

Risk-Averse Network Design with Behavioral Conditional Value-at-Risk for Hazardous Materials Transportation

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December 12, 2018

Abstract

We consider a road-ban problem in hazardous materials (hazmat) transportation. We formulate the problem as a network design problem to select a set of closed road segments for hazmat traffic and obtain a bi-level optimization problem. While modeling probabilistic route-choices of hazmat carriers by the random utility model (RUM) in the lower level, we consider a risk-averse measure called conditional value-at-risk (CVaR) in the upper level, instead of the widely used expected risk measure. Using RUM and CVaR, we quantify the risk of having hazmat accidents and large consequences, and design the network policy for road-bans accordingly. While CVaR has been used in hazmat routing problems, this paper is the first attempt to apply CVaR in risk averse hazmat network design problems considering stochastic route-choices of hazmat carriers. The resulting problem is a mixed integer nonlinear programming problem, for which we devise a line search approach combined with Benders decomposition. We demonstrate the efficiency of the proposed computational method with case studies. The average computation time for a network with 105 nodes and 268 arcs is 3 hours. Commercial solvers are inadequate to solve this problem, because the optimality gap is 99.9% after 24 hours just for a linear subproblem. By applying CVaR to the route-choice behavior of hazmat carriers, we protect the road network from undesirable route-choices that may lead to severe consequences. We define the Value of RUM-CVaR Solutions (VRCS) over the deterministic model based on shortest-path problems and the expected risk measure. Our case study shows that VRCS can range from 4.9% to 64.1% depending on the probability threshold used in the CVaR measure.

Keywords: transportation; hazardous materials; network design; conditional value-at-risk; Benders decomposition

1 Introduction

Hazardous materials (hazmat) are defined as materials that can pose an unreasonable threat to the public and the environment (Federal Motor Carrier Safety Administration, 2016) and about

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1 million shipments of hazmat crisscross the United States every day. While the *average* nature of hazmat accidents on highways is not very different from non-hazmat cargo accidents, hazmat accidents can bring in catastrophic consequences. The average cost for hazmat accident on nation's highways is about \$414,000 per accident, while non-hazmat cargo accidents are averaged about \$340,000 per accident (Federal Motor Carrier Safety Administration, 2001). When hazmat is released at the accident, the average cost increases to \$536,000. Furthermore, if a fire or an explosion is involved, the average cost of hazmat accidents increases to \$1,150,000 and \$2,070,000, respectively. An extreme example is an hazmat accident in 2017 causing damages of \$4,273,606 at Highway 410, Detroit, TX (Pipeline and Hazardous Materials Safety Administration, 2017). Hazmat accidents exhibit the characteristics of the low-probability high-consequence events. Reducing the impact of hazmat accidents via risk-averse approaches is important for the public safety and the environment protection.

On account of the large amount of hazmat transported and high accident consequences of hazmat trucks via roads, the government and transportation agencies often consider road-ban policies to protect the public and the environment from severe accident consequences of hazmat. In a road-ban policy for hazmat transportation, the government can close certain road segments for hazmat traffic. The decision-making problem of determining which road segments to close is called a hazmat network design problem (Verter and Kara, 2008; Sun et al., 2016). In addition, the government can design toll policies to regulate hazmat transportation (Marcotte et al., 2009; Esfandeh et al., 2016). While toll pricing can provide more flexible regulatory methods, it is easier to implement and modify road-ban policies without needs of additional toll-collection facilities. The current registry of hazmat route restriction on the U.S. highways is provided by the Federal Motor Carrier Safety Administration (2018).

For hazmat network design problems, modeling and predicting route choices of carriers is essential to determine the risk associated with transporting hazmat. Typically, hazmat network design problems for road-ban are formulated as bi-level optimization problems (Kara and Verter, 2004; Erkut and Gzara, 2008; Gzara, 2013; Fontaine and Minner, 2018; Sun et al., 2016, 2018). The upper level *selects* a set of road segments to be closed aiming at minimizing the risk level of hazmat transportation in the network. The lower level *predicts* the carriers' routes of transporting hazmat from origin-destination (OD) pairs. Most existing studies on hazmat transportation utilize the shortest path problem to model the route choices (Kara and Verter, 2004; Erkut and Gzara, 2008; Gzara, 2013; Fan et al., 2015; Esfandeh et al., 2017; Fontaine and Minner, 2018). In the lower-level shortest-path problem, the cost of carriers can be the travel time or a combination of the travel time and risk (List et al., 1991; Taslimi et al., 2017). There are, however, other factors that hazmat carriers consider for route choices. According to a hazmat routing survey (Battelle, 2006), hazmat carriers consider various factors such as tunnels, bridges, the population exposure, the state of regulations, and the directness of route, among others, to determine the route. This indicates that there would be factors that are *unobserved* by the government or a central authority. Even when multiple factors are modeled, the weights among various factors are difficult to determine.

Probabilistic approaches for modeling unobservable factors in route choice decision-making are abundant. The most popular model is arguably the Random Utility Model (RUM), which directly relates the probability of a route choice with the its utility. McFadden (1975) first proposed RUM to model general choice behaviors. In RUM, it is assumed that users’ utility depends on both a fixed effect and a random observation error. Williams (1977) proposed the Multinomial Logit (MNL) model by assuming that the observation errors are from Gumbel distribution as a system evaluation criterion. To model route choices of drivers in urban road networks, Daganzo and Sheffi (1977) presented the Multinomial Probit (MNP) model assuming that the observation errors are normally distributed. MNP model introduces a lack of tractability for researchers to perform further analysis, because it cannot provide an explicit formula which relates choice probabilities and known factors. The simple explicit form of MNL makes it incorporable with further analysis while describing users’ stochastic behavior. By using MNL in transportation, the route choice probabilities can directly relate to route costs. For general freight movements, RUM and its variants have been used to model route choices using GPS tracking data (Quattrone and Vitetta, 2011; Hess et al., 2015).

Despite the popularity and effectiveness of RUM in modeling probabilistic route choices of drivers, RUM has not been used in the hazmat transportation problems, in particular hazmat network design problems. To take account of unobservable factors in drivers’ route choices, Sun et al. (2018) proposed a suboptimal decision-making model based on satisficing and robust optimization for hazmat network design problems. This paper provides the first hazmat network design model considering probabilistic route choices.

We illustrate the importance of considering probabilistic route choices via a simple example with three arcs. We need to transport hazmat from node 1 to node 2. The data of travel cost and risk for three arcs are as follows:

Arc	Travel Cost	Hazmat Risk
1	10	1.0
2	5	1.1
3	11	50.0

Assume that we can close only one arc. The results of road-ban with the shortest-path problem (SPP) and probabilistic route choice are shown in Figure 1. Arc 1 is the only minimum risk arc (path) and the travel cost of arc 1 is higher than arc 2. With SPP to predict the hazmat routing, the optimal solution is to close arc 2. It assumes that hazmat carriers will only follow arc 1, because it provides the shortest path after closing arc 2. Note that the travel costs of arc 3 and arc 1 are very close, while arc 3 has a large risk of 50. In reality, hazmat carriers can choose arc 3 escalating the estimated risk that SPP captures. Under probabilistic route choice models, hazmat carriers are assumed to choose paths with some probabilities based on utilities, which can be represented by travel cost or other observed and unobserved factors. The optimal solution with probabilistic route choice is to close arc 3. Since the risk of arc 2 is 1.1 and the risk of arc 1 is 1, the risk for this

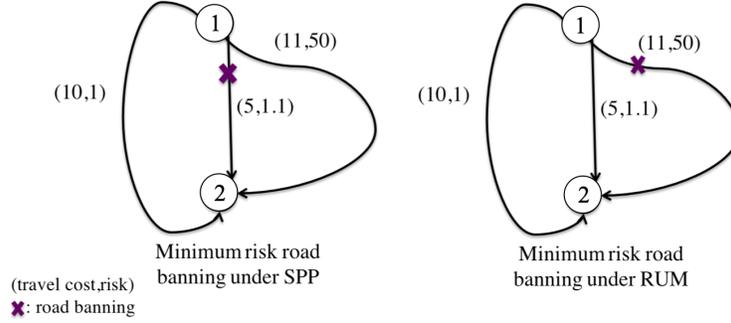


Figure 1: Road banning with SPP and probabilistic route choice models

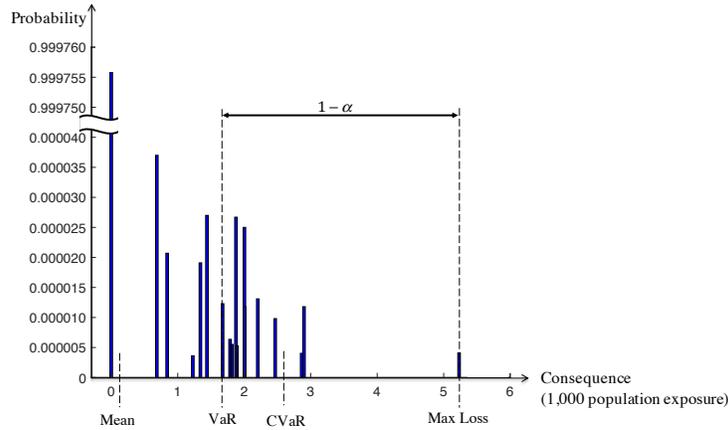


Figure 2: VaR and CVaR for a network only including a path (Su et al., 2017)

network is still around 1 considering all available paths. Illustrated by this example, the road-ban solution with probabilistic route choice can be more preferable than SPP, which motivates this paper.

The property of low probabilities but extreme accident consequences for hazmat transportation motivates researchers to consider an aversive risk measure when quantifying the risk to avoid catastrophic consequences (Erkut and Ingolfsson, 2000). Most hazmat transportation network designs consider simple risk measures such as the expectation of accident consequences. In risk management, value-at-risk (VaR), also known as α -quantile, once was commonly used to measure risk ignoring the left tail of loss distribution. Its lack of subadditivity and convexity, as discussed by Artzner et al. (1997, 1999), however, leads researchers' attention to a coherent measure: conditional value-at-risk (CVaR). While both VaR (Duffie and Pan, 1997) and CVaR (Rockafellar and Uryasev, 2000) have been popularly used in financial portfolio optimization problems, they have also been applied to hazmat routing (Kang et al., 2014; Toumazis et al., 2013; Toumazis and Kwon, 2013, 2016; Hosseini and Verma, 2018).

Figure 2 shows the mean value, VaR, CVaR, and the maximum loss value for the random risk of a typical path in a hazmat transportation network. While VaR only captures a quantile, CVaR

Table 1: Have *risk-averse* approaches been used in hazmat transportation problems? RO represents robust optimization. The ‘Data’ column represents uncertainty or inaccuracy in data for hazmat accident probability and consequence in each road segment.

Paper	Source of Uncertainty			
	Route-Choice	Accident Consequence	Data	Context
Erkut and Ingolfsson (2000)	-	Max Loss	-	Routing
Kang et al. (2014)	-	VaR	-	Routing
Toumazis et al. (2013)	-	VaR/CVaR	-	Routing
Kwon et al. (2013)	-	-	RO	Routing
Toumazis and Kwon (2016)	-	CVaR	RO	Routing
Sun et al. (2016)	-	-	RO	Network Design
Sun et al. (2018)	RO	-	-	Network Design
This Paper	CVaR	CVaR	-	Network Design

considers the expected risk (ER) beyond VaR; hence CVaR provides more risk-averse approach for mitigating tail risks. Both VaR and CVaR can be flexibly determined between the mean value and the maximum loss, depending on the probability threshold value α .

Our main contribution is that we introduce a *risk-averse* CVaR measure to both *probabilistic behavior* of hazmat carriers and *probabilistic consequences* from hazmat accidents in hazmat network design problems. To the best of our knowledge, this paper is the first attempt to mitigate both factors via an averse risk measure. While RUM is used in some urban network design problems (Davis, 1994; Liu and Wang, 2015), this is the first time to incorporate RUM in hazmat transportation network design problems. CVaR is an averse risk measure that focuses on high consequences. While CVaR has been used in hazmat routing, this is the first time for the hazmat network design problem. The CVaR measure captures high consequences stemming from probabilistic route choices of hazmat carriers as well as the nature of hazmat accidents.

Table 1 further highlights our main contribution and shows the differences between our work and other available risk-averse approaches in the literature. Risk-averse approaches focus on three sources of uncertainty in the literature: route-choice, accident consequence, and data. For uncertain route choices of hazmat carriers, Sun et al. (2018) considered their worst-case behavior using the notion of bounded rationality to derive a robust network design, while the ER as a risk-neutral measure was used to evaluate the risk from hazmat accidents. To overcome the limitation in the risk-neutral ER measure of accident consequences, however, VaR and CVaR have been used for hazmat routing (Toumazis et al., 2013; Toumazis and Kwon, 2016). In this paper, using CVaR, we consider both sources of uncertainty in route choices and accident consequences under the network design setting for the first time.

Most operations research approaches for hazmat routing assume the availability of two critical data: accident probability and accident consequence. In practice, those data are rough estimates, usually computed from the national average, if not unavailable. To manage risk from such data

uncertainty, robust optimization approaches have been used (Kwon et al., 2013; Toumazis and Kwon, 2016; Sun et al., 2016). In this paper, however, we assume that hazmat accident probabilities and consequences at each road segment are available. Considering all three sources of uncertainty will clearly be a next step.

We analyze the proposed CVaR minimization problem for hazmat network design theoretically and develop an efficient algorithm that combines line search with Benders decomposition to solve the problem. In addition, we provide case studies on realistic road networks to confirm the validity of CVaR concept incorporating probabilistic-route choices and the practicability of the proposed algorithms.

2 A Deterministic Model for Hazmat Network Design

In this section, we review a deterministic model for hazmat transportation network design. Later, we extend the deterministic model to consider CVaR and uncertain route choices.

Let us consider a transportation network $G = (\mathcal{N}, \mathcal{A})$ where \mathcal{N} is the set of nodes and \mathcal{A} is the set of arcs. In a multi-commodity transportation network, let \mathcal{S} denote the set of shipments. In practice, S specifies the OD pair, and the type of hazmat. Let N^s be the demand of shipment $s \in \mathcal{S}$ that represents the number of trucks carrying hazmat. Each arc (i, j) is known with the travel cost t_{ij} , the accident probability p_{ij} , and the accident consequence c_{ij}^s for each shipment $s \in \mathcal{S}$. Accidents caused by various kinds of hazmat can have different influences on a road network making it possible that different shipments can have different accident consequences. Let \mathcal{K}_s be the set of available paths for shipment $s \in \mathcal{S}$.

To transport shipment $s \in \mathcal{S}$, the approximated risk distribution for a single demand (truck) along path $k \in \mathcal{K}_s$ can be written as follows (Jin and Batta, 1997):

$$\Pr\{R^{sk} = x\} \approx \begin{cases} 1 - \sum_{(i,j) \in \mathcal{A}^k} p_{ij} & \text{if } x = 0 \\ p_{ij} & \text{if } x = c_{ij}^s \text{ for some } (i, j) \in \mathcal{A}^k \end{cases} \quad (1)$$

where \mathcal{A}^k is the set of arcs for path k .

One of the most common approaches that regulators use to measure the risk is expected value of consequences for potential hazmat truck accidents. It is a common assumption that hazmat carriers travel along the shortest path. We also assume that hazmat carriers only follow the shortest path in the deterministic model for hazmat transportation network design. Erkut and Gzara (2008) solved a bi-level hazmat transport network design problem based on an arc-based formulation. Verter and Kara (2008) proposed a path-based approach for hazmat transport network design by simplifying the shortest path problem with the closest assignment constraints. Similarly, the deterministic

path-based hazmat transportation network design is formulated as follows:

$$\min_{y,z} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} \sum_{(i,j) \in \mathcal{A}} N^s p_{ij} \delta_{ij}^{sk} c_{ij}^s \gamma^{sk} \quad (2)$$

$$\text{s.t. } z^{sk} \geq \sum_{(i,j) \in \mathcal{A}} \delta_{ij}^{sk} y_{ij} - \sum_{(i,j) \in \mathcal{A}} \delta_{ij}^{sk} + 1, \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}_s \quad (3)$$

$$z^{sk} \leq y_{ij} - \delta_{ij}^{sk} + 1, \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}_s, \forall (i,j) \in \mathcal{A} \quad (4)$$

$$\sum_{k \in \mathcal{K}_s} z^{sk} \geq 1, \quad \forall s \in \mathcal{S} \quad (5)$$

$$\gamma^{sk} \leq z^{sk}, \quad \forall s \in \mathcal{S}, k \in \mathcal{K}_s \quad (6)$$

$$\gamma^{sk} \geq z^{sk} - \sum_{j=1}^{k-1} z^{sj}, \quad \forall s \in \mathcal{S}, k \in \mathcal{K}_s \quad (7)$$

$$\sum_{(i,j) \in \mathcal{A}} (1 - y_{ij}) \leq N \quad (8)$$

$$\gamma^{sk}, z^{sk} \text{ binary}, \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}_s \quad (9)$$

$$y_{ij} \text{ binary}, \quad \forall (i,j) \in \mathcal{A} \quad (10)$$

where y is the design variable, z is the path availability variable and γ is the route-choice variable. If arc (i, j) is open for transportation of hazmat, $y_{ij} = 1$; otherwise, $y_{ij} = 0$. If path k is available for transportation of shipment $s \in \mathcal{S}$, $z^{sk} = 1$; otherwise, $z^{sk} = 0$. If path k is chosen for transportation of shipment $s \in \mathcal{S}$, $\gamma^{sk} = 1$; otherwise, $\gamma^{sk} = 0$. In addition, δ_{ij}^{sk} is the parameter to define a path. If $\delta_{ij}^{sk} = 1$, arc (i, j) is on path k for shipment s ; otherwise, $\delta_{ij}^{sk} = 0$.

In the single-level problem by a path-based formulation, the objective minimizes the ER as (2) shows. Path-based network design constraints are defined by (3)–(10). Constraints (3) and (4) define path availability for shipments. A path is available only when all arcs on the path are open. If there exist closed arcs on a path, the path is out of service. In addition, at least one path for a shipment is available to ensure transportation as (5) shows. Constraints (6) state that the chosen path for shipments must come from available paths. All paths for a shipment are sorted from 1 to k by lengths meaning that the length of path 1 for any shipment has shortest length among all possible paths. Constraints (7) guarantee that the available path with the smallest index is used for each shipment. Because of the sorted path data, (7) is equivalent to the shortest path problem in a path-based context. Due to the cost associated with closing arcs, (8) restricts the number of closed arcs. Constraints (9) and (10) are binaries for decision variables. The path-based hazmat transportation network design problem is a mixed-integer linear programming (MILP) problem.

3 Hazmat Risk Modeling with Probabilistic Route Choices

There are works that model the risk distribution for a hazmat transportation network by using shortest path problems. The route choice behavior of hazmat carriers, however, is not only resulted

from known factors such as travel cost (Ben-Akiva et al., 1984) thus making it possible that the shortest path problem (SPP) and other hazmat routing optimization may fail to predict the routing decision. To consider the uncertainty of route choices, probabilistic route choice models are used. In probabilistic route choice models, hazmat carriers choose an available path with a probability. The risk distribution for a hazmat transportation network is redefined to incorporate with probabilistic route choices. In this section, probabilistic route choice models are reviewed and utilized in risk distribution for hazmat transportation network.

3.1 Random Utility and Probabilistic Route Choice Models

RUM assumes that the utility of a choice that decision makers perceive comes from two sources: a deterministic (observable) component and a random (unobservable) component (Dial, 1971). In the context of route choices, the utility U^{sk} of path k for shipment $s \in \mathcal{S}$ is defined by:

$$U^{sk} = -\theta^s t^{sk} + \xi^{sk} \quad (11)$$

where t^{sk} is the generalized cost of observable attributes, θ^s is a positive parameter, and ξ^{sk} is a random variable for unobservable attributes. To consider both travel cost and risk in hazmat routing, the utility can also be formulated as:

$$U^{sk} = -\theta^s (t^{sk} + \beta \cdot \text{risk}^{sk}) + \xi^{sk} \quad (12)$$

Usually, t^{sk} is travel time. It is assumed to be additive with respect to arc costs.

$$t^{sk} = \sum_{(i,j) \in \mathcal{A}} t_{ij} \delta_{ij}^{sk} \quad (13)$$

where t_{ij} is the generalized travel cost associated with arc (i, j) , and $\delta_{ij}^{sk} = 1$ if arc (i, j) is on path k for shipment $s \in \mathcal{S}$ and 0 otherwise. Note that risk^{sk} can be the expected risk of path k for shipment $s \in \mathcal{S}$ or any other risk measure depending on their attitudes towards risk.

Different distributions for random components ξ^{sk} result in various forms of probabilities π^{sk} of choosing path $k \in \mathcal{K}_s$ for shipment $s \in \mathcal{S}$. By assuming that the random component ξ^{sk} are independently and identically from Gumbel distribution, the MNL model can be obtained as follows (Ben-Akiva et al., 1985):

$$\pi^{sk} = \frac{\rho^{sk}}{\sum_{l \in \mathcal{K}_s} \rho^{sl}} \quad (14)$$

$$\rho^{sk} = e^{-\theta^s (t^{sk} + \beta \cdot \text{risk}^{sk})} \quad (15)$$

for all $s \in \mathcal{S}, k \in \mathcal{K}_s$.

There exist other logit-type models with different formulations of ρ^{sk} (Cascetta et al., 1996; Ben-Akiva and Bierlaire, 1999; Ramming, 2001; Prashker and Bekhor, 2004). In C-logit model,

for example, a commonality factor is introduced while a path size is defined in path-size logit model. Path size is calculated based on the length of arcs within a path and the relative lengths of paths that share an arc. Both the commonality factor and the path size are used to measure the similarity among paths. They are used to adjust the utilities of paths and address issues caused by overlapping arcs. To obtain the commonality factor and the path size, however, we need to know the path set \mathcal{K}_s for shipment $s \in \mathcal{S}$ beforehand. Therefore, C-logit model and path-size model are computationally expensive to be applied in the hazmat network design problem. We use MNL of the form (14)–(15) to model the probabilistic route choices.

3.2 The Risk Distribution for Hazmat Transportation

In this section, the risk distribution for hazmat transportation is defined based on the probabilistic route choice model. Various shipments $s \in \mathcal{S}$ can have different accident consequences. Let \mathcal{A}^k denote the set of arcs for path $k \in \mathcal{K}_s$ to transport shipment $s \in \mathcal{S}$. It is assumed that hazmat carriers operate independently. Among N^s demands of hazmat for shipment $s \in \mathcal{S}$, demand (truck) 1 and demand (truck) 2 have the same risk distribution along path $k \in \mathcal{K}_s$. Choosing path $k \in \mathcal{K}_s$ to transport shipment $s \in \mathcal{S}$, the risk distribution for n -th truck can be approximated as follows (Jin and Batta, 1997):

$$R_n^{sk} = \begin{cases} 0 & \text{with probability } 1 - \sum_{(i,j) \in \mathcal{A}^k} p_{ij} \\ c_{ij}^s & \text{with probability } p_{ij} \text{ for } (i,j) \in \mathcal{A}^k \end{cases} \quad (16)$$

When there are multiple paths available for each truck to transport shipment $s \in \mathcal{S}$, we assume that a path is chosen with the probability described by the probabilistic route choice model introduced in Section 3.1. Let R_n^s be the random risk variable for n -th truck to transport $s \in \mathcal{S}$, distributed among all available paths in \mathcal{K}_s . Under the consideration of available paths, the probability of taking risk x of shipment $s \in \mathcal{S}$ by n -th truck is:

$$\Pr \left[R_n^s = x \right] = \sum_{k \in \mathcal{K}_s} \Pr \left[R_n^s = x \mid \text{path } k \text{ chosen} \right] \Pr \left[\text{path } k \text{ chosen for shipment } s \right] \quad (17)$$

$$= \sum_{k \in \mathcal{K}_s} \Pr \left[R_n^{sk} = x \right] \pi^{sk} \quad (18)$$

where π^{sk} is given in (14). Hence, R_n^s is distributed as follows:

$$R_n^s = \begin{cases} 0 & \text{with probability } 1 - \sum_{k \in \mathcal{K}_s} \sum_{(i,j) \in \mathcal{A}^k} \pi^{sk} p_{ij} \\ c_{ij}^s & \text{with probability } p_{ij} \sum_{k \in \mathcal{K}_s} \pi^{sk} \delta_{ij}^{sk} \text{ for } (i,j) \in \bigcup_{k \in \mathcal{K}_s} \mathcal{A}^k \end{cases} \quad (19)$$

where δ_{ij}^{sk} is the incidence parameter for $s \in \mathcal{S}, k \in \mathcal{K}_s, (i,j) \in \mathcal{A}$. If $\delta_{ij}^{sk} = 1$, arc (i,j) is on

path k for shipment s ; if $\delta_{ij}^{sk} = 0$, arc (i, j) is not on path k for shipment s . The risk for a given transportation network comes from all demands among all shipments. Therefore, the risk for a transportation network is:

$$R = \sum_{s \in \mathcal{S}} \sum_{n=1}^{N^s} R_n^s \quad (20)$$

Since different trucks are operated separately transporting multiple units of hazmat, we can assume that the risks for multiple units of hazmat among all shipments are independently distributed. According to the North America data on hazmat transportation accident statistics, the probabilities of an accident to take place are very small ranging from 10^{-8} to 10^{-6} (Abkowitz et al., 1992). Utilizing

$$p_{ij} p_{i'j'} \approx 0 \quad (21)$$

for all $(i, j), (i', j') \in \mathcal{A}$, we can obtain the probability that the risk variable becomes 0 as follows:

$$\begin{aligned} \Pr [R = 0] &= \prod_{s \in \mathcal{S}} \prod_{n=1}^{N^s} \Pr [R_n^s = 0] \\ &= \prod_{s \in \mathcal{S}} \prod_{n=1}^{N^s} \left(1 - \sum_{k \in \mathcal{K}_s} \sum_{(i,j) \in \mathcal{A}^k} \pi^{sk} p_{ij} \right) \\ &\approx \prod_{s \in \mathcal{S}} \left(1 - N^s \sum_{k \in \mathcal{K}_s} \sum_{(i,j) \in \mathcal{A}^k} \pi^{sk} p_{ij} \right) \\ &= 1 - \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} \sum_{(i,j) \in \mathcal{A}^k} N^s \pi^{sk} p_{ij} \end{aligned} \quad (22)$$

and for each $c_{ij}^s : s \in \mathcal{S}, (i, j) \in \mathcal{A}$:

$$\begin{aligned} \Pr [R = c_{ij}^s] &= \Pr \left[\sum_{s \in \mathcal{S}} \sum_{n=1}^{N^s} R_n^s = c_{ij}^s \right] \\ &\approx \sum_{n=1}^{N^s} \Pr [R_n^s = c_{ij}^s] \\ &= \sum_{n=1}^{N^s} p_{ij} \sum_{k \in \mathcal{K}_s} \pi^{sk} \delta_{ij}^{sk} \end{aligned} \quad (23)$$

$$= \sum_{k \in \mathcal{K}_s} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} \quad (24)$$

Therefore, the approximated risk distribution for hazmat transportation network is

$$R = \begin{cases} 0 & \text{with probability } 1 - \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} \sum_{(i,j) \in \mathcal{A}^k} N^s \pi^{sk} p_{ij} \\ c_{ij}^s & \text{with probability } \sum_{k \in \mathcal{K}_s} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} \text{ for } (i, j) \in \mathcal{A}, s \in \mathcal{S}. \end{cases} \quad (25)$$

4 The CVaR Minimization Model for Hazmat Network Design

In this section, a CVaR minimization network design model considering drivers' probabilistic route choices is proposed. It is well-known that CVaR is a general, coherent and risk-averse measure (Rockafellar and Uryasev, 2002). For any random loss X , the VaR and CVaR are introduced in Definitions 1 and 2, respectively. CVaR can also be redefined as an optimization problem as Theorem 1 shows.

Definition 1 (VaR Measure). *The value-at-risk (VaR) is defined as follows:*

$$\text{VaR}_p(X) = \inf\{x : \Pr[X \leq x] \geq p\} \quad (26)$$

where $p \in (0, 1)$ is a threshold probability.

Definition 2 (CVaR Measure). *The conditional value-at-risk (CVaR) is defined as follows:*

$$\text{CVaR}_\alpha(X) = \frac{1}{1-\alpha} \int_\alpha^1 \text{VaR}_p(X) dp \quad (27)$$

for a threshold probability $\alpha \in (0, 1)$ where $\text{VaR}_p(X)$ is shown in Definition 1.

Theorem 1 (Rockafellar and Uryasev, 2002). *For $r \in \mathbb{R}$, let us define*

$$\Phi_\alpha(r; X) = r + \frac{1}{1-\alpha} \mathbb{E}[X - r]^+,$$

where $[x]^+ = \max(x, 0)$. *Then the CVaR measure is equivalent to:*

$$\text{CVaR}_\alpha(X) = \min_{r \in \mathbb{R}} \Phi_\alpha(r; X) \quad (28)$$

4.1 Route-Choice Probabilities Depending on Network Design

To introduce the CVaR measure for hazmat transportation, the route-choice probabilities depending on network design are clarified. Let y be the path-based network design variables and z be the path availability variables here. If arc (i, j) is open for transportation of hazmat, $y_{ij} = 1$; otherwise, $y_{ij} = 0$. If path k is available for transportation of shipment $s \in \mathcal{S}$, $z^{sk} = 1$; otherwise, $z^{sk} = 0$. The route-choice probabilities are formulated as follows:

$$z^{sk} \geq \sum_{(i,j) \in \mathcal{A}} \delta_{ij}^{sk} y_{ij} - \sum_{(i,j) \in \mathcal{A}} \delta_{ij}^{sk} + 1, \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}_s \quad (29)$$

$$z^{sk} \leq y_{ij} - \delta_{ij}^{sk} + 1, \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}_s, \forall (i, j) \in \mathcal{A} \quad (30)$$

$$\sum_{k \in \mathcal{K}_s} z^{sk} \geq 1, \quad \forall s \in \mathcal{S} \quad (31)$$

$$\sum_{(i,j) \in \mathcal{A}} (1 - y_{ij}) \leq N \quad (32)$$

$$\pi^{sk} = \frac{\rho^{sk} z^{sk}}{\sum_{l \in \mathcal{K}_s} \rho^{sl} z^{sl}}, \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}_s \quad (33)$$

$$z^{sk} \text{ binary}, \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}_s \quad (34)$$

$$y_{ij} \text{ binary}, \quad \forall (i, j) \in \mathcal{A} \quad (35)$$

Equations (29) and (30) determine the path availabilities, similarly as in Section 2. Equation (31) constrains that there exists at least one path for shipment $s \in \mathcal{S}$. Equation (32) states that at most N arcs can be closed in the network.

Hazmat carriers, however, do not necessarily choose the shortest path or follow the optimal path of multi-objectives which are still within SPP in all cases. To model the uncertainty of driver behaviors, probabilistic route choice model is introduced. In the proposed model, we assume that carriers choose paths among all available paths by estimating their utilities. Then, we use RUM to model carriers' probabilistic behavior and MNL to further simplify the stochastic route-choice. Equation (33) shows the route-choice probabilities among all available paths. If path $k \in \mathcal{K}_s$ is unavailable for shipment $s \in \mathcal{S}$, namely $z^{sk} = 0$, its route-choice probability is 0; otherwise, the route-choice probability can be given by (14) and (15).

4.2 The CVaR Minimization Model

This section shows the CVaR minimization model for hazmat network design. The distribution of risk introduced in Section 3.2 and the route-choice probabilities in Section 4.1 can model the CVaR minimization network design problem. Let

$$\Phi_\alpha(r; \pi) = r + \frac{1}{1-\alpha} \mathbb{E}[R - r]^+ \quad (36)$$

$$\approx r + \frac{1}{1-\alpha} \left\{ \left(1 - \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} \sum_{(i,j) \in \mathcal{A}^k} N^s \pi^{sk} p_{ij} \right) [0 - r]^+ \right. \\ \left. + \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} [c_{ij}^s - r]^+ \right\} \quad (37)$$

$$\approx r + \frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} [c_{ij}^s - r]^+ \quad (38)$$

We use the optimization of CVaR in Theorem 1 to define the CVaR measure in hazmat transportation network,

$$\text{CVaR}_\alpha = \min_{r \in \mathbb{R}^+} \Phi_\alpha(r; \pi) \approx \min_{r \in \mathbb{R}^+} \left[r + \frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} [c_{ij}^s - r]^+ \right]. \quad (39)$$

Therefore, the CVaR minimization model is,

$$\min_{\pi \in \Omega} \text{CVaR}_\alpha = \min_{\pi \in \Omega, r \in \mathbb{R}^+} \Phi_\alpha(r; \pi) \quad (40)$$

$$\approx \min_{\pi \in \Omega, r \in \mathbb{R}^+} \left[r + \frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} [c_{ij}^s - r]^+ \right] \quad (41)$$

where Ω can be defined by

$$\Omega = \{ \pi : \exists y, z \text{ such that (29)–(35) hold} \}. \quad (42)$$

4.3 The Model Analysis

The CVaR minimization model for hazmat transportation network design (41) is a mixed integer nonlinear programming problem. If a network is complicated with a large demand of shipments, it becomes very difficult to solve the problem. In the proposed model, variable r only has an impact on the objective function and does not exist in constraints. Because the objective function is linear with r within each interval between two consecutive c_{ij}^s values, the optimal r value lies in $\Theta = \{0\} \cup \{c_{ij}^s : \forall (i, j) \in \mathcal{A}, s \in \mathcal{S}\}$ (Toumazis et al., 2013). The CVaR minimization model (41) is reformulated as:

$$\min_{r \in \Theta} f_\alpha(r) \quad (43)$$

where

$$f_\alpha(r) = \min_{\pi \in \Omega} \Phi_\alpha(r; \pi).$$

Given a large network with various kinds of hazmat, set Θ becomes large. To obtain the optimal solution of the proposed model, we should solve a large number of $f_\alpha(r)$. If some r values can be eliminated without solving optimization problems, the computation can be more efficient. Analysis is conducted to explore which r values can be eliminated from being optimal solutions for the proposed model. Let

$$0 = r_0 \leq r_1 \leq r_2 \leq \dots \leq r_{q-1} \leq r_q \leq r_{q+1} \leq \dots \leq r_{M_{\mathcal{A}}} \quad (44)$$

where r_q is the q -th smallest value in $\{c_{ij}^s : \forall (i, j) \in \mathcal{A}, s \in \mathcal{S}\}$ and $M_{\mathcal{A}}$ is the number of unique values in $\{c_{ij}^s : \forall (i, j) \in \mathcal{A}, s \in \mathcal{S}\}$. For each $q = 0, 1, \dots, M_{\mathcal{A}} - 1$, we have

$$\begin{aligned} \Phi_\alpha(r_{q+1}; \pi) - \Phi_\alpha(r_q; \pi) &= r_{q+1} + \frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} [c_{ij}^s - r_{q+1}]^+ \\ &\quad - r_q - \frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} [c_{ij}^s - r_q]^+ \\ &= r_{q+1} - r_q \\ &\quad - \frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{\substack{k \in \mathcal{K}_s \\ c_{ij}^s \geq r_{q+1}}} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} (r_{q+1} - r_q) \end{aligned}$$

$$= (r_{q+1} - r_q) \left(1 - \frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{\substack{k \in \mathcal{K}_s \\ c_{ij}^s \geq r_{q+1}}} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} \right). \quad (45)$$

Theorem 2. Consider an index $q \in \{0, 1, \dots, M_{\mathcal{A}}\}$ such that the following condition holds:

$$\frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{\substack{k \in \mathcal{K}_s \\ c_{ij}^s \geq r_{q+1}}} N^s p_{ij} \delta_{ij}^{sk} \leq 1 \quad (46)$$

Then we can show that

$$\Phi_{\alpha}(r_q; \pi) \leq \Phi_{\alpha}(r_{q+1}; \pi) \quad (47)$$

for all $\pi \in \Omega$. Further

$$f_{\alpha}(r_q) \leq f_{\alpha}(r_{q+1}) \leq \dots \leq f_{\alpha}(r_{M_{\mathcal{A}}}) \quad (48)$$

Proof of Theorem 2. Given condition (46), we have

$$\frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{\substack{k \in \mathcal{K}_s \\ c_{ij}^s \geq r_{q+1}}} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} \leq 1$$

for any π , since $\pi^{sk} \in [0, 1]$ is the probability associated with path $k \in \mathcal{K}_s$ for shipment $s \in \mathcal{S}$. Based on (45), for any route-choice probabilities $\pi \in \Omega$

$$\Phi_{\alpha}(r_q; \pi) \leq \Phi_{\alpha}(r_{q+1}; \pi). \quad (49)$$

Note that

$$\begin{aligned} \frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{\substack{k \in \mathcal{K}_s \\ c_{ij}^s \geq r_{M_{\mathcal{A}}}}} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} &\leq \dots \leq \frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{\substack{k \in \mathcal{K}_s \\ c_{ij}^s \geq r_{q+2}}} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} \\ &\leq \frac{1}{1-\alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{\substack{k \in \mathcal{K}_s \\ c_{ij}^s \geq r_{q+1}}} N^s p_{ij} \delta_{ij}^{sk} \leq 1. \end{aligned} \quad (50)$$

Consequently, we obtain

$$\Phi_{\alpha}(r_q; \pi) \leq \Phi_{\alpha}(r_{q+1}; \pi) \leq \dots \leq \Phi_{\alpha}(r_{M_{\mathcal{A}}}; \pi). \quad (51)$$

Let π_q be an optimal solution for $f_{\alpha}(r_q) = \min_{\pi \in \Omega} \Phi_{\alpha}(r_q; \pi)$; that is $f_{\alpha}(r_q) = \Phi_{\alpha}(r_q; \pi_q)$. Then, we have

$$f_{\alpha}(r_q) = \Phi_{\alpha}(r_q; \pi_q) \leq \Phi_{\alpha}(r_q; \pi_{q+1}) \leq \Phi_{\alpha}(r_{q+1}; \pi_{q+1}) = f_{\alpha}(r_{q+1}). \quad (52)$$

Similarly,

$$f_{\alpha}(r_q) \leq f_{\alpha}(r_{q+1}) \leq \dots \leq f_{\alpha}(r_{M_{\mathcal{A}}}). \quad (53)$$

This completes the proof. \square

Instead of considering all r values in Θ , we can narrow the searching range for r if there exist r values satisfying (46). Let \hat{q} be the smallest index to satisfy (46). By Theorem 2, it is proved that $f_\alpha(r_{\hat{q}}) \leq f_\alpha(r_{\hat{q}+1}) \leq \dots \leq f_\alpha(r_{M_A})$; thus excluding $r \in \{\hat{q} + 1, \dots, r_{M_A}\}$ to search the minimal $f_\alpha(r)$. The CVaR minimization model (43) can be rewritten as:

$$\min_{r \in \{r_0, r_1, \dots, r_{\hat{q}}\}} f_\alpha(r) \quad (54)$$

If (46) is not satisfied for any q , every $r \in \Theta$ should be considered.

5 A Computational Scheme for the CVaR Minimization Model

In this section, an efficient computational scheme to solve the CVaR minimization model for hazmat transportation network design is proposed. The proposed CVaR minimization model for network design is a nonlinear optimization model. Based on (43), the proposed network design model can be decomposed into two stages. At the first stage, we search r within a finite set. At the second stage, $f_\alpha(r)$ is solved to yield the network design solution.

$$f_\alpha(r) = \left\{ \min_{\pi, y, z} \left[r + \frac{1}{1 - \alpha} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} [c_{ij}^s - r]^+ \right] \text{ subject to (29)–(35)}. \right\} \quad (55)$$

Because of the nonlinearity to link the route-choice probabilities and path availabilities in (33), we linearize as follows:

$$\sum_{l \in \mathcal{K}_s} \rho^{sl} \phi^{skl} = \rho^{sk} z^{sk}, \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}_s \quad (56)$$

$$\phi^{skl} \leq z^{sl}, \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}_s, \forall l \in \mathcal{K}_s \quad (57)$$

$$\phi^{skl} \geq -(1 - z^{sl}) + \pi^{sk}, \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}_s, \forall l \in \mathcal{K}_s \quad (58)$$

$$0 \leq \phi^{skl} \leq \pi^{sk}, \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}_s, \forall l \in \mathcal{K}_s. \quad (59)$$

The parameter ρ can be computed with (15). Then, $f_\alpha(r)$ is reformulated as a MILP problem.

Despite the fact that we may use Theorem 2 to reduce the searching set for r variable, it is still time-consuming to compute $f_\alpha(r)$ given all potential r values if the scale of a network is large. Finding the optimal r can be accelerated by developing an efficient search scheme which depends on $f_\alpha(r)$. Besides, solving $f_\alpha(r)$ is very difficult when many path alternatives are considered for a complicated network. Sometimes, it is even impractical to obtain a good feasible solution for $f_\alpha(r)$.

We propose a line search with mapping to obtain optimal r as shown in Section 5.1 and show that Benders decomposition can generate upper and lower bounds of MILPs for given r values thus solving the $f_\alpha(r)$ problem in Section 5.2. Generating useful lower bounds by Benders decomposition, however, costs large computation efforts while good upper bounds can be obtained after a certain

number of iterations. In this case, we terminate the algorithm by some criteria and gain the best feasible solutions from upper bounds.

5.1 A Line Search with Mapping

To search the optimal r value for the proposed network design model, we only consider a narrowed range of values checked by Theorem 2. Initially, we can think of obtaining an optimal solution for network design problem by visiting every value in Θ . If Θ involves a large number of values, the computation for the problem can be time-consuming because we need to solve a large number of MILPs. A searching mechanism for r based on line search methods are proposed in order to solve the problem efficiently. We use the Golden Section method. When it is applied to a strictly quasiconvex function, the Golden Section method can find a global minimal solution. The essence of the Golden Section method is to reuse one searching point in previous iteration and compare with an updated point derived by the golden ratio to reduce computations. Note that the golden ratio is 0.618.

We use the same idea to develop a discrete version of the Golden Section method, which only evaluates a limited number of r values in Θ . Usually, a line section method minimizes a nonlinear optimization problem over the interval $[a_0, b_0]$. The optimal r value lies in Θ , so $a_0 = 0$ and b_0 would be the smallest r value satisfying (46) by Theorem 2.

A line search algorithm usually copes with a continuous variable from a certain interval. In the proposed model, optimal r value is from a finite set. We map the updated point in iterations to value in the finite set using a simple mechanism. The simple mechanism can guarantee the correctness of searching interval. The procedures for searching optimal r for the proposed model are shown in Algorithm 1.

5.2 Benders Decomposition for $f_\alpha(r)$

The line search for r highly depends on obtaining optimal objective values for MILPs. As the size of the network increases, the computation time for solving $f_\alpha(r)$ given r goes up exponentially. When we solved $f_\alpha(r)$ given r with CPLEX solver of version 12.6 for the Ravenna network (Bonvicini and Spadoni, 2008; Erkut and Gzara, 2008), which has 105 nodes, 134 undirected arcs, 31 OD pairs, and 50 available paths for each OD pair, the optimality gap is 99.9% after 24 hours. This motivates us to develop an efficient algorithm solving $f_\alpha(r)$ given r . We can benefit from generating upper and lower bounds for $f_\alpha(r)$ and solving the problem iteratively rather than directly solves a large MILP with CPLEX. Seen from the structure of the MILPs, it is found that $f_\alpha(r)$ can be decomposed into: (1) optimizing network design (2) analyzing probabilities assigned for paths.

Benders decomposition is a popular algorithm framework to deal with complicating variables and large-scale optimization problems in which variables and constraints are decomposed into a master problem and subproblems. The algorithm employs cutting-planes procedures for the master problem based on subproblems until it converges. There are two categories of cuts in Benders composition. When a subproblem reaches an optimal solution but its optimal objective value is

Algorithm 1 A line search with mapping

- 1: **Initialization:** Check the largest q (q^*) which satisfies (46). Let $k \leftarrow 0$ and $a_k \leftarrow 0$, $b_k \leftarrow r_{q^*}$. $\lambda_k = a_k + (1 - \varphi)(b_k - a_k)$ and $\mu_k = a_k + \varphi(b_k - a_k)$. Find the left-closest value to λ_k (λ_{left}) and the right-closest value to μ_k (μ_{right}) among Θ . Let $\lambda_k = \lambda_{\text{left}}$, $\mu_k = \mu_{\text{right}}$ and

$$f_\alpha(\lambda_k) = \min_{\pi \in \Omega} \Phi_\alpha(\lambda_k; \pi)$$

$$f_\alpha(\mu_k) = \min_{\pi \in \Omega} \Phi_\alpha(\mu_k; \pi)$$

- 2: **Convergence check:** If $a_k = r_q$ and $b_k = r_q$ or r_{q+1} for any $q = 0, 1, \dots, (q^* - 1)$, go to Step 6; otherwise, continue estimating $f_\alpha(\lambda_k)$ and $f_\alpha(\mu_k)$. If $f_\alpha(\lambda_k) > f_\alpha(\mu_k)$, go to Step 3; if $f_\alpha(\lambda_k) \leq f_\alpha(\mu_k)$, go to Step 4.

- 3: **Reuse μ_k :** Find the right-closest value to λ_k in Θ (λ_{right}) and let $a_{k+1} = \lambda_{\text{right}}$ and $b_{k+1} = b_k$. If $\mu_k - a_{k+1} \leq b_{k+1} - \mu_k$, let

$$\lambda_{k+1} = \mu_k, f_\alpha(\lambda_{k+1}) = f_\alpha(\mu_k)$$

$$\mu_{k+1} = \frac{\mu_k + b_{k+1}}{2}.$$

Find the right-closest value to μ_{k+1} in Θ (μ_{right}) and let $\mu_{k+1} = \mu_{\text{right}}$. Evaluate $f_\alpha(\mu_{k+1})$.

If $\mu_k - a_{k+1} > b_{k+1} - \mu_k$, let

$$\mu_{k+1} = \mu_k, f_\alpha(\mu_{k+1}) = f_\alpha(\mu_k)$$

$$\lambda_{k+1} = \frac{a_{k+1} + \mu_k}{2}.$$

Find the left-closest value to λ_{k+1} in Θ (λ_{left}) and let $\lambda_{k+1} = \lambda_{\text{left}}$. Evaluate $f_\alpha(\lambda_{k+1})$.

Go to Step 5.

- 4: **Reuse λ_k :** Find the left-closest value to μ_k in Θ (μ_{left}) and let $a_{k+1} = a_k$ and $b_{k+1} = \mu_{\text{left}}$. If $\lambda_k - a_{k+1} \leq b_{k+1} - \lambda_k$, let

$$\lambda_{k+1} = \lambda_k, f_\alpha(\lambda_{k+1}) = f_\alpha(\lambda_k)$$

$$\mu_{k+1} = \frac{\lambda_k + b_{k+1}}{2}.$$

Find the right-closest value to μ_{k+1} in Θ (μ_{right}) and let $\mu_{k+1} = \mu_{\text{right}}$. Evaluate $f_\alpha(\mu_{k+1})$.

If $\lambda_k - a_{k+1} > b_{k+1} - \lambda_k$, let

$$\mu_{k+1} = \lambda_k, f_\alpha(\mu_{k+1}) = f_\alpha(\lambda_k)$$

$$\lambda_{k+1} = \frac{a_{k+1} + \lambda_k}{2}.$$

Find the left-closest value to λ_{k+1} in Θ (λ_{left}) and let $\lambda_{k+1} = \lambda_{\text{left}}$. Evaluate $f_\alpha(\lambda_{k+1})$.

Go to Step 5.

- 5: **Iteration update:** $k \leftarrow k + 1$ and go to Step 2.

- 6: **Determine optimal solution:** Evaluate for $f_\alpha(a_k)$ and $f_\alpha(b_k)$. If $f_\alpha(a_k) \leq f_\alpha(b_k)$, $r^* = a_k$; otherwise, $r^* = b_k$. Stop.
-

not consistent with the master problem's, an optimality cut based on dual of a subproblem is generated. On the other hand, a feasibility cut can be generated if a subproblem is infeasible. Taking advantage of the extreme ray for the dual of a infeasible subproblem can help to generate a feasibility cut. Theories and applications for Benders decomposition are developed widely. Geoffrion (1972) generalized Benders' approach to a broader class of programs in which subproblems are not restricted to linear programs. Stochastic programming problems, which is well known as its stage structure can be solved efficiently by Benders decomposition (Santoso et al., 2005).

We implement Benders decomposition to solve MILPs and obtain $f_\alpha(r)$. The network design y and path availability z are master problem variables while the probabilities related variables including π and ϕ are in subproblems.

With Benders decomposition, we present the master problem as follows:

$$\text{(master) } \min_{g,y,z} \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}_s} g^{sk} \quad (60)$$

$$\text{s.t. } (29)-(32), (34)-(35)$$

$$g^{stkt} \geq \rho^{stkt} z^{stkt} \lambda_t + \sum_{l \in \mathcal{K}_{s_t}} z^{stl} \mu_t^l + \sum_{l \in \mathcal{K}_{s_t}} (-1 + z^{stl}) v_t^l, \quad \forall t = 1, 2, \dots \quad (61)$$

where t denotes the number of cuts generated by the t -th iteration of Benders decomposition. Constraints (61) are further explained by subproblem duals later.

The subproblems which analyze the route-choice probabilities (33) are decomposed by $s \in \mathcal{S}, k \in \mathcal{K}_s$ with dual variables $(\lambda, \mu^l, v^l, \omega^l)$ as follows:

$$\min_{\pi^{sk}} \sum_{(i,j) \in \mathcal{A}} N^s \pi^{sk} p_{ij} \delta_{ij}^{sk} [c_{ij}^s - r]^+ \quad (62)$$

$$\text{s.t. } \sum_{l \in \mathcal{K}_s} \rho^{sl} \phi^{skl} = \rho^{sk} z^{sk} \quad (\lambda) \quad (63)$$

$$\phi^{skl} \leq z^{sl}, \quad \forall l \in \mathcal{K}_s \quad (\mu^l \leq 0) \quad (64)$$

$$\phi^{skl} \geq -(1 - z^{sl}) + \pi^{sk}, \quad \forall l \in \mathcal{K}_s \quad (v^l \geq 0) \quad (65)$$

$$\phi^{skl} \leq \pi^{sk}, \quad \forall l \in \mathcal{K}_s \quad (\omega^l \leq 0) \quad (66)$$

$$\pi^{sk} \text{ free}, \quad (67)$$

$$\phi^{skl} \geq 0, \quad \forall l \in \mathcal{K}_s \quad (68)$$

Feed with master problem variables, route-choice probabilities can be estimated from subproblems. Therefore, subproblems are feasible making it only necessary to generate optimality cuts from subproblem duals. The subproblem dual is defined as follows:

$$(\text{SD}^{sk}) \hat{g}^{sk} = \max_{\lambda, \mu, v, \omega} \rho^{sk} z^{sk} \lambda + \sum_{l \in \mathcal{K}_s} z^{sl} \mu^l + \sum_{l \in \mathcal{K}_s} (-1 + z^{sl}) v^l \quad (69)$$

$$\text{s.t. } - \sum_{l \in \mathcal{K}_s} \mu^l - \sum_{l \in \mathcal{K}_s} v^l = \sum_{(i,j) \in \mathcal{A}} N^s p_{ij} \delta_{ij}^{sk} [c_{ij}^s - r]^+ \quad (70)$$

$$\rho^{sl} \lambda + \mu^l + v^l + \omega^l \leq 0, \forall l \in \mathcal{K}_s. \quad (71)$$

In subproblem duals, we can obtain a (s_t, k_t) with the objective value $\hat{g}^{s_t k_t}$ and the solution $(\lambda_t, \mu_t^l, v_t^l, \omega_t^l)$ accordingly. Let $\tilde{g}^{s_t k_t}$ be an optimal solution for the master problem. If $\hat{g}^{s_t k_t}$ is greater than $\tilde{g}^{s_t k_t}$, an optimality cut can be generated as (61) using (69). The algorithm is summarized in Algorithm 2. In Algorithm 2, ϵ is a small positive parameter. Besides, I is used to indicate

Algorithm 2 Benders decomposition for $f_\alpha(r)$

- 1: **Initialization:** Set $t = 0$, upper bound $\text{UB} = \infty$ and lower bound $\text{LB} = 0$. Go to Step 2.
 - 2: **Solve master problem:** Solve the master problem and obtain the optimal solution $(\tilde{g}, \tilde{y}, \tilde{z})$. Let $\text{LB} = \sum_{s \in S} \sum_{k \in \mathcal{K}_s} \tilde{g}^{sk}$ and $I = 0$. Go to Step 3.
 - 3: **Solve subproblem:** For (s, k) , solve SD^{sk} problem based on \tilde{z} and obtain optimal solution $(\hat{\lambda}, \hat{\mu}^l, \hat{v}^l, \hat{\omega}^l)$. The optimal objective value for the subproblem is \hat{g}^{sk} . If all (s, k) are visited, go to Step 5; otherwise, go to Step 4.
 - 4: **Generate an optimality cut:** If $I = 1$ go to Step 2; otherwise, compare \tilde{g}^{sk} and \hat{g}^{sk} . If $\hat{g}^{sk} - \tilde{g}^{sk} \geq \epsilon$, update $I \leftarrow 1, t \leftarrow t + 1, s_t \leftarrow s, k_t \leftarrow k, \lambda_t \leftarrow \hat{\lambda}, \mu_t \leftarrow \hat{\mu}, v_t \leftarrow \hat{v}$ and an optimality cut is generated; otherwise, update (s, k) and go to Step 3.
 - 5: **Convergence check:** If $\text{UB} > \sum_{s \in S} \sum_{k \in \mathcal{K}_s} \hat{g}^{sk}$, set $\text{UB} = \sum_{s \in S} \sum_{k \in \mathcal{K}_s} \hat{g}^{sk}$. If $\text{UB} - \text{LB} \leq \epsilon$, terminate; otherwise go to Step 1.
-

whether an optimality cut is generated. Based on an optimal solution for the master problem, we can generate multiple optimality cuts from different subproblems. The master problem becomes very difficult to solve if too many cuts are added at a time, which makes hard to obtain an upper bound. In order to produce upper bounds effectively, we only add one optimality cut after solving the master problem until the algorithm terminates.

We implement Benders decomposition on the Ravenna network in (Bonvicini and Spadoni, 2008; Erkut and Gzara, 2008) with 105 nodes and 134 undirected arcs. Four kinds of hazardous materials are considered including methanol, chlorine, gasoline and LPG. There are 31 shipments and each shipment defines a certain demand of a hazmat transported from an OD pair. For each shipment, we generate 50 paths using k-shortest path approach to test the performance of the proposed framework. The computation process for solving $f_{0.95}(0.454)$ is shown in Figure 3. We terminate the algorithm when the optimality gap is less than 5%. In this example, we can see that a good feasible solution is achieved within a small number of iterations. The improvement of lower bound, however, is very slow. Besides, it becomes more difficult to solve the master problem as iteration proceeds. It indicates that the time spent on the iteration close to the optimal solution can be far more than early iterations. An optimal solution is obtained when the upper bound and the lower bound are close.

Since we can obtain feasible solutions and useful upper bounds before reaching the convergence of Benders decomposition, a close optimal solution generated by a set of feasible solutions is used. When the upper bound does not improve, we terminate the algorithm. Different stopping criteria

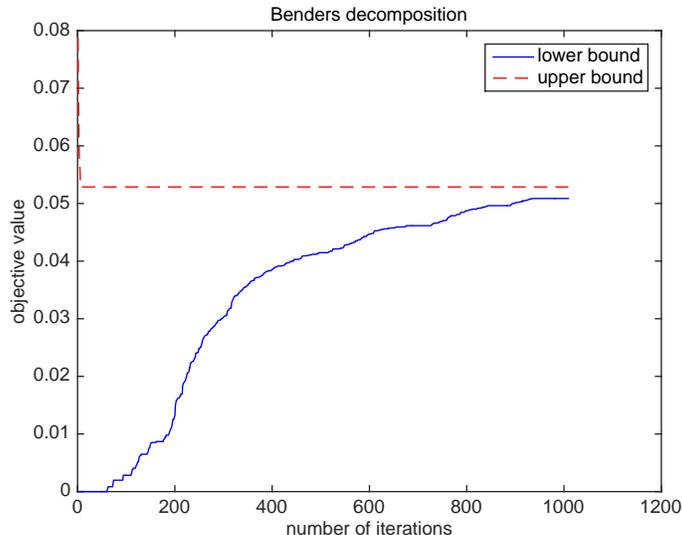


Figure 3: Lower bounds and upper bounds for a MILP given $r = 0.454$ and $\alpha = 0.95$ by Benders decomposition for the Ravenna network.

such as the total time limit of algorithm, the total number of iterations and the number of iterations that upper bound does not improve can be set. The local optimality can be guaranteed for the best feasible solution thus providing a practical approach. Besides, the effectiveness to terminate at a good feasible solution for $f_\alpha(r)$ accelerates the solving process.

5.3 Performance of Algorithm 1 Depending on Algorithm 2

This section discusses the performance of Algorithm 1 depending on Algorithm 2. The Ravenna network with 20 paths for each shipment are used for experiments in this section. Let $\alpha = 0.95$ and the maximum number of closed arcs $N = 10$. To solve the proposed CVaR minimization model for hazmat network design, we incorporate the searching scheme for r in Section 5.1 with evaluations of $f_\alpha(r)$ using Algorithm 2. We can either use the optimal or the best feasible solution obtained from Algorithm 2 to proceed the searching of r in Algorithm 1. The searching process of r is shown in Figure 4.

It can be seen that an optimal network design is achieved when solving MILP with $r = 0.687$ and the minimum risk equals to 0.732. In Figure 4, it is found that the optimal r value is 0.699 and the approximated minimum risk is 0.742 by the best feasible solution of $f_\alpha(r)$. Accordingly, the network design results are shown in Figure 5. The number of closed arcs in both cases is 10 with 8 of which are the same. It indicates that both network designs are similar. Given the best feasible design, the CVaR is the minimum value for $\Phi(r; \pi)$ through all r values. Hence, the risk for best feasible network design is less than or equal to 0.742. The best feasible solution by Algorithm 2 yields a network design with the objective function value no greater than 1.35% of the optimal solution. This shows that the line search for r with a best feasible solution for $f_\alpha(r)$ is close to

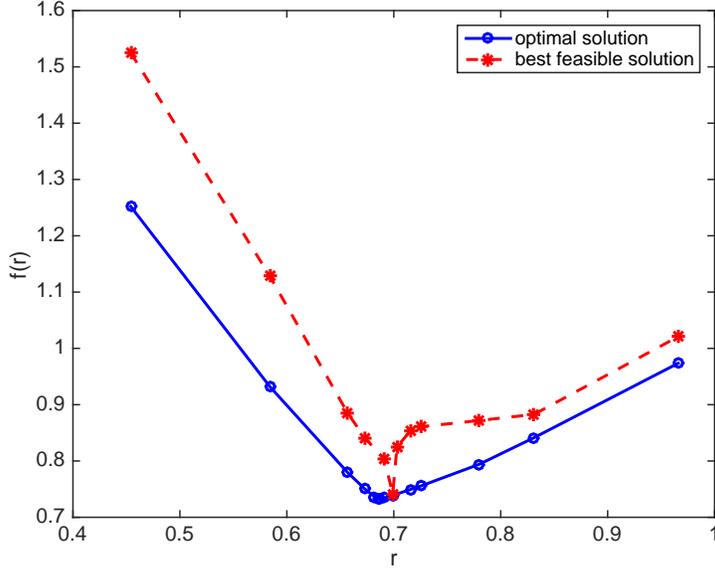


Figure 4: Searching of r based on the optimal and the best feasible solution by Algorithm 2

the optimal solution. In addition, it costs 3 hours to compute an optimal hazmat network design depending on the exact value of $f_\alpha(r)$, while the best feasible design is obtained in 1 hour and 33 minutes. To improve the computation efficiency while ensuring the solution quality, we incorporate the line search for r with a best feasible solution for $f_\alpha(r)$ in Section 6.

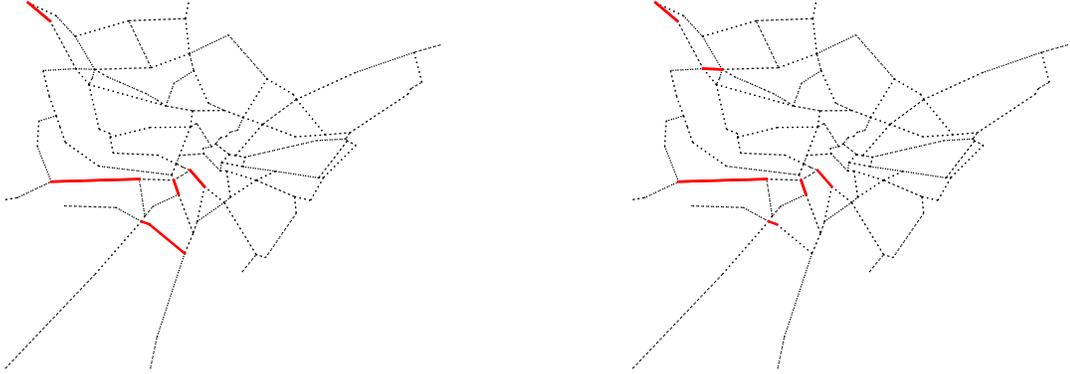
6 Numerical Experiments

In this section, applications of the proposed model are shown. All numerical experiments in this section are conducted using the Ravenna (Bonvicini and Spadoni, 2008; Erkut and Gzara, 2008) network. Four kinds of hazardous materials are considered: methanol, chlorine, gasoline and LPG. There are 31 shipments transported through the networks. The data set includes the length of each arc, the population that each kind of hazmat can influence on each arc, the OD pairs for each kind of hazmat and the demand of hazmat accordingly.

6.1 Data Analysis

For a transportation network, we can obtain the network structure and related data including arc length l_{ij} and the population density τ_{ij} . For each arc (i, j) , the accident probability p_{ij} and the accident consequence c_{ij}^s to transport hazmat s should also be provided. To specify the accident probability p_{ij} , we can use

$$p_{ij} = 3.19922 \times 10^{-7} \times l_{ij} \quad (72)$$



(a) An optimal solution

(b) The best feasible solution

Figure 5: Network designs based on the optimal and the best feasible solution by Algorithm 2

where 3.19922×10^{-7} is the hazmat accident rate per mile/vehicle (Federal Motor Carrier Safety Administration, 2001). The accident consequence for an arc is quantified by the population exposure impacted if that accident happens. We use the following formula to estimate c_{ij}^s (Toumazis and Kwon, 2016)

$$c_{ij}^s = 3.14159 \times d_s^2 \times \tau_{ij} \quad (73)$$

where 3.14159 is the ratio of a circle's circumference to its diameter. For different kinds of hazmat, the impacted radius of an accident is different. The impacted radius d_s is selected based on the recommendations of the Emergency Response Guidebook (2012) for the length of the evacuation radius in the case of an accident involving hazmat, which ranges between 0.5 and 1 mile depending on the type of the hazmat of shipment s . τ_{ij} is the population density along arc (i, j) .

Our proposed model is a path-based hazmat network design model which requires specified path alternatives by hazmat carriers. The k shortest path algorithm of Yen (1971) is used to generate path sets. Despite the modifications or improvements of k shortest path algorithm, this approach rarely emphasizes on accident consequences of arcs. If the set of path alternatives obtained by k shortest path algorithm is very small, for example, only five paths for each shipment, some important arcs with high chosen probabilities and high risks can be left out. On the other hand, it is nearly impossible to solve our proposed model enumerating all paths for all shipments due to the tremendous model size. For example, there are more than 30,000 variables and 100,000 constraints for a network with 100 arcs, 3 shipments and 100 paths available for each shipment. Hazmat carriers can be restricted to some roads due to massive weights, large heights and long lengths for trucks. Usually, hazmat carriers select a route within a limited number of path alternatives. We use k shortest path algorithm to enumerate a list of paths that includes the shortest 50 paths for each shipment.

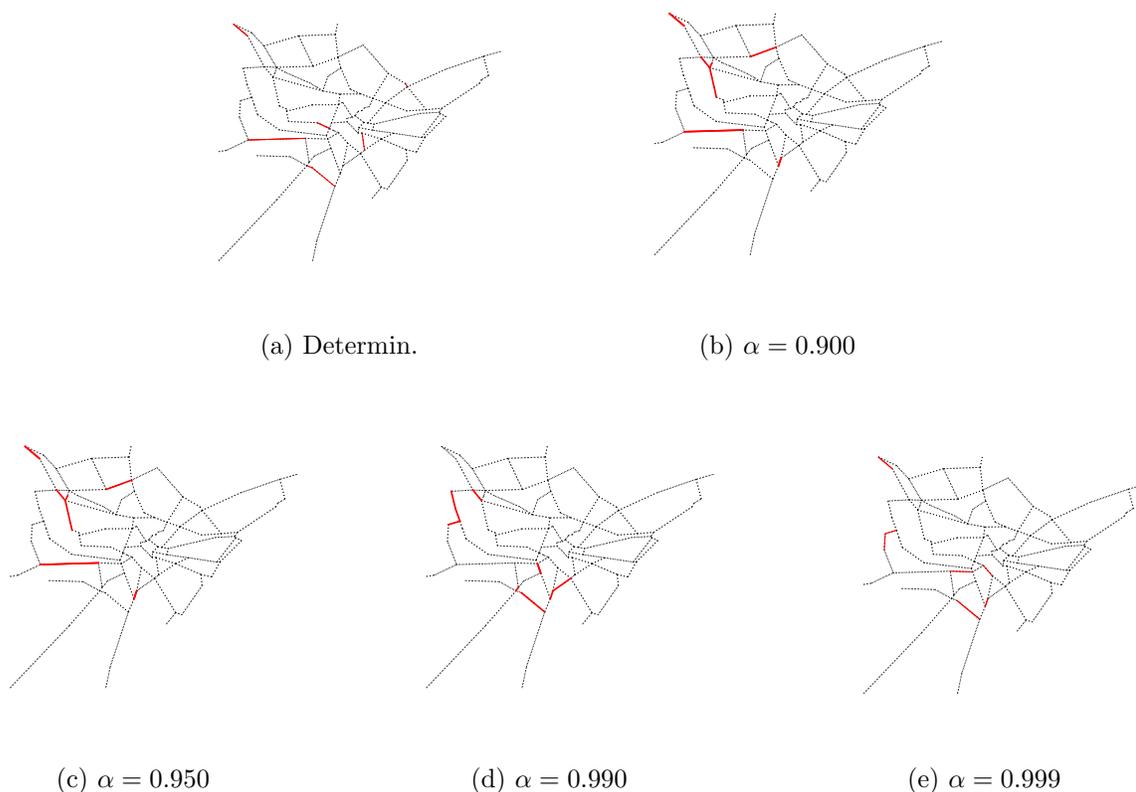


Figure 6: Ravenna road-ban with different probability threshold value α

6.2 Computation Performance

The computational scheme in Section 5 is coded in the Julia Language with the JuMP.jl package (Dunning et al., 2017) and CPLEX solver of version 12.6 is used. The experiments are implemented on a computer with 8GB of RAM and a 2.7GHz processor.

Table 2 shows the computation time and objective values using Algorithm 1 and Algorithm 2. To accelerate the computation, Algorithm 2 is terminated when the solution is not improved within the next fifty iterations. With the proposed algorithms, the average computation time to solve CVaR minimization problem of Ravenna network is 3 hours for different probability threshold values. The exact algorithm that uses CPLEX solver of version 12.6 to solve $f_\alpha(r)$ for every r is inadequate to solve the problem, because the optimality gap solving $f_\alpha(r)$ with CPLEX is 99.9% after 24 hours.

For the Ravenna network, the road-ban decisions with different probability threshold values can be seen in Figure 6. For example, when $\alpha = 0.900$ and $\alpha = 0.950$, the optimal network designs are the same. Regulators for hazmat transportation have different attitudes towards risks but may end up with the same optimal network design. Theoretically, the higher the probability threshold α is, the more we focus on severe accident consequences. With the increasing of the probability threshold value, the optimal network design for the proposed model can vary a lot. The optimal

Table 2: Numerical results for different probability threshold value α in Ravenna network

α	Computation time	Closed arcs
0.900	2hr 53min	(17, 19), (22, 38), (9, 10), (78, 74), (105, 106), (20, 10), (38, 22), (74, 78), (10, 5), (106, 105)
0.910	3hr 20min	(17, 19), (22, 38), (9, 10), (78, 74), (105, 106), (20, 10), (38, 22), (74, 78), (10, 5), (106, 105)
0.920	3hr 22min	(17, 19), (22, 38), (9, 10), (78, 74), (105, 106), (20, 10), (38, 22), (74, 78), (10, 5), (106, 105)
0.930	3hr 26min	(17, 19), (22, 38), (9, 10), (78, 74), (105, 106), (20, 10), (38, 22), (74, 78), (10, 5), (106, 105)
0.940	3hr 17min	(17, 19), (22, 38), (9, 10), (78, 74), (105, 106), (20, 10), (38, 22), (74, 78), (10, 5), (106, 105)
0.950	3hr 22min	(17, 19), (22, 38), (9, 10), (78, 74), (105, 106), (20, 10), (38, 22), (74, 78), (10, 5), (106, 105)
0.960	3hr 26min	(17, 19), (22, 38), (9, 10), (78, 74), (105, 106), (20, 10), (38, 22), (74, 78), (10, 5), (106, 105)
0.970	3hr 52min	(2, 7), (4, 17), (22, 38), (62, 54), (74, 69), (74, 76), (105, 106), (38, 22), (74, 78), (106, 105)
0.980	3hr 46min	(3, 6), (6, 11), (62, 54), (83, 66), (60, 58), (78, 74), (5, 10), (54, 62), (8, 11), (76, 74)
0.990	2hr 44min	(15, 22), (74, 76), (75, 76), (6, 3), (54, 62), (66, 83), (76, 75), (74, 78), (10, 5), (106, 105)
0.991	2hr 37min	(15, 22), (74, 76), (75, 76), (6, 3), (54, 62), (66, 83), (76, 75), (74, 78), (10, 5), (106, 105)
0.992	2hr 38min	(15, 22), (74, 76), (75, 76), (6, 3), (54, 62), (66, 83), (76, 75), (74, 78), (10, 5), (106, 105)
0.993	2hr 38min	(15, 22), (74, 76), (75, 76), (6, 3), (54, 62), (66, 83), (76, 75), (74, 78), (10, 5), (106, 105)
0.994	3hr 2min	(15, 22), (74, 76), (75, 76), (6, 3), (54, 62), (66, 83), (76, 75), (74, 78), (10, 5), (106, 105)
0.995	3hr 1min	(15, 22), (74, 76), (75, 76), (6, 3), (54, 62), (66, 83), (76, 75), (74, 78), (10, 5), (106, 105)
0.996	3hr 2min	(15, 22), (74, 76), (75, 76), (6, 3), (54, 62), (66, 83), (76, 75), (74, 78), (10, 5), (106, 105)
0.997	2hr 4min	(8, 15), (69, 56), (83, 66), (78, 74), (105, 106), (66, 83), (8, 11), (74, 78), (38, 54), (106, 105)
0.998	2hr 35min	(8, 15), (69, 56), (83, 66), (78, 74), (105, 106), (66, 83), (8, 11), (74, 78), (38, 54), (106, 105)
0.999	2hr 4min	(8, 15), (69, 56), (83, 66), (78, 74), (105, 106), (66, 83), (8, 11), (74, 78), (38, 54), (106, 105)

Table 3: Comparisons of algorithms to obtain $f_\alpha(r) = r + \frac{1}{1-\alpha}h(r)$ for the Ravenna network

r value ($\times 10^3$ population exposure)	$h(r)$		$\frac{h(r)_{\text{CPLEX}} - h(r)_{\text{Algorithm 2}}}{h(r)_{\text{Algorithm 2}}}$
	Algorithm 2	CPLEX	
0.454	0.046	0.079	72.8 %
0.265	0.091	0.152	67.7 %
0.427	0.053	0.086	64.2 %
0.130	0.167	0.255	52.9 %
0.352	0.068	0.110	62.7 %
0.199	0.120	0.191	59.3 %
0.165	0.133	0.213	60.7 %
0.231	0.109	0.171	57.8 %
0.183	0.124	0.201	61.4 %
0.174	0.130	0.207	59.3 %
0.191	0.121	0.196	62.1 %
0.197	0.122	0.192	57.7 %
0.186	0.123	0.199	61.1 %
0.193	0.123	0.194	58.1 %
0.189	0.123	0.197	59.8 %
0.188	0.123	0.198	60.7 %

network design of $\alpha = 0.990$ only has two common closed arc – arc (78, 74) and (106, 105) with $\alpha = 0.900$ and $\alpha = 0.950$. For example, closing arc (3, 6) plays a significant role in reducing risk with $\alpha = 0.990$ but not in $\alpha = 0.900$ and $\alpha = 0.950$ cases. If we close arc (3, 6), the large accident consequences by hazmat within 1% chance to happen can be avoided while it may not be effective to reduce the risk brought by 10% potential hazmat truck accidents.

6.3 Comparisons for Algorithms

To solve the CVaR minimization network design model, Algorithms 1 and 2 are proposed. Algorithm 1 and the CPLEX solver can also be used to solve the problem. Algorithm 2 and the CPLEX solver are compared for the values of $f_\alpha(r)$ obtained. CPLEX may not return an optimal solution for $f_\alpha(r)$ within limited time. We have restricted 30 minutes as time limit for both Algorithm 2 and CPLEX. Seen from (55), $f_\alpha(r)$ can be estimated by solving an MILP which only relates to r . Let $f_\alpha(r) = r + \frac{1}{1-\alpha}h(r)$ and $h(r)$ denote the objective value of the MILP. Given r values, Table 3 shows the obtained $h(r)$ by using Algorithm 2 and CPLEX solver.

It can be seen that Algorithm 2 performs better than CPLEX in solving the MILPs and obtaining $f_\alpha(r)$. Algorithm 2 can provide acceptable solutions for $f_\alpha(r)$ thus proceeding Algorithm 1.

6.4 Comparisons of Models

To show the value of our model, we compare SPP and RUM route-choice models with various CVaR measures in Table 4. Note that when $\alpha = 0$, the CVaR_α measure is equivalent to the ER measure. When using the SPP model with $\alpha = 0$, it is equivalent to the deterministic model described in in Section 2. As the probability threshold value α increases, it is preferred to close short arcs in SPP- CVaR_α while there is no pattern for our proposed model. In addition, our proposed RUM- CVaR_α model tends to close higher risk (population density) arc than SPP- CVaR_α model does with the same probability threshold value α .

To show the value of our proposed model using RUM and CVaR, we first define various measures that are similar to the values of stochastic solutions (VSS) used to compare the performance of stochastic solutions with the performance of the deterministic solutions in stochastic environments (Birge, 1982). We define the Value of the RUM Solutions (VRS) over SPP solutions. When ER is used as the risk measure, we define VRS as follows

$$\text{VRS} = \frac{(\text{RUM-ER measure of the SPP-ER solution}) - (\text{Optimal RUM-ER})}{(\text{Optimal RUM-ER})} \quad (74)$$

and when CVaR_α is used as the risk measure,

$$\text{VRS} = \frac{(\text{RUM-CVaR}_\alpha \text{ of the SPP-CVaR}_\alpha \text{ solution}) - (\text{Optimal RUM-CVaR}_\alpha)}{(\text{Optimal RUM-CVaR}_\alpha)} \quad (75)$$

VRS measures how much we gain by considering RUM compared to SPP. Figure 7 presents VRS with various α values. We observe that as α increases VRS tends to increase. This shows that the value of probabilistic modeling becomes significant, when we are interested in low-probability high-consequence outcomes and more risk averse. On the other hand, for mid-range α values, VRS is not significant. Note that for some α values, VRS value becomes negative, which happens since our algorithm finds a suboptimal solution in general.

Similarly, we define the Value of the CVaR_α Solutions (VCS_α) over ER solutions. When SPP is used for route-choice modeling, we define VCS_α as follows:

$$\text{VCS}_\alpha = \frac{(\text{SPP-CVaR}_\alpha \text{ of the SPP-ER solution}) - (\text{Optimal SPP-CVaR}_\alpha)}{(\text{Optimal SPP-CVaR}_\alpha)} \quad (76)$$

When RUM is used for route-choice modeling,

$$\text{VCS}_\alpha = \frac{(\text{RUM-CVaR}_\alpha \text{ of the RUM-ER solution}) - (\text{Optimal RUM-CVaR}_\alpha)}{(\text{Optimal RUM-CVaR}_\alpha)} \quad (77)$$

VCS measures how much we gain by considering CVaR_α compared to ER. Figure 8 shows VCS with various α values for both cases with SPP and RUM for route-choice modeling. We observe that VCS increases significantly as α increases. In all α values, the value of CVaR solutions becomes more apparent when RUM is used for route-choice modeling.

Table 4: Comparisons of SPP-CVaR and RUM-CVaR models for the Ravenna network

Model	Probability threshold α	Risk Measure Values				Closed Arcs (in average)	
		SPP ER	RUM ER	SPP CVaR $_{\alpha}$	RUM CVaR $_{\alpha}$	Length (mile)	Population density (/mile ²)
SPP CVaR $_{\alpha}$	0	358.1	419.7	358.1	419.7	8.10	207.1
	0.9	364.8	396.8	698.1	783.4	6.17	215.2
	0.91	364.8	396.8	700.3	794.3	6.17	215.2
	0.92	364.8	389.9	702.4	769.3	6.42	295.6
	0.93	364.8	389.9	704.6	781.1	6.42	295.6
	0.94	364.8	389.9	707.5	796.7	6.42	295.6
	0.95	364.8	389.9	711.6	818.7	6.42	295.6
	0.96	364.8	389.9	717.8	851.6	6.42	295.6
	0.97	364.8	389.9	728.0	906.5	6.42	295.6
	0.98	364.8	389.9	748.5	1016.2	6.42	295.6
	0.99	374.2	424.9	808.5	1456.9	6.11	215.2
	0.991	374.2	424.9	820.2	1494.6	6.11	215.2
	0.992	397.5	474.8	832.4	1660.2	4.62	244.6
	0.993	397.5	474.8	839.9	1711.0	4.62	244.6
	0.994	397.5	474.8	849.8	1778.9	4.62	244.6
	0.995	397.5	474.8	863.8	1849.6	4.62	244.6
	0.996	397.4	459.8	884.2	1935.7	4.17	229.3
	0.997	397.4	459.8	911.7	2032.4	4.17	229.3
	0.998	398.1	423.4	965.9	2113.0	3.55	184.5
0.999	398.1	423.4	1113.8	2434.2	3.55	184.5	
RUM CVaR $_{\alpha}$	0	363.5	376.4	363.5	376.4	6.78	324.8
	0.9	372.1	405.4	718.7	762.4	7.83	345.9
	0.91	372.1	405.4	723.0	767.1	7.83	345.9
	0.92	372.1	405.4	727.5	768.1	7.83	345.9
	0.93	372.1	405.4	733.3	769.5	7.83	345.9
	0.94	372.1	405.4	741.0	812.7	7.83	345.9
	0.95	372.1	405.4	751.8	837.9	7.83	345.9
	0.96	372.1	405.4	768.0	875.6	7.83	345.9
	0.97	385.1	408.2	771.3	890.1	8.88	376.4
	0.98	402.4	423.4	846.4	970.0	4.59	181.5
	0.99	402.2	444.3	833.7	1165.4	5.08	298.9
	0.991	402.2	444.3	839.7	1205.8	5.08	298.9
	0.992	402.2	444.3	847.1	1255.0	5.08	298.9
	0.993	402.2	444.3	856.7	1315.1	5.08	298.9
	0.994	402.2	444.3	869.5	1388.1	5.08	298.9
	0.995	402.2	444.3	887.4	1442.6	5.08	298.9
	0.996	402.2	444.3	913.2	1450.3	5.08	298.9
	0.997	389.2	435.7	1239.2	1459.6	5.80	247.4
	0.998	389.2	435.7	1272.9	1463.4	5.80	247.4
0.999	389.2	435.7	1373.7	1487.0	5.80	247.4	

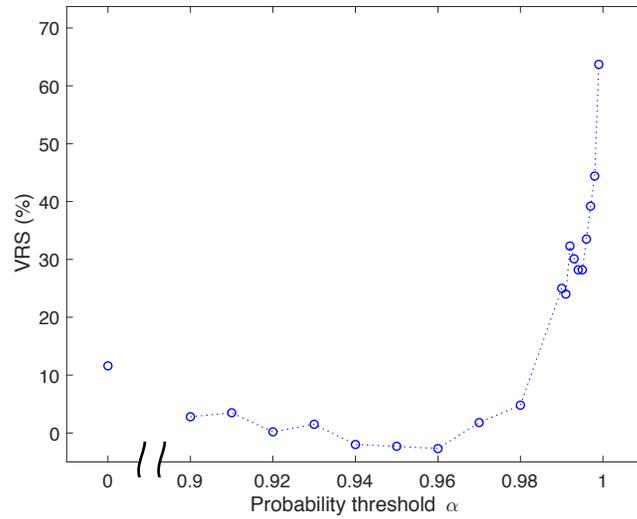


Figure 7: The value of RUM solutions over SPP solutions

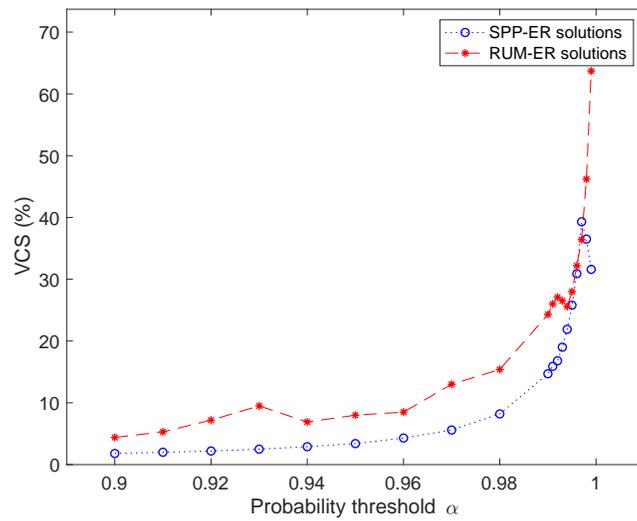


Figure 8: The value of CVaR_α solutions over ER solutions with SPP and RUM for route-choice modeling

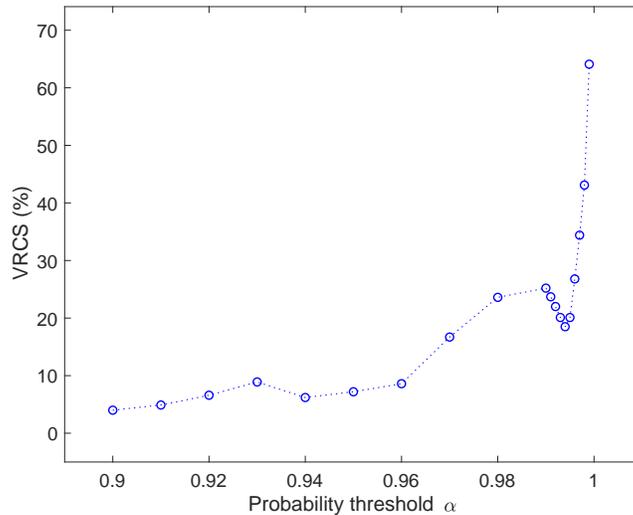


Figure 9: The value of RUM-CVaR $_{\alpha}$ solutions over SPP-ER solutions

Finally, we also define the Value of the RUM-CVaR $_{\alpha}$ Solutions (VRCS $_{\alpha}$), over SPP-ER solutions:

$$\text{VRCS}_{\alpha} = \frac{(\text{RUM-CVaR}_{\alpha} \text{ of the SPP-ER solution}) - (\text{Optimal RUM-CVaR}_{\alpha})}{(\text{Optimal RUM-CVaR}_{\alpha})} \quad (78)$$

Figure 9 shows VRCS with various α values. For lower α values, while the overall value of RUM-CVaR $_{\alpha}$ solution is marginal in the range of 4.9% to 8.6%, it clearly gives advantages if compared with VCS in Figure 7. For higher α values, however, we find VRCS in the range of 16.7% to 64.1%. By using both RUM and CVaR $_{\alpha}$ in decision-making, we conclude that risk-averse hazmat network designers can obtain clear merits for all probability threshold values, especially for higher values.

In the Ravenna network, the optimal network designs by the proposed model and the deterministic model yield different available paths for shipments with which lead to different risks. The comparisons of available paths for transporting methanol from node 110 to node 105 by both models are shown in Table 5. For each path, the length, the ER to transport methanol and the probability to be chosen by hazmat carriers are given. It is found that two of the available paths are the same while the rest of them are different either in length or ER for both models. The lowest ER path in both models is Path 1. It can be seen that Path 1 has 24.6% chance to be traveled in SPP-ER model while it has the probability of 34.5% to be traveled in the proposed model. The proposed model assigns larger probabilities for low risk paths than SPP-ER model does.

7 Concluding Remarks

In this paper, we formulate a road-ban problem in hazardous hazmat transportation considering the uncertainty of routing behavior and a risk-averse measure for hazmat accident consequences. With the probabilistic route choice, the risk distribution for hazmat transportation incorporates with

Table 5: Comparisons of available paths for transporting methanol from node 110 to node 105 between RUM-CVaR and the deterministic (SPP-ER) model for the Ravenna network

Model	Path	Length	ER	Prob
RUM-CVaR _{0.99}	1: 110 → 104 → 83 → 78 → 62 → 54 → 45 → 31 → 98 → 14 → 11 → 6 → 3 → 5 → 105	18.24	0.0149	0.345
	14: 110 → 104 → 83 → 78 → 62 → 54 → 45 → 43 → 36 → 30 → 26 → 24 → 20 → 10 → 5 → 105	26.04	0.0169	0.158
	19: 110 → 104 → 83 → 78 → 62 → 54 → 56 → 46 → 43 → 36 → 30 → 26 → 24 → 20 → 10 → 5 → 105	26.87	0.0168	0.146
	26: 110 → 104 → 83 → 78 → 62 → 54 → 45 → 43 → 36 → 30 → 26 → 24 → 20 → 10 → 9 → 7 → 5 → 105	28.17	0.0172	0.128
	34: 110 → 104 → 83 → 78 → 62 → 54 → 56 → 46 → 43 → 36 → 30 → 26 → 24 → 20 → 10 → 9 → 7 → 5 → 105	29.00	0.0172	0.118
	44: 110 → 104 → 83 → 78 → 62 → 54 → 56 → 46 → 43 → 45 → 31 → 98 → 14 → 11 → 6 → 3 → 5 → 105	30.18	0.0171	0.105
SPP-ER	1: 110 → 104 → 83 → 78 → 62 → 54 → 45 → 31 → 98 → 14 → 11 → 6 → 3 → 5 → 105	18.24	0.0149	0.246
	3: 110 → 104 → 83 → 78 → 62 → 57 → 58 → 38 → 54 → 45 → 31 → 98 → 14 → 11 → 6 → 3 → 5 → 105	19.93	0.0169	0.207
	5: 110 → 104 → 83 → 78 → 74 → 76 → 69 → 56 → 54 → 45 → 31 → 98 → 14 → 11 → 6 → 3 → 5 → 105	23.26	0.0157	0.149
	8: 110 → 104 → 83 → 78 → 74 → 69 → 56 → 54 → 45 → 31 → 98 → 14 → 11 → 6 → 3 → 5 → 105	24.77	0.0160	0.128
	18: 110 → 104 → 83 → 78 → 74 → 76 → 69 → 56 → 46 → 43 → 45 → 31 → 98 → 14 → 11 → 6 → 3 → 5 → 105	26.69	0.0164	0.106
	27: 110 → 104 → 83 → 78 → 74 → 69 → 56 → 46 → 43 → 45 → 31 → 98 → 14 → 11 → 6 → 3 → 5 → 105	28.20	0.0167	0.091
	44: 110 → 104 → 83 → 78 → 62 → 54 → 56 → 46 → 43 → 45 → 31 → 98 → 14 → 11 → 6 → 3 → 5 → 105	30.18	0.0171	0.073

not only the road accident probability but also the carriers routing behavior. Following Toumazis et al. (2013), we introduce conditional value-at-risk (CVaR) as a general, coherent, and risk-averse approach. We present a CVaR minimization model for hazmat network design problems. The proposed model is a mixed-integer nonlinear program, which can be decomposed into two stages: (1) searching the optimal solution for a nonnegative variable; (2) solving MILPs given the nonnegative variable. We develop a line search with mapping based on Benders decomposition for solving MILP sub-problems and obtain quality network design solutions.

We present case studies in the real road network of Ravenna. The comparisons of algorithms show that the proposed methods can solve the CVaR minimization network design problem efficiently and generate quality network design solutions. To highlight the value of our model, comparisons of the deterministic model and the proposed model are conducted. When the confidence level of CVaR is small, it indicates that decision makers and regulators for the transportation network pay limited attention on sever accidents.

With the proposed algorithms, the average computation time to solve CVaR minimization problem of a 105-node and 268-arc network was 3 hours for different probability threshold α values. The exact algorithm that uses CPLEX solver of version 12.6 was not inadequate to solve even a linear subproblem, because the optimality gap solving $f_\alpha(r)$ with CPLEX was 99.9% after 24 hours.

While the proposed algorithm was shown to be effective, for large-scale urban networks, we will need a faster algorithm. Since the proposed algorithm relies on solutions of multiple MILP problems, it is not suitable for large-scale networks. Developing a fast heuristic algorithm is a potential future research direction.

There are some other limitations of the proposed model and method. The first limitation is that we did not consider any uncertainty from data sources and hazmat travel demand. Considering these additional sources of uncertainty will make the network design problem more challenging. Second, the RUM used in this paper is not the most advanced random route-choice model. More advanced RUM approaches are available, which may be incorporated within the network design problem suggested in this paper, at the cost of computational time increases. Third, we did not consider equity in the network design. Risk equity among zones and cost equity among hazmat carriers should be considered for fair management of the road infrastructure. Addressing these limitations are promising future research directions.

Another interesting extension of the proposed method in this paper will be the consideration of multiples modes of hazmat transportation as in multimodal or intermodal transportation. The discrete-choice model in RUM should be extended to consider mode choices in multi-modal transportation. This will create a nested logit model, for example, which adds significant computational and analytical complexity in the model.

Acknowledgment

This manuscript is based upon work funded partially by a grant from the U.S. Department of Transportation's University Transportation Centers Program. However, the U.S. Government assumes no liability for the contents or use thereof. The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the information presented herein.

References

- Abkowitz, M. D., M. Lepofsky, P. Cheng. 1992. Selecting criteria for designating hazardous materials highway routes. *Transportation Research Record* (1333).
- Artzner, P., F. Delbaen, J.-M. Eber, D. Heath. 1997. Thinking coherently: generalised scenarios rather than VaR should be used when calculating regulatory capital. *Risk-London-Risk Magazine Limited* **10** 68–71.
- Artzner, P., F. Delbaen, J.-M. Eber, D. Heath. 1999. Coherent measures of risk. *Mathematical Finance* **9**(3) 203–228.
- Battelle. 2006. Hazardous materials routing survey analysis. URL <https://www.fmcsa.dot.gov/sites/fmcsa.dot.gov/files/docs/HM-Highway-Routing-Route-Plan-Guidance-Report-and-Appendices-FINAL-March-2009.pdf>.
- Ben-Akiva, M., M. Bergman, A. J. Daly, R. Ramaswamy. 1984. Modeling inter-urban route choice behaviour. *Proceedings of the 9th international symposium on transportation and traffic theory*. VNU Science Press Utrecht, The Netherlands, 299–330.
- Ben-Akiva, M., M. Bierlaire. 1999. Discrete choice methods and their applications to short term travel decisions. *Handbook of Transportation Science*. Springer, 5–33.
- Ben-Akiva, M. E., S. R. Lerman, S. R. Lerman. 1985. *Discrete choice analysis: theory and application to travel demand*, vol. 9. MIT press.
- Birge, J. R. 1982. The value of the stochastic solution in stochastic linear programs with fixed recourse. *Mathematical Programming* **24**(1) 314–325.
- Bonvicini, S., G. Spadoni. 2008. A hazmat multi-commodity routing model satisfying risk criteria: a case study. *Journal of Loss Prevention in the Process Industries* **21**(4) 345–358.
- Cascetta, E., A. Nuzzolo, F. Russo, A. Vitetta. 1996. A modified logit route choice model overcoming path overlapping problems: specification and some calibration results for interurban networks. *Proceedings of the 13th International Symposium on Transportation and Traffic Theory*. Pergamon Oxford, NY, USA, 697–711.

- Daganzo, C. F., Y. Sheffi. 1977. On stochastic models of traffic assignment. *Transportation Science* **11**(3) 253–274.
- Davis, G. A. 1994. Exact local solution of the continuous network design problem via stochastic user equilibrium assignment. *Transportation Research Part B: Methodological* **28**(1) 61–75.
- Dial, R. B. 1971. A probabilistic multipath traffic assignment model which obviates path enumeration. *Transportation Research* **5**(2) 83–111.
- Duffie, D., J. Pan. 1997. An overview of value at risk. *The Journal of Derivatives* **4**(3) 7–49.
- Dunning, I., J. Huchette, M. Lubin. 2017. JuMP: a modeling language for mathematical optimization. *SIAM Review* **59**(2) 295–320. doi: 10.1137/15M1020575.
- Emergency Response Guidebook. 2012. Emergency response guidebook : A guidebook for first responders during the initial phase of a dangerous goods/hazardous materials transportation incident. URL http://phmsa.dot.gov/pv_obj_cache/pv_obj_id_7410989F4294AE44A2EBF6A80ADB640BCA8E4200/filename/ERG2012.pdf.
- Erkut, E., F. Gzara. 2008. Solving the hazmat transport network design problem. *Computers & Operations Research* **35**(7) 2234–2247.
- Erkut, E., A. Ingolfsson. 2000. Catastrophe avoidance models for hazardous materials route planning. *Transportation Science* **34**(2) 165–179.
- Esfandeh, T., R. Batta, C. Kwon. 2017. Time-dependent hazardous-materials network design problem. *Transportation Science* **52**(2) 454–473.
- Esfandeh, T., C. Kwon, R. Batta. 2016. Regulating hazardous materials transportation by dual toll pricing. *Transportation Research Part B: Methodological* **83** 20–35.
- Fan, T., W.-C. Chiang, R. Russell. 2015. Modeling urban hazmat transportation with road closure consideration. *Transportation Research Part D: Transport and Environment* **35** 104–115.
- Federal Motor Carrier Safety Administration. 2001. Comparative risks of hazardous materials and nonhazardous materials truck shipment accidents/incidents. URL <https://www.hsd1.org/?abstract&did=233019>.
- Federal Motor Carrier Safety Administration. 2016. Federal motor carrier safety regulations. URL <https://www.fmcsa.dot.gov/regulations/hazardous-materials/how-comply-federal-hazardous-materials-regulations>.
- Federal Motor Carrier Safety Administration. 2018. National hazardous materials route registry. URL <https://www.fmcsa.dot.gov/regulations/hazardous-materials/national-hazardous-materials-route-registry>.

- Fontaine, P., S. Minner. 2018. Benders decomposition for the hazmat transport network design problem. *European Journal of Operational Research* **267**(3) 996–1002.
- Geoffrion, A. M. 1972. Generalized benders decomposition. *Journal of Optimization Theory and Applications* **10**(4) 237–260.
- Gzara, F. 2013. A cutting plane approach for bilevel hazardous material transport network design. *Operations Research Letters* **41**(1) 40–46.
- Hess, S., M. Quddus, N. Rieser-Schüssler, A. Daly. 2015. Developing advanced route choice models for heavy goods vehicles using gps data. *Transportation Research Part E: Logistics and Transportation Review* **77** 29–44.
- Hosseini, S. D., M. Verma. 2018. Conditional value-at-risk (CVaR) methodology to optimal train configuration and routing of rail hazmat shipments. *Transportation Research Part B: Methodological* **110** 79–103.
- Jin, H., R. Batta. 1997. Objectives derived from viewing hazmat shipments as a sequence of independent Bernoulli trials. *Transportation Science* **31**(3) 252–261.
- Kang, Y., R. Batta, C. Kwon. 2014. Value-at-risk model for hazardous material transportation. *Annals of Operations Research* **222**(1) 361–387.
- Kara, B. Y., V. Verter. 2004. Designing a road network for hazardous materials transportation. *Transportation Science* **38**(2) 188–196.
- Kwon, C., T. Lee, P. Berglund. 2013. Robust shortest path problems with two uncertain multiplicative cost coefficients. *Naval Research Logistics* **60**(5) 375–394.
- List, G. F., P. B. Mirchandani, M. A. Turnquist, K. G. Zografos. 1991. Modeling and analysis for hazardous materials transportation: Risk analysis, routing/scheduling and facility location. *Transportation Science* **25**(2) 100–114.
- Liu, H., D. Z. Wang. 2015. Global optimization method for network design problem with stochastic user equilibrium. *Transportation Research Part B: Methodological* **72** 20–39.
- Marcotte, P., A. Mercier, G. Savard, V. Verter. 2009. Toll policies for mitigating hazardous materials transport risk. *Transportation Science* **43**(2) 228–243.
- McFadden, D. 1975. The revealed preferences of a government bureaucracy: theory. *The Bell Journal of Economics* 401–416.
- Pipeline and Hazardous Materials Safety Administration. 2017. Hazmat incident report: 10 year incident summary reports. URL <https://hip.phmsa.dot.gov>.
- Prashker, J. N., S. Bekhor. 2004. Route choice models used in the stochastic user equilibrium problem: a review. *Transport Reviews* **24**(4) 437–463.

- Quattrone, A., A. Vitetta. 2011. Random and fuzzy utility models for road route choice. *Transportation Research Part E: Logistics and Transportation Review* **47**(6) 1126–1139.
- Ramming, M. S. 2001. Network knowledge and route choice. Ph.D. thesis, Massachusetts Institute of Technology.
- Rockafellar, R. T., S. Uryasev. 2000. Optimization of conditional value-at-risk. *Journal of Risk* **2** 21–42.
- Rockafellar, R. T., S. Uryasev. 2002. Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance* **26**(7) 1443–1471.
- Santoso, T., S. Ahmed, M. Goetschalckx, A. Shapiro. 2005. A stochastic programming approach for supply chain network design under uncertainty. *European Journal of Operational Research* **167**(1) 96–115.
- Su, L., L. Sun, M. Karwan, C. Kwon. 2017. Spectral risk measure minimization in hazardous materials transportation. *IIEE Transactions* Accepted.
- Sun, L., M. H. Karwan, C. Kwon. 2016. Robust hazmat network design problems considering risk uncertainty. *Transportation Science* **50**(4) 1188–1203.
- Sun, L., M. H. Karwan, C. Kwon. 2018. Generalized bounded rationality and robust multi-commodity network design. *Operations Research* **66**(1) 42–57.
- Taslimi, M., R. Batta, C. Kwon. 2017. A comprehensive modeling framework for hazmat network design, hazmat response team location, and equity of risk. *Computers & Operations Research* **79** 119–130.
- Toumazis, I., C. Kwon. 2013. Routing hazardous materials on time-dependent networks using conditional value-at-risk. *Transportation Research Part C: Emerging Technologies* **37** 73–92.
- Toumazis, I., C. Kwon. 2016. Worst-case conditional value-at-risk minimization for hazardous materials transportation. *Transportation Science* **50**(4) 1174–1187.
- Toumazis, I., C. Kwon, R. Batta. 2013. Value-at-risk and conditional value-at-risk minimization for hazardous materials routing. *Handbook of OR/MS Models in Hazardous Materials Transportation*. Springer, 127–154.
- Verter, V., B. Y. Kara. 2008. A path-based approach for hazmat transport network design. *Management Science* **54**(1) 29–40.
- Williams, H. C. 1977. On the formation of travel demand models and economic evaluation measures of user benefit. *Environment and Planning A* **9**(3) 285–344.
- Yen, J. Y. 1971. Finding the K shortest loopless paths in a network. *Management Science* **17**(11) 712–716.