $\|\hat{M}^i_\alpha\| = \hat{m}_i$
$\hat{P}_\alpha$	Set of dissimilar paths for a shipment given confidence level α, $\hat{P}_\alpha \subseteq \hat{M}_\alpha$
$\hat{P}^i_\alpha$	Set of dissimilar paths for O-D pair $i \in I$, $\hat{P}^i_\alpha \subseteq \hat{M}^i_\alpha$, $\|\hat{P}^i_\alpha\| = \hat{p}^i$
A^l	link set for path $l \in \mathcal{P}$, $\|A^l\| = m^l$
$\mathcal{C}^l$	set of ascending-order sorted link consequences for path $l \in \mathcal{P}$, $\|\mathcal{C}^l\| = \overline{m}^l$
Parameters	
α	confidence level to control the worst risk brought by hazmat transportation
VaR^l_α	cutoff risk for a path $l \in \mathcal{P}$ given confidence level α, with definition $\Pr R^l > VaR^l_\alpha \leq 1-\alpha$
β_α	VaR value for a shipment under confidence level α
$\beta^{j(h)}_{\alpha i}$	VaR value brought by a shipment with hazmat type $h \in H$ on the path $j \in \hat{P}_i$ of O-D pair $i \in I$ given confidence level α
$\hat{\beta}^h_{\alpha,k}(i,j)$	VaR value brought by a shipment with hazmat type $h \in H$ on the path $j \in \hat{P}_i$ of O-D pair $i \in I$ to a geographical zone $k \in K$ given confidence level α
n_{hi}	number of shipments assigned to O-D pair $i \in I$ with hazmat type $h \in H$
$w_{ij_1ij_2}$	dissimilarity index between two candidate routes l_{ij_1} and l_{ij_2}
$f(\Delta_{\hat{i}\hat{j}})$	dissimilarity function of a pair of routes $(\hat{i},\hat{j})$
μ	threshold for risk equity constraint
η	Lagrangean multipliers
Variables	
R^l	risk brought by path $l \in \mathcal{P}$
x^j_α	binary variable indicating whether path j is selected in the candidate set $\hat{M}^\alpha$
y^j_α	binary variable indicating whether path j is selected in the dissimilar path subset $\hat{P}^\alpha$
$x^{j(h)}_{\alpha i}$	number of shipments assigned to path $j \in \hat{P}_i$ selected for shipment with hazmat type $h \in H$ on O-D pair $i \in I$ or not given confidence level α

for each hazmat and origin-destination combination, each of which can be solved using the algorithm in Kang et al. (2011). There are many different ways to ensure that one obtains an equitable set of routes. For example, Gopalan et al. (1990b) bound the maximum risk sustained by any zone within a set threshold. This constraint offers the advantage of being able to enforce equity between those zones that offer reasonable transportation alternatives. We adopt this risk equity formulation in our model to spread the risk into different zones, but with a different definition of the risk equity parameters $\hat{\beta}^h_{\alpha k}(i,j)$. In our case, the risk of a shipment to a zone is the maximal cutoff risk that its path will experience within that zone under the given confidence level α. After segmenting the road segments of the path in each zone, we can calculate the zone VaR value $\hat{\beta}^h_{\alpha k}(i,j)$ with the same method for path VaR value $\beta^{j(h)}_{\alpha i}$. The detailed calculation method has been described in Kang et al. (2011).

4 Determination of Candidate Paths

For (P) to deliver an optimal solution to the hazmat routing problem, every possible path between every O-D pair must be included. This leads to an unmanageable formulation that requires path enumeration. To circumvent this difficulty we study in this section the problem of finding a reasonable set of paths for each O-D pair for inclusion in the optimization problem (P). We start by assuming that there are a number of feasible routes available for a given O-D pair, say, set $\mathcal{P}$. Each path in the set $\mathcal{P}$ represents an option that is acceptable to the carrier for routing a certain hazmat between the O-D pair. Suppose that we are able to select a candidate set $\hat{M} \subseteq \mathcal{P}$ containing k-minimal VaR paths given α.

$$\hat{M}_\alpha = \{l^k : VaR^k_\alpha < VaR^w_\alpha, \forall w \in Q, Q \subseteq \mathcal{P} - \{l^1, l^2, \cdots, l^{k-1}\}\} \qquad (5)$$

From the candidate set $\hat{M}_\alpha$, we will determine a subset $\hat{P}_\alpha \subseteq \hat{M}_\alpha$ for each O-D pair such that every pair of paths in the subset $\hat{P}_\alpha$ are as dissimilar from each other as possible. These candidate routes represent routes (set of links that connect a particular O-D pair) that are spatially dissimilar to one another (dissimilar with respect to a distance metric). Define x^j_α as the binary variable indicating whether path j is selected in the candidate set $\hat{M}_\alpha$, and y^j_α as the binary variable indicating whether path j is selected in the dissimilar path subset $\hat{P}_\alpha$ or not. The problem is then equivalent to solving:

$$\begin{aligned} &\min_x \textstyle\sum_{j\in\mathcal{P}} \beta_\alpha^j x_\alpha^j \\ &\max_y f_{\hat{P}_\alpha \subseteq \hat{M}_\alpha}(\Delta_{\hat{i}\hat{j}}) \end{aligned} \tag{6}$$

subject to

$$\sum_{j\in\mathcal{P}} x_\alpha^j = |\hat{M}_\alpha| \tag{7}$$

$$\sum_{j\in\hat{M}_\alpha} y_\alpha^j = |\hat{P}_\alpha| \tag{8}$$

$$y_\alpha^j \leq x_\alpha^j \quad \forall j \in \mathcal{P} \tag{9}$$

$$x_\alpha^j \in \{0,1\} \quad \forall j \in \mathcal{P} \tag{10}$$

$$y_\alpha^j \in \{0,1\} \quad \forall j \in \mathcal{P} \tag{11}$$

Here $f(\Delta_{\hat{i}\hat{j}})$ represents the dissimilarity of a pair of routes $(\hat{i},\hat{j})$. Methods to measure the path dissimilarity have been developed by many researchers as discussed in Section 2.3.

Let $\hat{M}_\alpha^i$ be the set of minimal VaR routes for O-D pair i with $|\hat{M}_\alpha^i| = \hat{m}_i$. $\hat{P}_\alpha^i \subseteq \hat{M}_\alpha^i$ is the subset of $\hat{p}_i$ dissimilar VaR paths out of $\hat{M}_\alpha^i$. Let i represent the O-D pair, j represent the candidate path in set $\hat{P}_\alpha^i$. The term $w_{ij_1ij_2}$ represents the dissimilarity index between two candidate routes l_{ij_1} and l_{ij_2}. Thus the dissimilar VaR path problem can be formulated as

$$\max_{\hat{P}_\alpha^i \subseteq \hat{M}_\alpha^i} \min_{l_{ij_1} \neq l_{ij_2}, j_1, j_2 \in \hat{P}_\alpha^i} w_{ij_1ij_2} \tag{12}$$

Given route set $\hat{M}_\alpha^i$, $\hat{p}_i$ routes are chosen so that the minimum dissimilarity between any two selected routes is maximized. In other words, we minimize the maximum similarity of any two routes in the set $\hat{P}_\alpha^i$.

If there are a set of O-D pairs I in the network G, with $|I| = \bar{i}$, there would be $\bar{i}$ candidate sets of minimal VaR routes and $\bar{i}$ subsets of dissimilar VaR routes selected from them. Let these be labeled as $\hat{P}_\alpha^1, \hat{P}_\alpha^2, \cdots, \hat{P}_\alpha^{\bar{i}}$, where $\hat{P}_\alpha^1 \subseteq \hat{M}_\alpha^1, \hat{P}_\alpha^2 \subseteq \hat{M}_\alpha^2, \cdots, \hat{P}_\alpha^{\bar{i}} \subseteq \hat{M}_\alpha^{\bar{i}}$. The key problem is how to generate the sets $\hat{M}_\alpha^1, \hat{M}_\alpha^2, \cdots, \hat{M}_\alpha^{\bar{i}}$. Once the candidate sets $\hat{M}_\alpha$s are generated, the dissimilar path subset $\hat{P}_\alpha^1, \hat{P}_\alpha^2, \cdots, \hat{P}_\alpha^{\bar{i}}$ can be selected from candidate set $\hat{M}_\alpha^1, \hat{M}_\alpha^2, \cdots, \hat{M}_\alpha^{\bar{i}}$.

Candidate Path Set Determination The candidate set $\hat{M}_\alpha$ aims to sort and select from the available path set for a shipment from origin (O) to destination (D) with defined criteria, shortest distance or minimal risk. In the dissimilar hazmat VaR transportation problem, we intend to find the first k minimal VaR paths. $\hat{M}_\alpha$ can be constructed in many different ways. But generally speaking, no matter how those algorithms vary, they can be divided into two categories: Removing Path algorithm and Deviation Path algorithm.

The Removing Path algorithm is proposed by Martins (1984). The main idea of the algorithm can be summarized into four steps:

- Find the path with minimum VaR value in G^k, labeled by l^k;
- Remove l^k from G^k;
- Construct a new network G^{k+1};
- Determine the shortest path l^k in the resulting network G^{k+1}.

The Deviation Path algorithm is proposed by Eppstein (1998). The main idea of this algorithm is to obtain the k-shortest path l^k from a set of candidate paths composed by the deviation paths of $l^1, l^2, \cdots, l^{k-1}$. Starting with the determination of shortest tree τ_D, and the candidate set for the next shortest path X, the general steps of this algorithm consist of:

- let l^k be the path with minimum VaR value in X^k;
- remove l^k from X^k;
- join all deviation paths from l^k to X^{k+1}.

The Deviation Path algorithm is an effective method to solve the k-shortest path problem, because it does not need to reconstruct graphs by adding or removing links or nodes like the Removing Path algorithm. It is frequently used by researchers who study the k-shortest path problem, e.g. (Yen 1971). For years, much effort has been made to improve its efficiency and effectiveness. However, the efficiency of the Deviation Path algorithm is built based on the construction of partial search space for the $k+1^{th}$ path —- the set of deviation paths from the neighborhood of the k^{th} "best" path. Due to this fact, it is not effective in solving problems with objectives other than transportation cost. For the $\hat{p}_\alpha$ Dissimilar VaR path problem, because the objective is VaR rather than transportation cost, the 2^{nd} minimal VaR path is not necessarily contained in the neighborhood of the 1^{st} minimal VaR path. By contrast, the Removing Path algorithm is much more effective in such a situation. Given a certain confidence level α, by removing the 1^{st} minimal VaR path from the network G^1, we can always find the 2^{nd} minimal VaR path from the resulting graph G^2. Similarly, by removing the 2^{nd}, 3^{rd}, $\cdots$, $(k-1)^{st}$ paths, the k^{th} minimal VaR path can be generated from the newly constructed network G^k. The candidate path set $\hat{M}_\alpha$ can be defined as the set of minimal VaR paths in each newly constructed graph G^k for O-D pair $i \in I$,

$$\hat{M}_\alpha = \{l^k : VaR^k_\alpha = \min_{j \in Q} VaR^k_\alpha, \quad \text{where} \quad Q \in \mathcal{P} - \{l_1, \cdots, l_{k-1}\}\} \tag{13}$$

Dis/similarity Index Definition and Spatially Dissimilar Paths Selection With the candidate path set $\hat{M}_\alpha$ determined, we can select the dissimilar VaR path subset $\hat{P}_\alpha$ with any method in the literature review. We have discussed many methods in Section 2.3 to solve the dissimilar path problem in different ways, with each of them having a similarity or dissimilarity function. Our model utilizes the p-dispersion method proposed by Akgün et al. (2000) to generate the $\hat{p}_\alpha$ spatially dissimilar paths.

General Procedure to Determine Spatially Dissimilar VaR Paths Our solution to find dissimilar VaR paths for a single-shipment multi-path hazmat shipment can be described as a two-step solution:

1. Constructing the Candidate Set of Minimal VaR Paths Using Removing Paths Procedure

   (a) Remove Paths Procedure

   - Step0: Assuming the shortest path $l = \{s_1, (s_1, s_2), \cdots, s_{m-1}, (s_{m-1}, s_m), s_m = s_n\}$, set $flag = d(s_1, s_2)$;
   - Step1: Delete (s_1, s_2) from $G(N, A)$;
     - If $s_2 = s_n$, stop;
     - otherwise, go to Step 2;
   - Step2: $\forall j = 2, \cdots, m-1$, adjoin to N a node $s_j^{'}$;

- Step3: $\forall j = 2, \cdots, m-1$, and $\forall (s_j, x) \in A$ such that $x \neq s_{j+1}$, adjoin $(s_j^{'}, x)$ to A and set $d(s_j^{'}, x) = d(s_j, x)$;
- Step4: $\forall j = 2, \cdots, m-2$, adjoin $(s_j^{'}, s_{j+1}^{'})$ to A and set $d_{i+1}(s_j^{'}, s_{j+1}^{'}) = d(s_j, s_{j+1})$;
- Step5: Adjoin $(s_1, s_2^{'})$ to A and set $d(s_1, s_2^{'}) = flag$.

(b) Minimal VaR Path Algorithm

- Step0: Set $k = 1$;
- Step1: Determine the shortest path $l_k^{'}$ in $G_k(N_k, A_k)$;
  - If $l_k^{'}$ is not found in $G_k(N_k, A_k)$, stop, all paths have been determined;
  - Otherwise, identify $l_k^{'}$ with its corresponding path l_k in $G_1(N_1, A_1) - l_k$, as the kth shortest path in $G_1(N_1, A_1)$;
- Step2: If $k = |\hat{M}_\alpha|$, finish the algorithm; Otherwise go to step 3;
- Step3: Use one of the stated procedures to delete $l_k^{'}$ from $G_k(N_k, A_k)$, to obtain $G_{k+1}(N_{k+1}, A_{k+1})$;
- Step4: Set $k = k + 1$, return to Step 1.

2. Selecting Spatially Dissimilar VaR Paths

Define $\hat{M}_\alpha$ to be the set of candidate VaR paths with $|\hat{M}_\alpha| = \hat{m}$, $\hat{P}_\alpha$ to be a subset of $\hat{M}_\alpha$ with $|\hat{P}_\alpha| = \hat{p}_\alpha$, and $w_{\hat{i}\hat{j}}$ to be the distance between candidate paths $\hat{i}$ and $\hat{j}$. The $\hat{p}_\alpha$-dispersion problem can be expressed as follows:

$$\max_{\hat{P}_\alpha \subseteq \hat{M}_\alpha} \min_{\substack{\hat{i} \neq \hat{j} \\ \hat{i}, \hat{j} \in \hat{P}_\alpha}} w_{\hat{i}\hat{j}} \tag{14}$$

In our problem, $w_{\hat{i}\hat{j}}$ represents the dissimilarity index between any pair of paths.

$$w_{\hat{i}\hat{j}} = 1 - \frac{[d(\hat{i}, \hat{j})/l_{\hat{i}} + d(\hat{i}, \hat{j})/l_{\hat{j}}]}{2},$$

where l_i denotes the length of path i and $d(\cdot, \cdot)$ denotes the length of the shared portion of two paths.

The objective of the $\hat{p}_\alpha$-dispersion method is to maximize the minimum distance between any two of the selected paths. This problem can be solved in a two-phase heuristic, by constructing an initial solution in a semi-greedy fashion, and then perform a local search to improve the initial solution (Erkut 1990). Akgün et al. (2000) has done a detailed comparison with another three methods like IPM, GSP and Minimax. They empirically demonstrate that the p-disperse method is more effective in generating the set of dissimilar paths comparing with the other methods. However, a point worth noting is that, unlike the other three methods, the solution generated by this method does not necessarily include the least risk path. It may be difficult to persuade any decision maker to accept a set of routes that does not include the "best" path. This drawback can be improved by placing restrictions on the size of $|\hat{M}_\alpha|$ and $|\hat{P}_\alpha|$, or by enforcing constraints on the risk threshold.

5 Lagrangean Relaxation Solution Method

With the VaR value of each path and risk equity parameters of each path in each zone determined, this problem can be considered as an integer programming problem. It can be easily solved with software like CPLEX12.1 for small- or medium-sized road network. Table 2 displays the results of our computational experiments with Java1.6+CPLEX12.1 on a 3.40GHz CPU, 2.00GB RAM computer system:

Table 2: Computational Results From Data Sets By CPLEX

O-D Pairs	Paths for Each O-D Pair	Hazmat Types	Geographical Zone	Presolve Time (seconds)	Total Time
1	1	1	2	0.00	0.01seconds
2	2	2	2	0.00	0.01seconds
10	10	10	10	0.01	2seconds
20	20	20	20	0.02	4 minutes 24 seconds
30	30	30	30	0.03	102 minutes 2 seconds

In order to solve the hazmat VaR problem on a large-sized road network, we apply a branch and bound procedure, converting this problem into a simple traffic flow assignment problem by relaxing the risk equity constraint in a Lagrangean manner, and appending them to the objective function with Lagrangean multipliers (penalties) η_{ab}. We obtain the following Lagrangean relaxation problem:

$$g(\eta) = \min_x \sum_{h\in H}\sum_{i\in S}\sum_{j\in \hat{P}^i_\alpha} \beta'^{j(h)}_{\alpha i} x^{j(h)}_{\alpha i} - \sum_{a\in K}\sum_{b\in K} \mu\eta_{ab} \tag{15}$$

subject to

$$\sum_{j\in \hat{P}^i_\alpha} x^{j(h)}_{\alpha i} = n_{hi}, \quad \forall \quad i \in I, \quad \forall \quad h \in H \tag{16}$$

$$x^{j(h)}_{\alpha i} \quad \text{integer}, \quad \forall \quad j \in \hat{P}^i_\alpha \quad \forall \quad i \in I \quad \forall h \in H \tag{17}$$

where $\beta'^{j(h)}_{\alpha i} = \beta^{j(h)}_{\alpha i} + \sum_{a\in K}\sum_{b\in K}\eta_{ab}\{\hat{\beta}^h_{\alpha a}(i,j) - \hat{\beta}^h_{\alpha b}(i,j)\}$.

With a given set of Lagrangean multipliers, the relaxed problem equals to a traffic flow assignment problem and can be easily solved by picking j that minimize $\beta'^{j(h)}_{\alpha i}$ and set $x^{j(h)}_{\alpha i}$ to n_{hi}, and all other variables $x^{t(h)}_{\alpha i}$ such that $t \neq j, t \in \hat{P}^i_\alpha$ as 0. Usually Lagrangean relaxation can be used to generate a lower bound for the integer programming problem, however, the Lagrangean relaxation of our hazmat shipment problem owns the integrality property, that means, a solution to the Lagrangean relaxation is naturally integral. Therefore, the Lagrangean formulation can be replaced by the linear programming relaxation, i.e., the additional complexity of obtaining a bound from the Lagrangean dual leads to no improvement over the standard bound obtained from the linear programming relaxation of problem (1). In our solution procedure, we utilize linear programming relaxation of original problem (1) as lower bound, while applying Subgradient Search Algorithm (Held, Wolfe, and Crowder 1974) to obtain an upper bound of our problem. Whenever the solution obtained in the Lagrangean relaxation scheme is feasible to the original problem we compute its objective function value with the original objective coefficients and see if it provides an improved upper bound (i.e., it is better than the best solution found so far). The most widely used and successful methods for finding improved multipliers use the concepts of gradient and subgradient search. We start with all penalties $\{\eta^0_{ab}\}$ equal to zero, and at each iteration $r > 1$, update them as

$$\eta^{r+1} = \eta^r + \lambda_r \times \frac{\sum_{a\in K}\sum_{b\in K}\eta^r_{ab}\{\pi^h_a(i,j) - \pi^h_b(i,j)\} - \mu}{||\sum_{a\in K}\sum_{b\in K}\eta^r_{ab}\{\pi^h_a(i,j) - \pi^h_b(i,j)\} - \mu||}$$

where $\{\lambda_r\}$ is a sequence such that

$$\lambda_r \geq 0 \quad \forall r \tag{18}$$

$$\lim_{r \to \infty} \lambda_r = 0 \tag{19}$$

$$\sum_{r=1}^{\infty} \lambda_r = \infty \tag{20}$$

Our computational analysis in Section 6.2 demonstrates that for large-sized network as $1000 \times 10 \times 10 \times 10$ O-D pair $\times$ candidate paths $\times$ hazmat types $\times$ zones, the lagrangean relaxation solution can solve the problem within 5 minutes, with a gap within 0.5% from the LP bound.

In summary, the solution to the generalized Hazmat VaR problem can be summarized in the following flowchart Figure 1:

6 Case Study and Experimental Analysis

We develop our computational experience based upon the county of Albany, New York, and its nearby highway transportation network. We chose this region because it is a key junction of major interstates and is a hub of hazmat transportation activity. There are altogether seven Interstate and US routes traversing the Albany area and its neighborhoods Rensselaer, Saratoga and Montgomery: I-90, I-890, I-87, I-787, US-20, US-9 and US-9W. These exhibit a highly variant population density among these areas and a dense transportation network along those routes.

We divide this network into six geographical zones according to the nearby townships in Albany county – Knox, Guilderland, Colonie, Albany City, Bethlehem, and Coeymans. These townships exhibit a highly variant population density and the transportation network is dense throughout the county. The transportation network in Albany county was pruned to 46 nodes and 70 arcs. See Figure 2 for a map of the pruned network. Two types of hazmats were chosen for the purpose of risk estimation, Benzyl Chloride and Toluene. Different sets of data were developed, representing various combination of O-D pairs, geographical zones, different types of hazardous materials, and assumed radius of spread considered.

The link accident probabilities are calculated according to road segment lengths as stated in

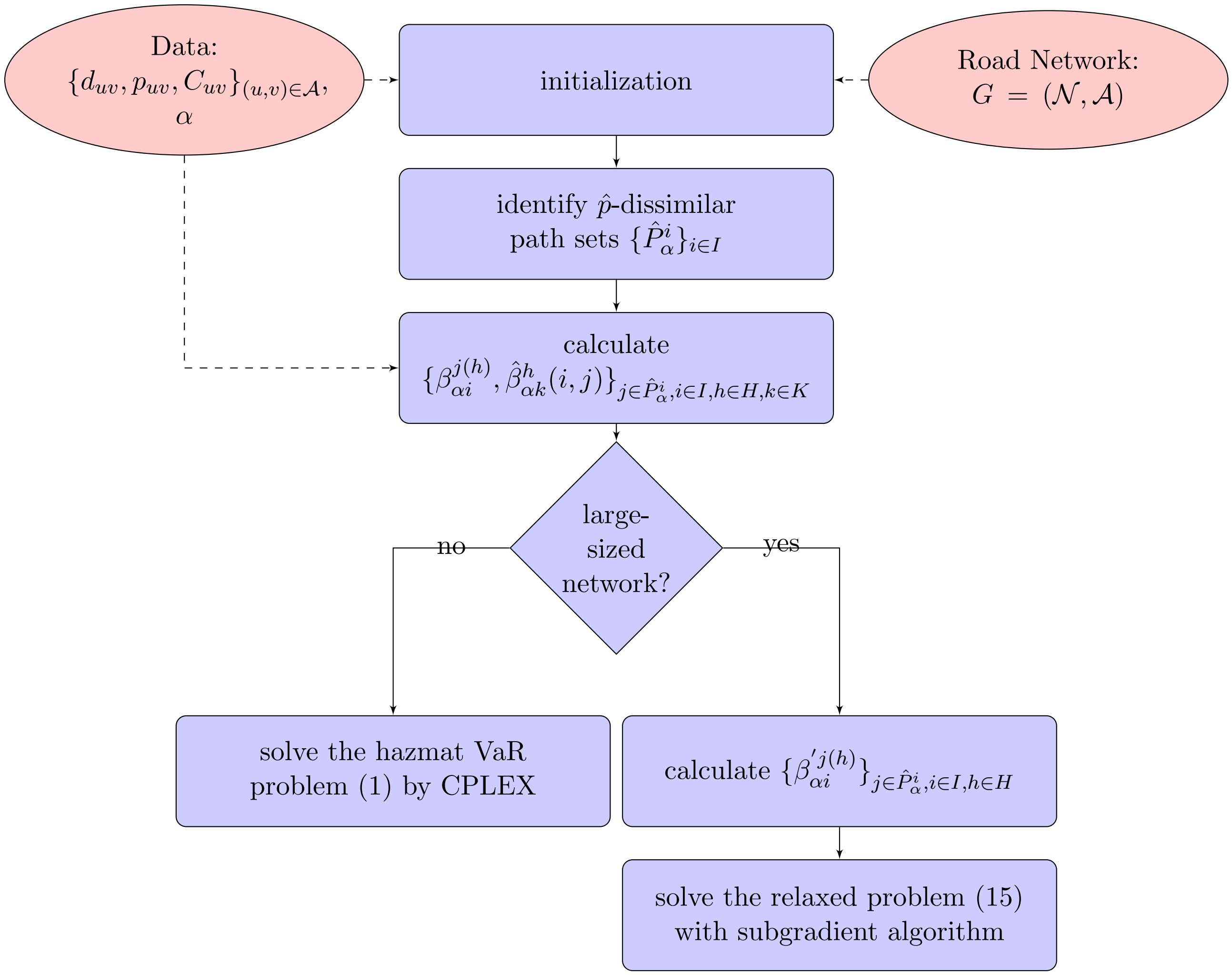

Figure 1: Generalized Hazmat VaR Problem Solution Flowchart

Abkowitz and Cheng (1988):

$$p = 10^{-6} \times l, \tag{21}$$

where l is in miles.

The link consequences are calculated according to population density within the neighborhood around the links. With different radii of spread λ from different hazmats, the endangered area can be described with a whole λ-neighborhood which is a concept developed by Batta and Chiu (1988). Here we simply compute the link consequence as the function:

$$C = (\pi\lambda^2 + 2 \times \lambda \times l) \times \rho, \tag{22}$$

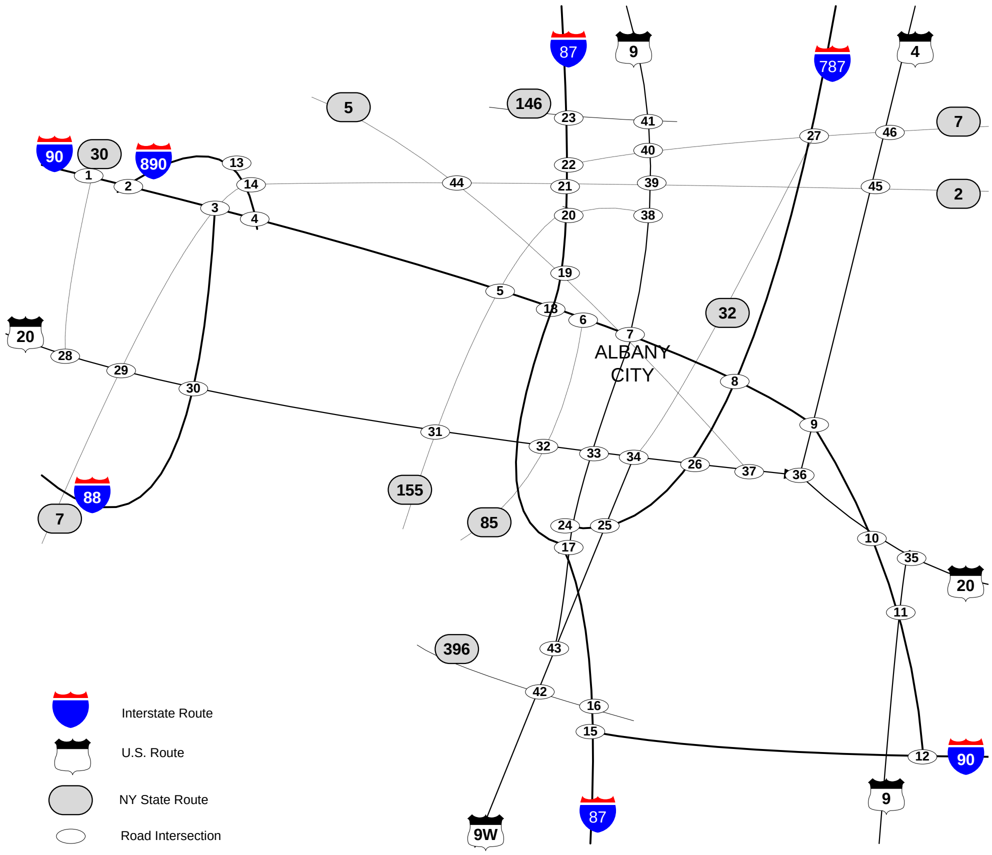

Figure 2: Albany Area Highway Roadmap

where ρ represents the population density in the neighborhood along the road segment (persons per mile-sq). We collect the population data of the town where the road segment is located and distribute the risk uniformly to the area. The road lengths and population statistics come from Department of Transportation and Department of Commerce websites.

6.1 Candidate Path Generation

Generation of dissimilar candidate set $\hat{P}_\alpha$ affects the computation quality and search speed of the route choices. We apply the modified p-dispersion method (Akgün et al. 2000) to generate $\hat{p}_\alpha$ dissimilar paths out of the $\hat{M}_\alpha$ minimal VaR path set for each O-D pair. We compare the effect of the size of the number of dissimilar paths selected, $|\hat{P}_\alpha|$, and the size of the number of candidate dissimilar paths, $|\hat{M}_\alpha|$. To do this comparison, we use four indicators: Average VaR value (AvVaR), Average Length (AvLen), Average Dissimilarity (AvDi) and Minimal Dissimilarity (MiDi).

6.1.1 Varying $\hat{P}_\alpha$ for Fixed $\hat{M}_\alpha$

We study the case of O-D pair (1,12). Table 3 displays the computational results for $\hat{p} = 2, 5, 10, 20$, from the same set of candidate path set $\hat{M}_{0.999994}$. The values of $|\hat{M}_{0.999994}|$ considered are 10, 20 and 30. For each candidate set $\hat{M}$, we have the same basic result: We obtain better solutions when $\hat{P}_\alpha$ is smaller.

Table 3: Computational Results for Dissimilar VaR Hazmat Problem Under $\alpha = 0.999994$, O-D Pair (1, 12)

# $\hat{M}$	# $\hat{P}$	MiVaR	Rank in $\hat{M}$	Gap (from Opt)	AvVaR	AvLen	AvDi	MiDi	Time for $\hat{M}$	Time for $\hat{P}$	Total Time (sec)
10	2	27240.34	1	0%	27240.34	45.79	38.68	38.68	31.44	0.41	32
	5	27240.34	1	0%	27240.34	45.73	11.24	20.88	31.84	0.51	32
20	2	27240.34	1	0%	31881.06	49.20	99.22	99.22	106.84	0.72	108
	10	27240.34	1	0%	41306.28	50.43	9.22	51.98	108.49	1.00	110
30	2	27240.34	1	0%	31881.06	49.20	99.22	99.22	256.17	1.08	257
	10	27240.34	1	0%	41306.28	50.43	9.22	50.82	257.53	1.41	259
	20	27240.34	1	0%	48435.11	47.01	2.23	45.74	255.78	1.70	258

6.1.2 Varying $\hat{M}_\alpha$ for Fixed $\hat{P}_\alpha$

Once again, we consider the case of O-D Pair (1,12). Table 4 compares the computational results for generating 10 dissimilar paths from 20, 30, 50 and 80 candidate paths $\hat{M}_{0.999994}$. From this table we conclude that a larger number of candidate paths yields a superior value of path dissimilarity.

However, a larger number of candidate paths also results in a large VaR value. We also observe that the generation of candidate paths is computationally expensive using the Removing Path

Algorithm. The decision maker needs to balance the computational efficiency and the quality of their objectives. A small number of candidate paths allows smaller VaR candidate paths and needs less computational effort, but may exclude good dissimilar paths. In contrast, a larger number of candidate paths needs more computational effort and may result in a large VaR path, but includes good dissimilar paths.

As shown in Table 4, the 10 out of 80 set generates solutions with large minimal path dissimilarity (MiDi), but its average path dissimilarity (AvDi) is rather small when compared with other data sets. This is because the objective of the p-dispersion method is to maximize the minimum distance between any two of the selected points, rather than to maximize the average distance. Besides, we note that the optimal solution to the p-dispersion problem does not necessarily contain the minimal VaR path (observe the result from 10 out of 80 set). This can be viewed as another drawback of the existing methods.

Table 4: Computational Results for Dissimilar VaR Hazmat Problem Under $\alpha = 0.999994$, O-D Pair (1, 12)

# $\hat{P}$	# $\hat{M}$	MiVaR	Rank in $\hat{M}$	Gap (from Opt)	AvVaR	AvLen	MiDi	AvDi	Time for $\hat{M}$	Time for $\hat{P}$	Total Time (sec)
10	20	27240.34	1	0%	41306.28	50.43	9.22	51.98	108.49	1.00	110
	30	27240.34	1	0%	41306.28	50.43	9.22	50.82	257.53	1.41	259
	50	27240.34	1	0%	41306.28	49.93	9.22	48.65	773.06	2.34	775
	80	74406.18	57	173.15%	74406.18	45.25	11.26	29.44	2058.36	3.36	2062

6.1.3 Ten O-D Pair Observation

Table 5 presents the 10 out of 30 p-dispersion solutions for ten different O-D pairs. In this computational experiment, we verify that, with rational size of candidate sets and dissimilar path sets, most O-D pairs can obtain the dissimilar path solution with relatively satisfactory quality within 5 minutes. Table 6 displays the ten dissimilar paths between O-D pair (1,35). More detailed solution of the ten sets of dissimilar paths as well as the risk they bring to their neighborhoods are displayed in Appendix B and compared with the optimal path of each O-D pair in Table 12.

Table 5: Computational Results for Dissimilar VaR Hazmat Problem Under $\alpha = 0.999994$, $|\hat{M}_{0.999994}| = 30, |\hat{P}_{0.999994}| = 10$

O-D Pair	MiVaR	Rank of MiVaR Path in $\hat{M}$	Gap	AvVaR	MiDi	AvDi	Total Time (sec)
(1,35)	27240.34	5	48.96%	35047.55	7.53	25.88	266
(23,15)	20917.48	1	0%	22464.71	9.45	50.41	139
(13,12)	41718.30	9	31.46%	42188.79	8.29	37.20	236
(2,15)	12572.11	1	0%	26866.89	11.46	45.86	209
(44,11)	42241.07	1	0%	42241.07	8.62	31.61	189
(22,12)	5569.55	1	0%	8012.68	11.49	58.59	73
(13,15)	42241.07	7	40.92%	42241.07	3.45	27.33	232
(2,10)	27240.34	4	48.95%	35047.55	9.98	35.64	181
(1,42)	12572.11	1	0%	28623.08	6.35	26.59	263
(23,12)	20917.48	1	0%	20917.48	16.07	39.75	154

Table 6: Ten Dissimilar Paths for O-D Pair (1,35) Under $\alpha = 0.999994$

O-D Pair	Dissimilar Path	Path VaR	Zone VaR					
			Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,35)	[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 35]	27240.34	11801.07	0	0	0	0	0
	[1, 2, 3, 4, 5, 6, 7, 8, 9, 36, 10, 35]	27240.34	0	0	0	0	0	0
	[1, 2, 13, 14, 4, 5, 6, 32, 33, 34, 26, 37, 36, 10, 35]	31733.24	0	0	0	0	0	0
	[1, 2, 3, 14, 4, 5, 6, 32, 33, 34, 26, 37, 36, 10, 35]	31733.24	7824.03	27240.34	0	0	0	0
	[1, 2, 3, 14, 4, 5, 6, 7, 33, 34, 26, 37, 36, 10, 35]	38754.73	0	0	0	0	0	0
	[1, 2, 3, 14, 4, 5, 6, 7, 8, 9, 10, 35]	38754.73	0	0	0	0	0	0
	[1, 2, 3, 14, 4, 5, 6, 7, 8, 9, 36, 10, 35]	38754.73	7824.03	27240.34	0	0	0	0
	[1, 2, 13, 14, 4, 5, 6, 7, 8, 9, 10, 35]	38754.73	0	0	0	0	0	0
	[1, 2, 13, 14, 4, 5, 6, 7, 8, 9, 36, 10, 35]	38754.73	0	0	0	0	0	0
	[1, 2, 13, 14, 4, 5, 6, 7, 33, 34, 26, 37, 36, 10, 35]	38754.73	0	0	44739.44	0	0	0

6.2 Solution for Generalized Hazmat VaR Problem

We use benzyl chloride and toluene as the two types of hazmat for the purpose of risk estimation, assuming a radius of spread of 1 mile and 3 miles (separately) once accidents occur. In order to compare the sensitivity of the size of the candidate set $\hat{P}_\alpha$ to the result, we build two sets of $\hat{P}_\alpha$, with 5 out of 30 and 10 out of 30 individually for each type of hazmat between each O-D pair in our case. In each of the six geographical zones, we recount the road segments of each path and calculate the corresponding VaR values they bring to each zone. The data sets are formed by using various combinations of dissimilar path sets of and types of hamzat shipped by the ten O-D pairs within the six geographical zones. Appendix A displays the path VaR values and zone VaR values of each candidate path of the 10 O-D pairs.

6.2.1 Impact of Size of Candidate Path Set

Table 7 compares the results generated by the dissimilar candidate path sets $\hat{P}_\alpha$, with 5 out of 30 and 10 out of 30 individually, under different risk threshold μ. From the table we observe that, the fewer paths allowed in the candidate path sets, the lower the total VaR we can get for the whole network. This is because small dissimilar path sets would exclude high risk paths from being selectable. But as we discussed in Section 6.1, large dissimilar path sets would allow generating a better risk dispersion solution.

6.2.2 Impact of Risk Equity Constraint μ

From Table 7, we have another important observation regarding the risk equity constraints. Too loose μ gives small global risk but will most likely lead to large risk difference in different geographical zones. By contrast, picking too tight values for μ gives a good risk dispersion solution, but may result in feasibility problems and may not permit the selection of low risk paths.

From the results shown in Table 8, we can see the optimal allocation of shipments on each candidate path under different μ thresholds. We notice that with stricter μ, more shipments are pushed to high risk paths in order to obtain better risk equity. With $\mu = 500000$, almost all shipments are assigned to the minimal VaR paths of the 10 O-D pairs. Table 9 and Table 10 give the cumulative VaR values in each geographical zone and the VaR difference between each zone pair. These two tables further demonstrate that the tight threshold ($\mu = 235000$) generates solutions with risk dispersed more evenly than a looser threshold ($\mu = 500000$). That means, $\mu = 235000$ gives much smaller VaR differences between any zone pair than $\mu = 500000$.

Table 7: Computational Results for Generalized VaR Hazmat Problem Under $\alpha = 0.999994$

Equity μ	Total VaR (# Candidate Paths/OD Pair = 5)	Total VaR (# Candidate Paths/OD Pair = 10)
50	infeasible	
50000	infeasible	
100000	infeasible	
235000	4048198.87	4062115.91
240000	4048198.87	4054794.92
245000	4040877.88	4054794.92
250000	4019875.28	4054794.92
255000	4012554.29	4054794.92
260000	3991551.68	4021941.83
265000	3991551.68	4021941.83
270000	3963228.09	3993618.23
275000	3963228.09	3993618.23
280000	3955907.10	3993618.23
285000	3934904.49	3993618.23
290000	3927583.50	3986297.25
295000	3906580.90	3986297.25
300000	3906580.90	3986297.25
350000	3872917.03	3964334.28
400000	3872917.03	3964334.28
450000	3872917.03	3964334.28
500000	3872917.03	3964334.28

6.2.3 Computational Analysis

Given the candidate path sets, with path VaR values and zone VaR values of each path preprocessed, CPLEX finds the solution quickly. On the average, it takes no more than 1 seconds to solve the problem. But as we discussed in Section 5, CPLEX solves the small-sized network problem efficiently, but for large-sized network, CPLEX is not a good solver for the VaR hazmat problem. The Lagrangean relaxation heuristic provides an effective method to solve the large-sized problem. Table 11 displays the computational results of Lagrangean relaxation and its gap from the Linear Programming lower bound, as well as comparison with the CPLEX computational effort on the original integer programming problem. It demonstrates the efficiency of Lagrangean relaxation as a solution method for this problem.

Table 8: Computational Results for Generalized VaR Hazmat Problem With 5 Candidate Path Sets for 10 O-D Pairs Under $\alpha = 0.999994$

Equity μ	Hazmat Type	Candidate Path	O-D Pair 1	2	3	4	5	6	7	8	9	10
235000	benzyl chloride	1	0	0	0	0	0	0	1	2	0	2
		2	0	0	0	0	0	5	0	0	0	0
		3	0	0	0	0	3	0	0	0	0	0
		4	0	0	0	4	0	0	0	0	0	0
		5	5	0	0	0	0	0	0	0	1	0
	toluene	1	1	2	0	1	4	0	5	2	0	0
		2	0	0	0	0	0	2	0	0	0	0
		3	0	0	0	0	0	0	0	0	0	0
		4	0	0	0	0	0	0	0	0	0	0
		5	1	0	0	0	0	0	0	0	0	0
			Total VaR = 4048198.87									
285000	benzyl chloride	1	4	0	0	0	0	0	1	2	0	2
		2	0	0	0	0	0	5	0	0	0	0
		3	0	0	0	0	3	0	0	0	0	0
		4	0	0	0	4	0	0	0	0	0	0
		5	1	0	0	0	0	0	0	0	1	0
	toluene	1	1	2	0	1	4	0	5	2	0	0
		2	0	0	0	0	0	2	0	0	0	0
		3	0	0	0	0	0	0	0	0	0	0
		4	0	0	0	0	0	0	0	0	0	0
		5	1	0	0	0	0	0	0	0	0	0
			Total VaR = 3934904.49									
500000	benzyl chloride	1	5	0	0	0	3	5	1	2	1	2
		2	0	0	0	0	0	0	0	0	0	0
		3	0	0	0	0	0	0	0	0	0	0
		4	0	0	0	4	0	0	0	0	0	0
		5	0	0	0	0	0	0	0	0	0	0
	toluene	1	2	2	0	1	4	2	5	2	0	0
		2	0	0	0	0	0	0	0	0	0	0
		3	0	0	0	0	0	0	0	0	0	0
		4	0	0	0	0	0	0	0	0	0	0
		5	0	0	0	0	0	0	0	0	0	0
			Total VaR = 3872917.03									

A potential problem of the Lagrangean relaxation approach lies in the fact that relaxing constraint set (3) in a Lagrangian fashion and solving the Lagrangian relaxation tends to result in all the shipments with the same O-D pair carrying the same type of hazmat on an identical route. It is intuitively clear that a solution with identical routes is undesirable for a multi-shipment prob-

Table 9: Zone VaR Comparison for Generalized VaR Hazmat Problem With 5 Candidate Path Sets for 10 O-D Pairs Under $\alpha = 0.999994$

Equity μ	Hazmat Type	Geographical Zone 1	2	3	4	5	6
235000	benzyl chloride	91478.50	0	39677.77	21721.90	0	0
	toluene	142059.95	195806.08	0	0	0	0
500000	benzyl chloride	158602.16	0	63484.44	21721.90	0	0
	toluene	155945.16	0	79555.79	0	0	0

Table 10: Zone Pair VaR Difference for Generalized Hazmat VaR Problem With 5 Candidate Path Sets for 10 O-D Pairs Under $\alpha = 0.999994$

Equity μ	Geographical Zone	Geographical Zone 1	2	3	4	5	6
235000	1	0	37732.37	193860.67	211816.55	233538.45	233538.45
	2	37732.37	0	156128.30	174084.18	195806.08	195806.08
	3	193860.67	156128.30	0	17955.87	39677.77	39677.77
	4	211816.55	174084.18	17955.87	0	21721.90	21721.90
	5	233538.45	195806.08	39677.77	21721.90	0	0
	6	233538.45	195806.08	39677.77	21721.90	0	0
500000	1	0	314547.32	171507.09	292825.42	314547.32	314547.32
	2	314547.32	0	143040.23	21721.90	0	0
	3	171507.09	143040.23	0	121318.33	143040.23	143040.23
	4	292825.42	21721.90	121318.33	0	21721.90	21721.90
	5	314547.32	0	143040.23	21721.90	0	0
	6	314547.32	0	143040.23	21721.90	0	0

lem, as it exposes population near the so-called optimal path multiple times and does not expose other areas of the network to any risk. But with good design of the dissimilar candidate path sets for each O-D pair carrying each type of hazmat, we have dispersed the risk brought by different candidate paths. Besides, a proper selection of the risk equity threshold μ can help direct the shipments between the same O-D pair carrying the same type of hazmat onto different dissimilar candidate paths as we observed in Table 8. In the model proposed by Gopalan et al. (1990b), they provide a heuristic solution procedure to avoid assigning all shipments to the same optimal path by iteratively using the solution procedure of the single-shipment problem.

Table 11: Computational Results Comparison Before vs. After Lagrangean Relaxation

# O-D Pairs	Objective Function Value			Gap	Total Time		
	IP	LB	UB	(UB vs. LB)	IP	LB	UB
100	10069.8	10069.48	10084.92	0.15%	3min 37sec	2min 42 sec	1min 4sec
200	19955.34	19954.87	19955.34	0.00%	15min 32sec	10min 40sec	1min 37sec
400	–	39568.62	39813.12	0.62%	–	38min 16sec	2min 9sec
800	–	80035.03	80481.75	0.56%	–	153min 39sec	3min 52sec
1000	–	99444.86	99914.45	0.47%	–	240min 40sec	4min 34sec

Note: IP – Integer Programming Solution by CPLEX
LB – Lower Bound by Linear Programming Relaxation
UB - Uppper Bound by Lagrangean Relaxation
– too slow to finish within allowable time
# of Paths for Each O-D Pair $\times$# of Hazmat Types $\times$# of Geographical Zones = $10 \times 10 \times 10$

7 Summary and Future Research

This paper aims to generate a generalized method for the multi-trip hazmat routing plan problem with a newly proposed VaR risk model and risk equity constraints. Different cases are discussed for both small-sized and large-sized road network, with corresponding solution methods provided. This generalized hazmat VaR problem represents the general hazmat routing problems in real life, as it allows multiple O-D pairs, multiple hazmat types and multiple shipments. It also allows for equity of risk consideration. The solution methods used include finding the k best VaR paths, p-dispersion, and lagrangean relaxation.

One direction of future research is to allow curfews and road bans to regulate hazmat movement. Our current research assumes that there is no time-varying impacts. Another direction for future research is to develop an efficient solution procedure for an alternative formulation with CVaR (Conditional VaR) model, which is a popular method in finance and economics.

Appendices

A Table of Optimal Path Sets for 11 O-D Pairs in Albany Case Study of Hazmat VaR Problem

Given any confidence level $\alpha \in (0, 1]$, the optimal path for a shipment between any O-D pair can be effectively found with the solution framework proposed by Kang et al. (2011). Table (12) displays the optimal paths for eleven O-D pairs.

Table 12: Computational Results for Single-Shipment Single-Path VaR Hazmat Problem Under $\alpha = 0.999994$

O-D Pair	Optimal Path	VaR value
(2,15)	[2, 3, 30, 31, 32, 33, 24, 17, 16, 15]	12572.11
(44,11)	[44, 22, 21, 20, 38, 7, 8, 9, 10, 11]	42241.07
(22,12)	[22, 39, 38, 7, 8, 9, 36, 10, 11, 12]	5569.55
(13,15)	[13, 14, 4, 5, 6, 32, 33, 34, 26, 25, 24, 17, 16, 15]	29973.77
(2,10)	[2, 3, 30, 31, 32, 33, 34, 26, 37, 36, 10]	18287.35
(1,42)	[1, 28, 29, 30, 31, 32, 33, 24, 17, 43, 42]	12572.11
(23,12)	[23, 22, 39, 38, 7, 33, 34, 26, 37, 36, 10, 11, 12]	20917.48
(23,15)	[23, 22, 39, 38, 7, 33, 24, 17, 16, 15]	20917.48
(1,35)	[1, 28, 29, 30, 31, 32, 33, 34, 26, 37, 36, 10, 35]	18287.39
(13,12)	[13, 14, 4, 5, 6, 32, 33, 34, 26, 37, 36, 10, 11, 12]	31733.24
(1,12)	[1, 28, 29, 30, 31, 32, 33, 34, 26, 37, 36, 10, 11, 12]	27240.34

B Table of Dissimilar Path Sets for 10 O-D Pairs in Albany Case Study of Hazmat VaR Problem

Table 13: Computational Results for Single-Shipment Multiple-Path Hazmat VaR Problem Under $\alpha = 0.999994$

O-D Pair	Dissimilar Path	Path VaR	Zone VaR Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(2,15)	[2, 3, 29, 30, 31, 32, 33, 34, 26, 25, 24, 17, 16, 15]	12572.11	0	0	7935.56	0	0	0
	[2, 3, 4, 5, 6, 32, 33, 24, 17, 16, 15]	27240.34	0	0	0	0	0	0
	[2, 3, 4, 5, 6, 7, 8, 26, 25, 24, 17, 16, 15]	27240.34	0	0	0	0	0	0
	[2, 3, 4, 5, 6, 7, 33, 24, 17, 43, 42, 16, 15]	27240.34	7824.03	27240.34	0	0	0	0
	[2, 3, 4, 5, 6, 32, 33, 24, 17, 43, 42, 16, 15]	27240.34	0	0	0	0	0	0
	[2, 3, 4, 5, 6, 7, 8, 26, 25, 24, 17, 43, 42, 16, 15]	27240.34	7824.03	27240.34	7935.56	0	0	0
	[2, 3, 14, 4, 5, 6, 32, 33, 34, 26, 25, 24, 17, 16, 15]	29973.76	7824.03	27240.34	0	0	0	0
	[2, 13, 14, 4, 5, 6, 32, 33, 34, 26, 25, 24, 17, 16, 15]	29973.76	0	0	0	0	0	0
	[2, 13, 14, 4, 5, 6, 32, 33, 34, 26, 25, 24, 17, 43, 42, 16, 15]	29973.76	0	0	7935.55	0	0	0
	[2, 3, 14, 4, 5, 6, 32, 33, 34, 26, 25, 24, 17, 43, 42, 16, 15]	29973.76	0	0	0	0	0	0

Continued on next page

Table 13 – continued from previous page

O-D Pair	Dissimilar Path	Path VaR	Zone VaR					
			Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(44,11)	[44, 22, 39, 38, 7, 33, 34, 26, 37, 36, 10, 11]	42241.07	0	0	0	0	0	0
	[44, 22, 39, 38, 7, 33, 34, 26, 37, 36, 10, 35, 11]	42241.07	0	0	0	0	0	0
	[44, 22, 21, 20, 38, 7, 8, 26, 37, 36, 10, 11]	42241.07	0	0	0	0	0	0
	[44, 22, 39, 38, 7, 8, 26, 37, 36, 10, 35, 11]	42241.07	0	0	0	0	0	0
	[44, 22, 39, 38, 7, 8, 9, 10, 35, 11]	42241.07	0	0	0	0	0	0
	[44, 22, 21, 20, 38, 7, 8, 9, 10, 35, 11]	42241.07	0	0	0	0	0	0
	[44, 22, 39, 38, 7, 8, 9, 36, 10, 11]	42241.07	0	0	0	0	0	0
	[44, 22, 39, 38, 7, 8, 9, 36, 10, 35, 11]	42241.07	0	0	0	0	0	0
	[44, 22, 21, 20, 38, 7, 8, 9, 36, 10, 11]	42241.07	0	0	0	0	0	0
	[44, 22, 40, 39, 38, 7, 8, 9, 10, 11]	42241.07	0	0	0	0	0	0
(22,12)	[22, 39, 38, 7, 8, 9, 36, 10, 35, 11, 12]	5569.55	0	0	0	0	0	0
	[22, 40, 39, 45, 9, 10, 11, 12]	7935.55	0	0	0	0	0	0

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Table 13 – continued from previous page

O-D Pair	Dissimilar Path	Path VaR	Zone VaR					
			Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
	[22, 40, 39, 45, 9, 10, 35, 11, 12]	7935.55	0	0	7935.55	0	0	0
	[22, 40, 27, 46, 45, 9, 10, 11, 12]	7935.55	0	0	7935.55	0	0	0
	[22, 40, 27, 46, 45, 9, 10, 35, 11, 12]	7935.55	0	0	7935.55	0	0	0
	[22, 21, 20, 38, 7, 8, 9, 10, 11, 12]	8003.23	0	0	0	0	0	0
	[22, 21, 20, 38, 7, 8, 9, 36, 10, 11, 12]	8003.23	0	0	0	0	0	0
	[22, 21, 20, 38, 7, 8, 9, 36, 10, 35, 11, 12]	8003.23	0	0	0	0	0	0
	[22, 21, 20, 38, 7, 8, 9, 10, 35, 11, 12]	8003.23	0	0	7935.55	0	0	0
	[22, 21, 20, 38, 7, 33, 34, 26, 37, 36, 10, 11, 12]	10802.15	0	0	0	0	0	0
(13,15)	[13, 44, 22, 21, 20, 38, 7, 8, 26, 25, 24, 17, 16, 15]	42241.07	19919.35	0	0	0	0	0
	[13, 44, 22, 21, 20, 38, 7, 8, 26, 25, 24, 17, 43, 42, 16, 15]	42241.07	0	0	0	0	0	0
	[13, 44, 22, 39, 38, 7, 8, 26, 25, 24, 17, 16, 15]	42241.07	0	0	0	0	0	0

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Table 13 – continued from previous page

O-D Pair	Dissimilar Path	Path VaR	Zone VaR					
			Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
	[13, 44, 22, 39, 38, 7, 8, 26, 25, 24, 17, 43, 42, 16, 15]	42241.07	0	0	0	0	0	0
	[13, 44, 22, 39, 38, 7, 33, 34, 26, 25, 24, 17, 43, 42, 16, 15]	42241.07	0	0	0	0	0	0
	[13, 44, 22, 40, 39, 38, 7, 33, 24, 17, 16, 15]	42241.07	0	0	0	0	0	0
	[13, 44, 22, 21, 20, 38, 7, 33, 24, 17, 16, 15]	42241.07	0	0	0	0	0	0
	[13, 44, 22, 21, 20, 38, 7, 33, 24, 17, 43, 42, 16, 15]	42241.07	0	0	0	0	0	0
	[13, 44, 22, 39, 38, 7, 33, 24, 17, 16, 15]	42241.07	0	0	0	0	0	0
	[13, 44, 22, 39, 38, 7, 33, 24, 17, 43, 42, 16, 15]	42241.07	0	0	0	0	0	0
(2,10)	[2, 3, 4, 5, 6, 7, 8, 9, 10]	27240.34	0	0	7935.55	0	0	0
	[2, 3, 4, 5, 6, 7, 8, 9, 36, 10]	27240.34	0	0	7935.55	0	0	0
	[2, 3, 14, 4, 5, 6, 32, 33, 34, 26, 37, 36, 10]	31733.23	0	0	7935.55	0	0	0
	[2, 13, 14, 4, 5, 6, 32, 33, 34, 26, 37, 36, 10]	31733.23	0	0	0	0	0	0
	[2, 3, 14, 4, 5, 6, 7, 8, 9, 10]	38754.72	0	0	7935.55	0	0	0
	[2, 13, 14, 4, 5, 6, 7, 8, 9, 10]	38754.72	0	0	0	0	0	0

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Table 13 – continued from previous page

O-D Pair	Dissimilar Path	Path VaR	Zone VaR					
			Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
	[2, 13, 14, 4, 5, 6, 7, 33, 34, 26, 37, 36, 10]	38754.72	0	0	0	0	0	0
	[2, 3, 14, 4, 5, 6, 7, 33, 34, 26, 37, 36, 10]	38754.72	0	0	7935.55	0	0	0
	[2, 3, 14, 4, 5, 6, 7, 8, 9, 36, 10]	38754.72	0	0	0	0	0	0
	[2, 13, 14, 4, 5, 6, 7, 8, 9, 36, 10]	38754.72	0	0	0	0	0	0
(1,42)	[1, 2, 3, 29, 30, 31, 32, 33, 34, 26, 25, 24, 17, 43, 42]	12572.11	0	0	7935.55	0	0	0
	[1, 2, 3, 4, 5, 6, 7, 33, 34, 26, 25, 24, 17, 43, 42]	27240.34	0	0	7935.55	0	0	0
	[1, 2, 3, 4, 5, 6, 32, 33, 34, 26, 25, 24, 17, 43, 42]	27240.34	0	0	7935.55	0	0	0
	[1, 2, 3, 4, 5, 6, 7, 33, 24, 17, 43, 42]	27240.34	0	0	0	0	0	0
	[1, 2, 3, 4, 5, 6, 32, 33, 24, 17, 43, 42]	27240.34	0	0	0	0	0	0
	[1, 2, 3, 4, 5, 6, 7, 8, 26, 25, 24, 17, 43, 42]	27240.34	0	0	0	0	0	0
	[1, 2, 3, 14, 4, 5, 6, 32, 33, 34, 26, 25, 24, 17, 43, 42]	29973.77	0	0	0	0	0	0
	[1, 2, 13, 14, 4, 5, 6, 32, 33, 34, 26, 25, 24, 17, 43, 42]	29973.77	0	0	0	0	0	0

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Table 13 – continued from previous page

O-D Pair	Dissimilar Path	Path VaR	Zone VaR					
			Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
	[1, 2, 3, 14, 4, 5, 6, 7, 33, 34, 26, 25, 24, 17, 43, 42]	38754.73	0	0	0	0	0	0
	[1, 2, 13, 14, 4, 5, 6, 7, 33, 34, 26, 25, 24, 17, 43, 42]	38754.73	0	0	0	0	0	0
(23,12)	[23, 22, 40, 27, 46, 45, 9, 36, 10, 11, 12]	20917.48271	0	0	0	0	0	0
	[23, 22, 40, 27, 46, 45, 9, 36, 10, 35, 11, 12]	20917.48	0	0	0	0	0	0
	[23, 22, 40, 27, 46, 45, 9, 10, 35, 11, 12]	20917.48	0	0	0	0	0	0
	[23, 22, 40, 27, 8, 26, 37, 36, 10, 11, 12]	20917.48	0	0	0	0	0	0
	[23, 22, 40, 27, 8, 26, 37, 36, 10, 35, 11, 12]	20917.48	0	0	0	0	0	0
	[23, 22, 40, 27, 8, 9, 10, 11, 12]	20917.48	0	0	0	0	0	0
	[23, 22, 40, 27, 8, 9, 10, 35, 11, 12]	20917.48	0	0	0	0	0	0
	[23, 22, 40, 27, 8, 9, 36, 10, 11, 12]	20917.48	0	0	0	0	0	0
	[23, 22, 40, 27, 8, 9, 36, 10, 35, 11, 12]	20917.48	0	0	44739.44	0	0	0
	[23, 22, 40, 39, 38, 7, 8, 26, 37, 36, 10, 11, 12]	20917.48	0	0	44739.44	0	0	0

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Table 13 – continued from previous page

O-D Pair	Dissimilar Path	Path VaR	Zone VaR					
			Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(23,15)	[23, 22, 21, 20, 38, 7, 8, 26, 25, 24, 17, 43, 42, 16, 15]	20917.48	0	27240.34	0	0	0	0
	[23, 22, 40, 39, 38, 7, 8, 26, 25, 24, 17, 43, 42, 16, 15]	20917.48	0	0	0	0	0	0
	[23, 22, 21, 20, 19, 18, 17, 43, 42, 16, 15]	20917.48	0	0	0	0	0	0
	[23, 41, 40, 39, 38, 7, 33, 24, 17, 16, 15]	23496.19	0	0	7935.55	0	0	0
	[23, 41, 40, 39, 38, 7, 33, 34, 26, 25, 24, 17, 16, 15]	23496.19	0	0	0	0	0	0
	[23, 41, 40, 27, 8, 26, 25, 24, 17, 16, 15]	23496.19	0	27240.34	0	0	0	0
	[23, 41, 40, 39, 38, 7, 33, 24, 17, 43, 42, 16, 15]	23496.19	0	27240.34	7935.55	0	0	0
	[23, 41, 40, 39, 38, 7, 8, 26, 25, 24, 17, 16, 15]	23496.19	0	27240.34	0	0	0	0
	[23, 41, 40, 39, 38, 7, 33, 34, 26, 25, 24, 17, 43, 42, 16, 15]	23496.19	0	0	7935.55	0	0	0
	[23, 41, 40, 27, 8, 26, 25, 24, 17, 43, 42, 16, 15]	23496.19	0	0	0	0	0	0
(1,35)	[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 35]	27240.34	11801.07	0	0	0	0	0

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Table 13 – continued from previous page

O-D Pair	Dissimilar Path	Path VaR	Zone VaR					
			Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
	[1, 2, 3, 4, 5, 6, 7, 8, 9, 36, 10, 35]	27240.34	0	0	0	0	0	0
	[1, 2, 13, 14, 4, 5, 6, 32, 33, 34, 26, 37, 36, 10, 35]	31733.24	0	0	0	0	0	0
	[1, 2, 3, 14, 4, 5, 6, 32, 33, 34, 26, 37, 36, 10, 35]	31733.24	7824.03	27240.34	0	0	0	0
	[1, 2, 3, 14, 4, 5, 6, 7, 33, 34, 26, 37, 36, 10, 35]	38754.73	0	0	0	0	0	0
	[1, 2, 3, 14, 4, 5, 6, 7, 8, 9, 10, 35]	38754.73	0	0	0	0	0	0
	[1, 2, 3, 14, 4, 5, 6, 7, 8, 9, 36, 10, 35]	38754.73	7824.03	27240.34	0	0	0	0
	[1, 2, 13, 14, 4, 5, 6, 7, 8, 9, 10, 35]	38754.73	0	0	0	0	0	0
	[1, 2, 13, 14, 4, 5, 6, 7, 8, 9, 36, 10, 35]	38754.73	0	0	0	0	0	0
	[1, 2, 13, 14, 4, 5, 6, 7, 33, 34, 26, 37, 36, 10, 35]	38754.73	0	0	44739.44	0	0	0
(13,12)	[13, 14, 4, 5, 6, 7, 8, 26, 37, 36, 10, 11, 12]	41718.30	0	0	7935.55	0	0	0
	[13, 44, 22, 21, 20, 38, 7, 8, 26, 37, 36, 10, 11, 12]	42241.07	0	0	7935.55	0	0	0
	[13, 44, 22, 21, 20, 38, 7, 8, 26, 37, 36, 10, 35, 11, 12]	42241.07	0	0	7935.55	0	0	0

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Table 13 – continued from previous page

O-D Pair	Dissimilar Path	Path VaR	Zone VaR					
			Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
	[13, 44, 22, 40, 39, 38, 7, 33, 34, 26, 37, 36, 10, 11, 12]	42241.07	0	0	7935.55	0	0	0
	[13, 44, 22, 21, 20, 38, 7, 8, 9, 36, 10, 35, 11, 12]	42241.07	0	0	7935.55	0	0	0
	[13, 44, 22, 40, 39, 38, 7, 8, 9, 36, 10, 11, 12]	42241.07	0	0	0	0	0	0
	[13, 44, 22, 40, 39, 38, 7, 8, 9, 36, 10, 35, 11, 12]	42241.07	0	0	0	0	0	0
	[13, 44, 22, 40, 39, 38, 7, 8, 9, 10, 35, 11, 12]	42241.07	0	0	0	0	0	0
	[13, 44, 22, 40, 39, 38, 7, 33, 34, 26, 37, 36, 10, 35, 11, 12]	42241.07	0	0	0	0	0	0
	[13, 44, 22, 40, 39, 38, 7, 8, 26, 37, 36, 10, 11, 12]	42241.07	0	0	0	0	0	0

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