Incorporating Driver Behaviors in Network Design Problems: Challenges and Opportunities

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Abstract

The network design problem (NDP) is to make optimal decisions to achieve a certain objective such as minimizing the total travel cost in the network. A critical component to NDP is how travelers make their route choices. Researchers in transportation have adopted human decision theories to describe more accurate route choice behaviors. In this paper, we review the NDP with various route choice models: the random utility model (RUM), bounded rationality (BR), cumulative prospect theory (CPT), the fuzzy logic model (FLM) and search-information-learning-knowledge (SILK) theory. Additionally, we identify challenges in applying behavioral route choice models to NDP and opportunities for future research.

Keywords: network design; behavior route choice; random utility; bounded rationality; cumulative prospect theory; fuzzy logic; SILK theory

1 Introduction

In the network design problem (NDP) in which a central authority determines the network conditions, it is essential to predict and model the behavior of network users. In the context of

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of vehicular networks, the primary interest in NDP is which routes network users will choose. Figure 1 shows the typical structure of NDP. The objective of the shown NDP is to minimize the total travel time of all travelers. However, other objectives such as maximizing total tolls collected can also be used. The decision variables \( y \) can be tolls enforced on the links for congestion pricing problem, new streets to be built for discrete NDP, and the capacities of existing streets for continuous NDP among others. Given \( y \), the flow pattern \( l \) can be decided by a traffic assignment module which models how travelers make route choices and reach equilibrium. NDP optimizes a certain objective function by making decisions \( y \) based on the travelers’ traffic pattern.

Many NDPs include route choice models that assume drivers to be rational and homogeneous with perfect information. However, Zhu and Levinson (2010) empirically study commuters’ routes and find that most people do not choose the shortest path. Nakayama et al. (2001) compare different route choice rules of no switching, random switching, rational and experience based rules using simulation. Their results show that drivers are neither homogeneous nor fully rational. They point out the need for behavioral models of drivers’ route choice as the foundation of equilibrium analysis of network flow.

Many studies focus on modeling travelers’ route choice behavior and applications with more accurate behavioral assumptions are attracting more research. Among the earliest is the random utility model (Sheffi, 1985) which assumes travelers’ perceptions of path cost are inaccurate. With the development of human decision theory (Conlisk, 1996; Tversky and Kahneman, 1992; Zadeh, 1965), researchers in transportation develop models to describe human decision behaviors in choosing routes. The most important ones are the bounded rationality (BR) models (Mahmassani and Chang, 1987) which consider that travelers are satisfied if the route utility is within a
certain band; prospect theory (PT) and cumulative prospect theory (CPT) models (Avineri and Prashker, 2004) which account for travelers’ risk preferences in regard to different route utilities; fuzzy logic models (FLM) (Teodorović and Kikuchi, 1990); and search-information-learning-knowledge (SILK) theory (Zhang, 2007) which models the dynamic route decision process of travelers.

The route choice models are then incorporated into applications of traffic assignment problems, network design problems such as continuous NDP, discrete NDP and congestion toll pricing problem. Zhang (2011) explores the traffic assignment problem by comparing deterministic user equilibrium (DUE) and stochastic UE (SUE, based on RUM), bounded rationality user equilibrium (BRUE, based on BR) and SILK-UE (based on SILK theory). The SILK-UE is also referred as behavioral UE (BUE) in the literature. However, we adopt SILK-UE in order to differentiate it from UE under other behavioral models.

In this paper, we extend the work of Zhang (2011) by considering two additional classes of behavior models (CPT, FLM) with a particular focus on their applications in network design problems. We also discuss challenges in practical applications and implications on computability of network design problems. The contributions of the paper can be summarized as follows:

- We conduct an updated review of the major route choice models considering drivers’ behaviors;
- We review the NDP with user equilibrium based on different route choice models;
- We compare the models from various aspects to denote their benefits and challenges; and
- We make suggestions on applications of route choice models to NDP and further research opportunities on this topic.

The remainder of the paper is organized as follows. Section 2 reviews the literature on route choice models and Section 3 focuses on the applications to NDP. Then we compare the listed models in Section 4 and finally Section 5 gives conclusions and future research directions.
2 Review of Route Choice Models

Route choice modeling deals with which path travelers will choose for their certain trip plan. It is essential to model route choice behavior since it is the basis for any traffic planning problem. However, it can be hard due to uncertain network conditions (for example, how congested certain road segments are), the uncertainty about drivers’ perceptions of route characteristics and the preferences of travelers to uncertainty (risk taking, neutral, averse, or having an indifference band). Henn and Ottomanelli (2006) summarize the route choice process for the traffic assignment problem as follows: (1) traffic condition on a route $k$ for origin-destination (OD) pair $w$ induces some costs factors, (2) for a driver of a given population those factors are combined into a cost $c^w_k$, (3) this cost is perceived as $\tilde{c}^w_k$, (4) for each route, the grade of preference with regard to its cost is $\phi^w_k \in [0,1]$, (5) then the probability that driver chooses route $k$ is calculated, (6) traffic flow is assigned based on obtained probability. From this process, we can see that various uncertainties can occur at different levels: (1) the uncertainty of the road conditions, (2) the perceived cost for the real transportation cost, and (3) the uncertainty of the preference for each individual.

In this section, we will review the current literature in modeling route choice involving behavioral uncertainty. Various models have been proposed in order to address this problem. The most important ones are the random utility model (RUM), bounded rationality (BR), cumulative prospect theory (CPT), fuzzy logic model (FLM) and search-information-learning-knowledge (SILK) theory.

2.1 Random Utility and Discrete Choice Models

Random utility models assume that drivers still minimize their travel time but their perceived path costs are inaccurate. Then the utility of using the $k$-th path between an OD pair $w$, $U^w_k$ is given by

$$U^w_k = -\theta^w c^w_k + \xi^w_k, \quad \forall \ k, w,$$
where $c_k^w$ is the measured travel time, $\theta^w$ is a positive parameter, and $\xi_k^w$ is a random term. Then the probability of choosing path $k$ for OD pair $w$ is defined as

$$P_k^w = \Pr\left(U_k^w \geq \max_{l \neq k} U_l^w\right),$$

which is equivalent to the probability that $U_k^w$ is greater than or equal to all $U_l^w$ for $l \neq k$. If the random terms $\xi_k^w$ are independently and identically distributed Gumbel variates, according to Sheffi (1985), the choice probability is

$$P_k^w = e^{-\theta^w c_k^w} \sum_l e^{-\theta^w c_l^w}.$$

If the random error term of each utility is normally distributed, then it becomes a multinomial probit model. However, the calculation of the probit choice probability is not straightforward in the presence of more than two alternatives. The choice probability can be estimated by using either an analytical approximation or a Monte Carlo simulation (Sheffi, 1985).

Since there is a high degree of similarity among alternatives generated by RUM, the independence of alternatives is questioned by researchers. Many route choice models are then proposed in the literature; for example, the Logit structure models such as C-Logit (Cascetta et al., 1996) and Path Size Logit (Ben-Akiva and Bierlaire, 1999). The Logit structure models introduce a correction term within the deterministic part of the utility function to approximate the correlation among alternative routes.

General Extreme Value (GEV) models are also proposed to address the independence issue. These kinds of models account for similarities by allowing the random components of alternatives to be correlated. Examples include Paired Combinatorial Logit (Chu, 1989; Koppelman and Wen, 2000), Cross Nested Logit (Vovsha, 1997), Generalized Nested Logic (Wen and Koppelman, 2001) among others. Readers can refer to Prato (2009) for a more detailed introduction. Besides Logit structure and GEV models, Castillo et al. (2008) propose the multinomial weibit model by considering a Weibull perceived travel cost.
2.2 Bounded Rationality Models

The bounded rationality (BR) models replace utility maximization with an alternative-satisfying behavior paradigm (Mahmassani and Chang, 1987). Instead of trying to optimize their route utility, drivers are indifferent with near optimal routes. According to Lou et al. (2010), the drivers with bounded rationality can be defined as those who (1) always choose routes with no cycles and (2) do not necessarily switch to the shortest (cheapest) routes when the difference between the travel time (or costs) on their current routes and shortest one is no greater than a threshold value.

Psychology and economics provide wide-ranging evidence that bounded rationality is important and include bounded rationality into various models for greater accuracy. Conlisk (1996) surveys the economic literature and provides the reasons for incorporating bounded rationality into economic models. In the transportation field, Mahmassani and Chang (1987) first study bounded rationality user equilibrium (BRUE) in departure time decision problems. Commuters are assumed to behave as if they had an indifference band (IB) of tolerable schedule delay which can be defined as $[IB_{\text{min}}, IB_{\text{max}}]$. They investigate the existence and uniqueness of BRUE under an idealized commuting system with only one OD pair affected by a single bottleneck. Then they show how to extend BR to multiple origins, bottlenecks and multiple origin-destination cases. Simulation experiments are later proposed to test the bounded rationality assumption by (Hu and Mahmassani, 1997; Mahmassani and Liu, 1999) using the simulation tool DYNASMART (Jayakrishnan et al., 1994).

2.3 Prospect and Cumulative Prospect Theory

Before we introduce prospect theory (PT) and cumulative prospect theory (CPT), we first discuss expected utility theory. Expected utility theory (Von Neumann, 1944) states that a decision is made based on the expected utility value when involving uncertainty. For route choice models, an individual will try to minimize his or her expected cost. Assume $E_j$ denotes possible events, each of which occurs with probability $Pr(E_j)$. Let $o(E_j)$ be the outcome of event $j$, then the
The expected outcome is

$$E[o] = \sum_{j=1}^{n} Pr(E_j)o(E_j).$$

Henn and Ottomanelli (2006) argue that the expected utility model can capture randomness such as road condition uncertainty due to recurrent events in the route choice model. The expected utility model assumes drivers behave as if assigning probabilities to random travel times and choose a route that maximizes the expected outcome. The expected utility model assumes equal preference for different uncertainty outcomes. However, this is criticized by behavioral scientists (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) and also by transportation researchers (Avineri and Prashker, 2004). They find the expected utility model has two violations: the extreme under-weighting of high probabilities which is known as Allais paradox (Allais, 1979) and the inflation of small probabilities.

In order to deal with violations of the expected utility model, prospect theory (Kahneman and Tversky, 1979) and cumulative prospect theory (Tversky and Kahneman, 1992) are proposed. PT is first analyzed by lottery evaluations. The lotteries are first mapped to gains and losses using a reference point. Then an S-shaped value function $\varphi(x)$ and inverse S-shaped weighting functions $W^+(p)$, $W^-(p)$ (as shown in Figure 2) are used to evaluate the lotteries. $x$ is the mapped outcome and $p$ is the corresponding occurring probability. Finally the perceived value of utility $V$ can be calculated.
The following are the mathematical formulations for value and weight functions:

$$\Delta x_i = x_i - x_0,$$

$$\varphi(x) = \begin{cases} x^\alpha, & x \geq 0, \\ -\lambda(-x)^\beta, & x < 0, \end{cases}$$

$$W^+(p) = p^\gamma / [p^\gamma + (1 - p)^\gamma]^{1/\gamma},$$

$$W^-(p) = p^\delta / [p^\delta + (1 - p)^\delta]^{1/\delta},$$

$$V(x_1, p_1; x_2, p_2) = W^+(p_1)\varphi(\Delta x_1) + W^-(p_2)\varphi(\Delta x_2),$$

where $x_0$ is the reference point, $\Delta x_1$ is gain and $\Delta x_2$ is loss. Cumulative prospect theory employs cumulative rather than single decision weights. Suppose an alternative, denoted by the pair $(x; p)$, is composed of $m + n + 1$ possible outcomes as $x_{-m} < \cdots < x_0 < \cdots < x_n$, which occur with probabilities $p_{-m}, \cdots, p_n$. Then the cumulative weights are defined as follows:

$$\varpi^+(p_i) = W^+(p_i + \cdots + p_n) - W^+(p_{i+1} + \cdots + p_n); \quad 0 \leq i \leq n,$$

$$\varpi^-(p_{-j}) = W^-(p_{-m} + \cdots + p_{-j}) - W^-(p_{-m} + \cdots + p_{-j-1}); \quad m \leq -j < 0,$$

and the value of the utility can be calculated as

$$V(x; p) = \sum_{i=0}^{n} \varphi(\Delta x_i)\varpi^+(p_i) + \sum_{i=-m}^{1} \varphi(\Delta x_i)\varpi^-(p_i).$$

In transportation, Avineri and Prashker (2004) first examine the violations of the expected utility model and introduce prospect theory into the route choice model. They achieve this by conducting a survey using questionnaires inspired by Tversky and Kahneman (1992) to examine the choices of responders’ alternative routes from home to work. They use default parameters $\alpha = \beta = 0.88, \lambda = 2.25, \gamma = 0.61, \delta = 0.69$ from Tversky and Kahneman (1992) and find the CPT prediction results are consistent with the experimental ones. Senbil and Kitamura (2004) apply prospect theory to departure time choice of morning commuters. Schwanen and Ettema (2009) apply CPT to model how employed parents make choices in unreliable transport networks.
to pick up their child(ren) from a nursery. Gao et al. (2010) compare CPT and the expected utility model and find CPT is more accurate in modeling travelers’ route choice behavior. Ramos et al. (2014) compare prospect theory, utility theory and regret theory to investigate travelers’ behavior with travel time uncertainty. Additionally, a review of CPT theory can be found in Li and Hensher (2011).

Due to the difference in applications, researchers have argued the need for parameter estimation in travel route choice settings. Avineri and Bovy (2008) propose a method of estimating the loss aversion parameter $\lambda$ and argue the estimation of other parameters are dependent on the reference point value. Schwanen and Ettema (2009) utilizes a generic search to obtain the set of parameter values. Xu et al. (2011b) estimate the parameters by setting $\gamma$ to its default value. They found in transportation applications, commuters have a much greater degree of risk aversion when confronted with the prospect of gains, a greater degree of risk seeking when confronted with the prospect of losses and have a lower degree of relative sensitivity of losses to gains than the average individual. Jou and Chen (2013) collect 539 valid questionnaires of respondents choosing from two freeways given the predicted travel times and their corresponding probabilities. Then STATA is used to estimate the coefficients of the CPT model. They analyze the parameter difference based on the trip purpose (business or leisure) and the drivers’ characteristics.

Another issue for the application of CPT is choosing the reference point value. Avineri and Prashker (2004) use a reference point of 30 minutes travel time and perform sensitivity analysis on the reference point value. They find the route choice decision is sensitive to the reference point. Connors and Sumalee (2009) consider different reference values for the path cost and flow. Jou and Chen (2013) use the reference point value from the survey of respondents. de Moraes Ramos et al. (2013) consider the role of heterogeneity in the reference point. With no heterogeneity, they consider the following reference points: travel time of the fastest route, travel time of the most reliable route, the average travel time of all routes and the minimum travel time of the fastest route. Accounting for heterogeneity, travelers update expectations and consequently update their reference point after each route. Mathematically, Xu et al. (2011b)
propose an optimization model to set the reference point value to the time he or she budgeted to ensure his or her desired on-time arrival probability.

2.4 Fuzzy Logic Models

Incorporating fuzzy sets (Zadeh, 1965) into the route choice model is another way to deal with uncertainty. There are two different kinds of fuzzy route choice models: fuzzy rule based models and fuzzy cost based models (Henn and Ottomanelli, 2006).

For the fuzzy rule based models, Teodorović and Kikuchi (1990) are first to address the route choice problem using fuzzy inference techniques. Similar models are proposed in (Lotan and Koutsopoulos, 1993; Teodorović et al., 1998; Vythoulkas and Koutsopoulos, 2003; Murat and Uludag, 2008). Such models are based on linguistic rules using fuzzy sets (if the travel time on route 1 is very short and on route 2 is intermediate, then the driver will certainty choose route 1). But actually, the proposed models are only valid for one particular network and do not give any representation of how information (and, conversely, uncertainty) is carried out by drivers (Henn and Ottomanelli, 2006).

For fuzzy cost based models, more arithmetic approaches are proposed. Some route choice models (Henn, 2000, 2003; Henn and Ottomanelli, 2006) are based on a comparison of fuzzy numbers representing the costs of the routes. These costs are represented and compared using possibility theory, which is based on a max-axiom instead of an additive axiom of the expected theory. Quattrone and Vitetta (2011) specify and calibrate FLM to be applied in real-size systems and compare it with RUM. Other models use fuzzy logic to model the perceived link cost. Wang and Liao (1999) consider the node-arc incidence as fuzzy while Chang and Chen (2000) formulate link cost using a triangular fuzzy member function.

2.5 SILK Theory

SILK theory emphasizes the role of search, information, learning, and knowledge in travel decision making (Zhang, 2007). It is a positive approach of how travelers actually make route decisions, instead of assuming the travelers are rational, have perfect information and use utility maximization as in the normative way. Its underlying theories have been supported in the
literature. For dynamic learning, Mahmassani and Liu (1999) consider the perceived cost as a function of the weighted average of past travel times. Jha et al. (1998) consider using Bayesian learning models to update current perceived travel time by using information from the previous travel time periods. Nakayama et al. (2001) show that travelers are not fully rational and homogeneous with perfect information in route decision making.

Zhang (2006a) develops an agent-based model of spatial learning and route choice based on a positive theory of travel behavior. The model considers the route finding process in a dynamic day-to-day manner. It updates and utilizes the probability that a traveler perceives a better path once its information is available and the probability a traveler takes the path once it is learned. Zhang (2007) then proposes SILK theory and develops quantitative models for a traveler’s individual behavior. SILK theory suggests that spatial knowledge relevant to route choice can be quantified and improved though a Bayesian learning process. The expected gain from an additional search for alternative routes is based on the traveler’s unique knowledge and beliefs. Subjective search gain and perceived search cost jointly determine search starting and stopping conditions. The heuristics which individuals develop to search for routes and choose among known routes are represented by if-then rules which should be empirically derived and validated.

3 Review of NDP with Route Choice Models

The NDP is to make an optimal decision in order to achieve a certain objective in the network while accounting for the route choice behavior of network users. The network users are usually modelled by user equilibrium with different route choice models. Although route choice models have been studied widely, the applications of them in network design models are not well developed. In this section, we will review different user equilibrium models and its applications in NDP. Before that, we list some notation that will be used in Table 1.
\( \mathcal{N} \): set of nodes,
\( \mathcal{A} \): set of links,
\( \mathcal{W} \): set of origin-destination (OD) pairs,
\( q^w \): travel demand between OD pair \( w \in \mathcal{W} \),
\( \mathbf{q} \): column vector of all the OD travel demands, \( \mathbf{q} = (q^w, w \in \mathcal{W})^T \),
\( P^w \): set of all paths between OD pair \( w \in \mathcal{W} \),
\( \delta_{ijk}^w \): \( \delta_{ijk}^w = 1 \) if path \( k \in P^w \) between OD pair \( w \in \mathcal{W} \) traverse link \((i, j) \in \mathcal{A} \),
\( \delta_{ijk}^w = 0 \) otherwise,
\( \Delta^w \): link/path incidence matrix associated with OD pair \( w \in \mathcal{W} \)
\( \Delta \): link/path incidence matrix for the entire network, \( \Delta = (\Delta^w, w \in \mathcal{W}) \)
\( f_{kw} \): traffic flow on path \( k \in P^w \) between OD pair \( w \in \mathcal{W} \),
\( f^w \): column vector of traffic flows on all the paths between OD pair \( w \in \mathcal{W} \)
\( \Lambda \): OD pair/path incidence matrix, \( \Lambda = (\sigma_{wk} : w \in \mathcal{W}, k \in P^w) \), where \( \sigma_{wk} \) equals 1 if path \( k \in P^w \) and 0, otherwise,
\( \Theta \): node link incidence matrix,
\( D^w \): demand vector satisfying \( D_{o(w)}^w = q^w \), \( D_{d(w)}^w = -q^w \) and \( D_i^w = 0 \) for all other \( i \in \mathcal{N} \),
\( o(w) \) and \( d(w) \) are the origin and destination node for OD pair \( w \),
\( z_{ij}^w \): flow on link \((i, j) \in \mathcal{A} \) for OD pair \( w \),
\( z^w \): column vector of all link flows for OD pair \( w \)
\( v_{ij} \): traffic flow on link \((i, j) \in \mathcal{A} \),
\( v \): column vector of all the link traffic flows, \( v = (v_{ij}, (i, j) \in \mathcal{A})^T \),
\( t_{ij}(v_{ij}) \): travel time on link \((i, j) \in \mathcal{A} \), and it is a nonnegative, monotonically increasing and continuously differentiable function,
\( c_k^w(v) \): travel time on path \( k \in P^w \), and \( c_k^w = \sum_{(i, j) \in \mathcal{A}} t_{ij}(v_{ij}) \delta_{ijk}^w \),
\( c^w(v) \): column vector of travel times on all the paths connecting OD pair \( w \in \mathcal{W} \),
\( \tau_{ij} \): toll collected on link \((i, j) \in \mathcal{A} \),
\( \tau \): vector form of toll collected for all links \((i, j) \in \mathcal{A} \).

| Table 1: Notation for the Network Design Problem |

### 3.1 NDP with RUM

After significant initial research on random utility in transportation area, stochastic user equilibrium (SUE) is proposed to address the perceived cost uncertainty in Waldrop’s first principle. Prashker and Bekhor (2004) review the SUE based on various random utility models: logit based SUE (Fisk, 1980), C-logit based SUE (Zhou et al., 2012), path size logit based SUE (Chen et al., 2012), cross nested logit based SUE (Prashker and Bekhor, 2000) and general nested logic based SUE (Bekhor and Prashker, 2001). Additionally, Kitthamkesorn and Chen (2013) propose a weibit SUE considering a Weibull perceived cost uncertainty distribution.
RUM is the most applied method for NDP among all the behavioral route choice models. Davis (1994) considers the continuous NDP with an SUE based on the logit model, which leads to a differentiable and large-scale, but tractable version of NDP. Lim et al. (2005) suggest a heuristic solution algorithm for continuous NDP by directly using the derivative deduced from the logit route choice model. Uchida et al. (2007) study NDP for multi-model networks with probit-based SUE flow.

For congestion toll pricing problem, Yang (1999) demonstrates that the classical principle of marginal-cost pricing is still applicable in a network under SUE and is a good alternative to drive an SUE flow pattern toward system optimum. Chen et al. (2004) study a toll design problem based on stochastic route choice behavior for multiple user groups when only certain links are tollable. Liu et al. (2013) address the toll design problem with logit-based SUE with first-best pricing. Let the feasible path flow set and feasible link flow set be defined as

$$\Omega_f = \{f : q = \Lambda f, \ f \geq 0\},$$

$$\Omega_v = \{v : q = \Lambda f, \ v = \Delta f, \ f \geq 0\}.$$  

Then under logit-based user equilibrium, the feasible set of alternative tolls can be formulated as a variational inequality (VI) problem:

$$\sum_{w \in W} \sum_{k \in P^w} \left( \bar{c}_k^w + \frac{1}{\theta} \ln f_k^* \right) (f_{kw} - f_{kw}^*) \geq 0, \forall f \in \Omega_f,$$  

where $\bar{c}_k^w$ is the tolled path cost. According to Hearn and Ramana (1998), problem (6) can be formulated as a set of constraints. Then the congestion toll pricing problem with SUE can be solved with a two stage process. We first calculate the system optimal flow, then using these equivalent constraints based on the system optimal flow, it is possible to decide how to toll the network using different objectives such as minimizing the total tolls collected:

$$\min_{\tau_{ij}} \sum_{(i,j) \in A} \tau_{ij} v_{ij}.  \quad (7)$$

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Liu et al. (2013) also propose a toll design model using C-logit based user equilibrium. Similar models based on (6) and (7) can be formulated and solved using the two stage process.

### 3.2 NDP with BR

Since early work applying BR in transportation (Mahmassani and Chang, 1987), researchers in the transportation area have applied BR to model more accurate traveling behaviors. Szeto and Lo (2006) formulate a route choice bounded rationality dynamic user equilibrium problem as a discrete time nonlinear complementarity problem. Guo and Liu (2011) consider the irreversible network change using BRUE due to necessity in the existence of multiple network equilibria. Wu et al. (2013) propose a day-to-day dynamic evolution model with consideration of BR within an urban railway network. Travelers will not change their path if the difference between the previous day’s perceived cost and the actual cost does not exceed an acceptable value. Han et al. (2014) analyze the continuous-time simultaneous route-and-departure choice dynamic user equilibrium model that incorporates the boundedness of travelers’ rationality. Di et al. (2014) study the Braess paradox (Braess et al., 2005) in the setting of BRUE.

BR has also been applied to network design problems. Lou et al. (2010) use BR to model travelers’ route choice behavior in the congestion pricing problem. They first propose two models for formulating the user equilibrium using BR: flow-based and link-based. One flaw for the flow formulation is that it allows cycles in the resulting network. Here we only list the link-based model:

\[
t_{ij}(v_{ij}) + \pi^w_i - \pi^w_j - \varepsilon_{ij}^w = 0 \quad \forall (i,j) \in A, w \in W,
\]

\[
z_{ij}^w(-\varepsilon_{ij}^w + \mu_i^w - \mu_j^w) \geq 0 \quad \forall (i,j) \in A, w \in W,
\]

\[-\mu_{d(w)}^w \leq \varepsilon^w - \mu_{w(w)}^w \quad \forall w \in W,
\]

\[
\Theta \cdot z^w = D^w \quad \forall w \in W,
\]

\[
v_{ij} = \sum_{w \in W} z_{ij}^w \quad \forall (i,j) \in A,
\]

\[
z_{ij}^w, \varepsilon_{ij}^w \geq 0, \pi^w_i, \mu^w_i \text{ free,} \quad \forall (i,j) \in A, w \in W,
\]

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where $\bar{\epsilon}^w$ is the indifference threshold value, $\pi^w \in R^{|N|}$, $\epsilon^w \in R^{|\mathcal{A}|}$ and $\mu^w \in R^{|N|}$ are vectors of node potentials associated with travel time, travel times in excess of the minimum, and node potentials associated with the excess travel times respectively. Constraints (8)–(10) restrict the path utility within a certain band and constraints (11)–(13) make the flow feasible. Due to the non-convexity and non-uniqueness of the BRUE set, a common method for the toll network design problem is to use a robust objective such as minimizing the maximum total cost. Lou et al. (2010) consider a robust toll network design problem with the lower level problem as a BRUE set. The problem can be formulated as a min-max problem:

$$\min_{\tau} \max_{(v, z, \epsilon, \pi, \mu)} \sum_{(i,j) \in \mathcal{A}} t_{ij}(v_{ij})v_{ij},$$

s.t. $t_{ij}(v_{ij}) + \tau_{ij} - \epsilon^w_{ij} + \pi^w_i - \pi^w_j = 0 \quad \forall (i, j) \in \mathcal{A}, w \in \mathcal{W},$

$$0 \leq \tau \leq \tau_{\max},$$

(9) – (13),

where $\tau_{\max}$ is the maximum toll vector that can be possibly charged to each link. Additionally, a BRUE set with the restriction that the best performance of BRUE can achieve is the same as that with DUE is considered.

Guo (2013) develops a toll sequence method to guide users to system optimal equilibrium under boundedly rational drivers. Di et al. (2013) propose a methodology for constructing the BRUE set. They formulate the BRUE as a nonlinear complementarity problem which is equivalent to the path flow formulation defined by Lou et al. (2010). With the method for obtaining the BRUE set, we can then apply it with a certain objective to formulate and solve NDP.

3.3 NDP with CPT

Avineri (2006) examines the possibility of applying prospect theory for modeling stochastic network equilibria and presents an investigation of the effect of reference point values on such equilibria. The paper considers a two path illustrative example by applying CPT with discrete
utility outcomes. Connors and Sumalee (2009) extend CPT to the continuous case and consider
a network equilibrium model using CPT as travelers’ perception of travel time. Existence and
uniqueness of the user equilibrium is proven with the assumption that the link cost function,
value function, probability weighting function and cumulative density function of the outcomes
are continuous and strictly monotonic. They use the same value function as (2). But a different
weight function is used with
\[ W(p) = \exp \left( - \left[ -\log(p) \right]^{\zeta} \right) \]
(14)
to be guaranteed to be monotonic for all parameter settings \( 0 < \zeta < 1 \). For a route \( k \) of OD pair \( w \), its utility \( U_k^w \) is a random variable. Suppose its CDF is \( F_k^w(u) \) and \( u_0^w \) is the reference point
for all routes between OD pair \( w \), then the perceived cost value of this route can be calculated
by the following equations:
\[
c_k^w = \int_{-\infty}^{u_0^w} -\frac{dW(1-F_k^w(u))}{du} \psi_k^w(u) du + \int_{u_0^w}^{\infty} \frac{dW(F_k^w(u))}{du} \psi_k^w(u) du, \forall k \in P^w, w \in W, \quad (15)
\]
\[
\psi_k^w(u) = \begin{cases} 
(u_0^w - u)^\alpha, & u \leq u_0^w \\
-\eta(u - u_0^w)^\beta, & u > u_0^w 
\end{cases} \quad (16)
\]
Using equations (14)–(16), we obtain the vector form of the perceived cost \( c(f) \). Then the user
equilibrium under CPT can be formulated as a VI as follows:
\[
c(f^*)(f - f^*) \geq 0, \forall f \in \Omega_f. \quad (17)
\]
Tian et al. (2012) study the dynamic user equilibrium model using CPT to formulate the trav-
elers’ risk evaluation of arrival time. Note that the value function they consider is asymmetric
since it is disutility if travelers arrive early or late. They formulate the model as a VI in discrete
time and show that commuter’s risk preference has big influence on flow distribution.

Xu et al. (2011a) extend the prospect-based user equilibrium of Connors and Sumalee (2009)
with endogenous reference points. The reference point values are decided by an on-time arrival
probability, which is based on the method in Xu et al. (2011b). They consider different demand
portions with different on-time arrival probabilities and compare the flow results with standard UE and multiple endogenous reference points. Furthermore, they are the first to apply a CPT based user equilibrium to NDP. They consider two congestion pricing models. The first one is that travelers view tolls as a sure loss and consider the outcome as a gain only when the travel time they save can pay for the toll. Instead of using equation (16), the value function for this model can be written as follows:

$$\psi_k^w(u) = \begin{cases} 
(u_0^w - \tau_k^w - u)^\alpha, & u \leq u_0^w - \tau_k^w, \\
-\lambda(u + \tau_k^w - u_0^w)^\beta, & u > u_0^w - \tau_k^w,
\end{cases}$$

where $\tau_k^w$ is the tolls enforced along the route $k$ for OD pair $w$. Another view is that the travelers treat tolls the same as travel time. Then the network design problems can seek a link toll pattern that either maximizes the total travel prospect or minimizes the total expected travel time.

### 3.4 NDP with FLM

For user equilibrium using FLM, due to the lack of behavioral interpretation and UE models, we focus only on the cost based models. Henn (2000) models predicted cost for each path by a fuzzy subset and uses various comparison indices from possibility theory to represent the different possible natures (pessimistic/optimistic, risk-taking/risk-averting) of drivers for the traffic assignment problem. Ridwan (2004) utilizes a fuzzy preference based model of route choice in the traffic assignment problem. Henn and Ottomanelli (2006) consider using both expected utility and fuzzy logic to model both road condition uncertainty and perceived cost and preference uncertainty. These models adopt possibility theory to compare fuzzy quantities. Given the universe $X$ and two subsets $A$ and $B$, the possibility follows

$$\text{Poss}(X) = 1, \quad \text{Poss}(\emptyset) = 0,$$

$$\text{Poss}(A \cup B) = \max\{\text{Poss}(A); \text{Poss}(B)\}. $$

In addition, possibility measure has a dual necessity measure defined as $\text{Nec}(A) = 1 - \text{Poss}(\bar{A})$ where $\bar{A}$ is the complement of $A$. In possibility theory, there are four indices that can compare
two fuzzy sets $M$ and $N$:

$$I_1(M) = \text{Poss}(M \leq N), \quad I_2(M) = \text{Poss}(M < N),$$

$$I_3(M) = \text{Nec}(M \leq N), \quad I_4(M) = \text{Nec}(M < N).$$

Henn (2000) denotes the indices by human interpretations. $I_1$ and $I_2$ are based on possibility measures and consider the cost positively. On the other hand, $I_3$ and $I_4$ use necessity measures and consider things negatively: they diminish their value when costs are encountered. $I_2$ is interested in gains whereas $I_3$ compare costs more conservatively. Then based on certain indices, the probability that a driver chooses route $k$ for OD pair $w$ based on indices $i \in \{1, 2, 3, 4\}$ can be defined as

$$P^w_{k|} = \frac{I_i(c^w_{ik})}{\sum_{j \in \mathcal{P}^w} I_i(c^w_{ij})}.$$  \hspace{1cm} (18)

Using the calculated probability from (18), we are able to divide the flow of travelers accordingly. Ramazani et al. (2011) show how to construct the fuzzy function and apply a fuzzy logic model to the traffic assignment problem by quantifying the possibility theory. The user equilibrium is modeled as VI using possibility theory indices $I_3(M)$. The obtained formulation uses a link travel time of $t^M_{ij} + t^U_{ij}$, which is the summation of mode and upper limit travel time for a triangular member link cost function.

There are other papers which model user equilibrium using fuzzy logic function. Wang and Liao (1999) consider the node-arc incidence as fuzzy to solve a user equilibrium problem in traffic assignment. Chang and Chen (2000) use the variational inequality approach to formulate a link-based fuzzy user-optimal route choice problem embedding link interactions. In formulating the problem, a single point estimation for the perceived cost is usually used. For example, Chang and Chen (2000) uses

$$\bar{t}_{ij} = \frac{t^L_{ij} + 4t^M_{ij} + t^U_{ij}}{6}$$

as the perceived travel time where $t^L_{ij}$, $t^M_{ij}$ and $t^U_{ij}$ are the lower limit, mode and upper limit for a triangular function of the link cost. This estimation is similar to CPT but with only three different outcomes and weights.
For FLM, there is a lack of applications to NDP. It is possible to apply the VI formulation of Chang and Chen (2000) or Ramazani et al. (2011) with certain objectives to formulate a NDP. More research needs to be done to validate fuzzy cost-based FLM for further applications to NDP.

3.5 NDP with SILK Theory

Zhang (2007) introduces SILK-UE to model the travelers’ traffic pattern within an agent based simulation framework. The SILK-UE is reached on a network when all users with limited spatial knowledge stop searching for alternatives because for each user the perceived search cost exceeds the expected gain from an additional search. Mathematically, assume that an individual’s perception capabilities allow the separation of a specific route attributes such as travel time into \( I \) categories, and travel time \( T_i \) has been observed \( m_i \) times between an OD pair in previous experiences. The vector \( \mathbf{K}(m_1, \cdots, m_i, \cdots, m_I) \) represents an individual’s network knowledge about routes connecting this OD pair. Let the vector \( \mathbf{B}(b_1, \cdots, b_i, \cdots, b_I) \) represent an individual’s subjective beliefs, where \( b_i \) is the subjective belief probability that an additional route search would produce an alternative route with attribute \( T_i \). By assuming Bayesian learning with Dirichlet prior and posterior distributions, one can compute subjective probabilities as

\[
    b_i = \frac{m_i}{M},
\]

where \( M \) is the total number of searches conducted. Assuming an individual’s travel time on the route currently used is \( T \), then the subjective search gain in regard to travel time saving per trip from an additional search is

\[
    g = \sum_{i(T_i < T)} b_i (T - T_i).
\]

If an individual stops route search after \( n \) rounds of search, the perceived search cost for this individual must be within the range \([g_{n+1}, g_n]\). Therefore, the perceived search cost can be estimated empirically as \(0.5(g_{n+1} + g_n)\). Then individuals will make decisions under uncertainty following a set of designed rules, which can be derived by survey. For example, the traveler will consider an alternative route if its total saving time is \(0.13 \sim 0.21\%\) and the difference of the
time spent on a freeway is higher than 57%. The traveler will change routes if the total time saving is 39% or higher.

Zhang (2011) implemented DUE, SUE and SILK-UE on the Twin Cities road network, which has 7976 nodes, 20194 links and about 600000 travelers during peak hours. He showed that DUE underestimates the level of congestion on the most congested links (e.g. freeway, bottlenecks). Additionally, SILK-UE performs slightly better in this network than DUE and SUE. Xiong and Zhang (2013) apply SILK theory to departure time problem by modifying the search for alternative routes to departure times.

While SILK-UE models can be a good method for analyzing traffic assignment problems due to its way of modeling how the route choice is made in the actual process, it is not straightforward how to apply SILK-UE to NDP. Since it is an agent based simulation model, we would have to obtain the design decision values beforehand so that the model could be run. It is nearly impossible to enumerate all the possible values. However, we can apply optimization via simulation (OvS) techniques to obtain a good solution. A recent review of OvS can be found in Hong and Nelson (2009). The OvS problems can be mainly categorized as: (1) Ranking-and-Selection (R&S) problems when the feasible solution number is small (often less than 100); (2) Continuous OvS (COvS) problems when the feasible region is continuous and convex and (3) Discrete OvS (DOvS) problems when the feasible solutions are discrete values. For congestion pricing, we can apply the COvS since the feasible toll values region is $0 \leq \tau \leq \tau_{\text{max}}$. The continuous NDP is also similar. For discrete NDP, the DOvS can be applied. COvS is commonly solved with a stochastic search algorithm (similar to the steepest gradient search method) while DOvS is solved with random search. Take the congestion pricing problem for example. With the objective of $\min \limits_{\tau} G(\tau) = \sum_{(i,j) \in A} t_{ij}(v_{ij})\tau_{ij}$, which minimizes the total social costs, a possible procedure to solve the problem can be as follows:

**Step 1.** Obtain an initial toll plan $\tau_0$. Set a step size $a_n$ and let $n = 0$. The initial toll plan can be either obtained through the mathematical formulation of the congestion pricing problem or randomly chosen values.

**Step 2.** Run SILK theory based simulation to estimate the gradient $\nabla G(\tau)$ with $\hat{\nabla} G(\tau)$. 

20
Step 3. Let $\tau_{n+1} = \tau_n - a_n \hat{\nabla} G(\tau)$. Run SILK theory based simulation to obtain flow pattern $v$ and calculate the objective $G(\tau)$. If $||\tau_{n+1} - \tau_n|| \leq h$ where $h$ is a chosen small number, then stop otherwise go to Step 2.

Another possible method to apply SILK theory to NDP is surrogate-based optimization (Queipo et al., 2005). Surrogate-based optimization is proposed for engineering design when expensive analysis is needed. It first approximates the objective function by using a sampled set of design points, which are usually obtained by a design of experiment. Then it uses a search algorithm to find a new design point and refine the approximated function. This process continues until certain criteria are fulfilled. Chen et al. (2014) consider a surrogate-based optimization method to approximate the network performance to toll charges and solve the congestion pricing problem. They utilize an open source simulation software DynusT (Chiu et al., 2010) to model the dynamic traffic assignment and evaluate the objective using a mesoscopic vehicle simulation tool. We can apply SILK-UE instead to model the traffic pattern to solve the congestion toll pricing problem using surrogate-based optimization method.

4 Discussion

After reviewing the literature on different route choice and NDP models, we now compare them in the following aspects: behavioral assumptions, UE uniqueness, data requirements, computational effort and case studies. The behavioral assumptions which we are generally interested in are: are travelers fully rational, do they search for maximum utility, are their perceptions of link costs accurate and are they fully aware of the network structure and conditions.

4.1 Behavioral Assumptions

The route choice problem is a human decision process and modeling drivers’ behaviors is very important. Here we consider the behavioral aspects that are captured by the models we have reviewed.

RUM assumes that travelers still maximize their utilities but their perceptions of the travel costs are not accurate. RUM captures the uncertainty in the evaluation of route cost. The
probability of choosing one route between an OD pair can be calculated based on the probability that this route has the highest utility. However, RUM does not consider the imperfect knowledge and rationality of the travelers. It assumes that the distributions of travel cost are perfectly known to the travelers and travelers are fully rational.

BR considers bounded rationality of the travelers. The behavioral assumptions under BR have been studied in psychology and economics and are shown to be efficient in modeling human decision behavior (Conlisk, 1996). Due to the complexity of the environment, travelers are satisfied with the route if its utility falls into a certain bound. However, no cost perception uncertainty is considered and drivers are assumed to know the entire network structure.

CPT models human risk preference to different utility outcomes. Similar to the findings in Tversky and Kahneman (1992), travelers are risk averse when confronted with the prospect of gains, risk seeking when confronted with the prospect of losses, and more sensitive to losses than gains. CPT is another way of modeling travelers’ rationality. But still, drivers are fully aware of how cost uncertainty is distributed and have perfect knowledge of the network. They maximize the route utility by weighting different outcomes.

FLM uses possibility theory, fuzzy functions or fuzzy rules to model the choosing of routes. For fuzzy rule based models, travelers utilize fuzzy rules to compare routes. Possibility based FLM uses different indices which model travelers’ risk preferences. Aggregating link cost based on fuzzy member function such as Chang and Chen (2000) is the same as putting different weights on outcomes. These all model travelers as not being fully rational. Fuzzy member functions capture cost uncertainty. Based on the cost obtained, travelers then try to maximize their utilities.

SILK theory models the dynamic way finding process of travelers as it is in a positive way. It considers the imperfect knowledge by spatial learning, the bounded rationality by search process and the human decision process by if-then rules.

4.2 User Equilibrium

The UEs of all the reviewed route choice models, except BR, are unique given certain assumptions. For CPT, uniqueness is established if the link travel time, the value and decision weight
functions are strictly monotonic (Connors and Sumalee, 2009). The SILK-UE flow pattern is also unique given an initial system state based on deterministic search, learning and decision rules (Zhang, 2006b). However, for BRUE, due to bounded rationality, drivers can choose multiple routes between an OD pair. This results in the non-uniqueness of the user equilibrium, which brings more challenges to solving the NDP.

4.3 Data Requirements

For RUM, we require the uncertainty values’ standard deviation to decide the \( \theta^w \) value (inverse proportional to the standard deviation) in the route utility. It is the least demanding model regarding data requirements.

BR models need to decide the band for different OD pairs. In the literature such as (Lou et al., 2010), threshold values are usually calculated as a certain percentage of the travel time in the traffic assignment problem using DUE. However, this assumes travelers are homogeneous. In order to model more accurately considering heterogeneity, different band values should be considered for different individuals, which should be obtained using real data.

For FLM, the rule based models need experiments to design the fuzzy rules. For example, Teodorović et al. (1998) collect results of 26 drivers within a 30-day period to extract the possibility of choosing two alternative routes. For cost based models, fuzzy member functions regarding cost need to be validated. For a triangular fuzzy member function, the lower limit, mode value and the upper limit should be estimated based on data. For example, the lower limit can be the free flow time computed based on off-peak traffic volumes while the upper limit should be based on highly congested cases.

For CPT and SILK theory, more effort in data preparation and analysis is required. For CPT, various parameters need to be estimated. Although multiple papers use estimated values from Tversky and Kahneman (1992), it is better to estimate the parameter values empirically to be more accurate based on the specific network and transportation background. Additionally, data is also required for choosing reference points. This information can be, for example, the smallest, average, and largest times needed for commuters, on-time arrival probabilities and the
portion of demand for different reference points. These two issues have been studied in the
literature as shown in Section 2.3 and relatively complex to handle.

For SILK theory, the estimation of cost requires the sequence of route searches to extract the
underlying patterns, which can be obtained by actually observing certain travelers or collecting
questionnaires from them. Additionally, data is needed to obtain decision rules of finding alter-
natives and the rules for changing routes. Zhang (2007) collects data from 82 student drivers
at the University of Minnesota, Twin Cities regarding their socio-economic and demographic
characteristics, typical travel patterns and routes considered, and chosen for three different
OD pairs: home to university, home to a randomly-selected destination A, and A to another
randomly selected destination B. The procedure of collecting and analyzing data can be very
difficult.

4.4 Computational Efforts of NDP

Here we consider the computation required for NDP by considering different route choice models.
NDP is very hard to solve and the solution algorithm is usually heuristic. Take the congestion
toll pricing problem for example. With RUM, due to the similar closed form structure of SUE
to DUE, it can be solved using a two stage process. For NDP based on FLM, it is very hard to
adopt the rule based FLM due to the lack of quantitative UE. Solving of NDP under cost based
FLM is similar to RUM by using UE in Ramazani et al. (2011) or Chang and Chen (2000).
These two models are relative easy to solve.

However, for NDP based on BR, the non-uniqueness and non-convexity of the UE set increase
computational difficulty. A non-convexity illustration of BRUE can be found in Lou et al. (2010).
There are multiple UEs in the BRUE set as shown in Section 2.2. Thus a way of choosing a
certain UE among the BRUE set should be designed. A common strategy is to be conservative
and consider the worst case. This is like adding another level to the problem which makes it
hard to solve. In order to address this problem, more efficient bi-level algorithms or heuristics
should be developed.

For NDP with CPT, even though it has unique user equilibrium under certain weight and
value functions, the calculation of route utility is based on numerical integration. Thus for each
calculation of the flow pattern, a numerical integration is needed. This is required for both the LP relaxation if MIP is involved and any iteration of heuristic methods. So NDP based on CPT is hard to solve. Since each step is computationally expensive, we would prefer an algorithm which converge in a fewer number of steps. Additionally, due to humans’ lack of differentiating utilities if they are very close, it is possible to discretize the utility value function and weight functions to reduce the computation needed for numerical integration.

As the SILK-UE is a simulation model, incorporating SLIK theory into NDP is very computationally demanding. We may employ search methods for network design variables, which will require solving a SILK-UE problem in each objective function evaluation. To lessen the computational burden, it is important to reduce the search space. We introduced two classes of search methods based on optimization-via-simulation techniques in Section 3.5: OvS-based methods and surrogate-based methods. For the OvS-based methods, at each iteration, the gradient and the objective value are obtained by running SILK theory based simulation models. For the surrogate-based methods, multiple initial toll plans should be evaluated and more should be tested in the following iterations.

4.5 Case studies

For RUM and SILK theory, Zhang (2011) compares DUE, SUE and SILK-UE for convergence speed for the Twin Cities road network with 7976 nodes and 20194 links. For FLM, Ramazani et al. (2011) apply FLM based UE for the traffic assignment problem to the city of Mashhas with 935 nodes, 2538 links and 7157 OD pairs. However, small networks are usually used in case studies of NDPs for these models.

The applications of CPT and BR have been applied to only small networks. Connors and Sumalee (2009) apply CPT based UE to a four link network. Xu et al. (2011a) apply the CPT based congestion pricing problem to the Nguyen and Dupui’s network with 13 nodes and 19 links. Lou et al. (2010) apply congestion pricing with BRUE and Han et al. (2014) apply dynamic BRUE to the Sioux Falls network. This is due to the high computational requirements by BRUE and CPT based UE.
<table>
<thead>
<tr>
<th>Feature</th>
<th>RUM</th>
<th>BR</th>
<th>CPT</th>
<th>FLM</th>
<th>SILK</th>
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<tbody>
<tr>
<td>Fully rational</td>
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<td>No</td>
<td>No</td>
<td>No</td>
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<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>UE uniqueness</td>
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<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
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<tr>
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<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Large network application of NDP</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 2: Comparison of features of studied models

For behavioral assumption validation, Avineri and Prashker (2004) and Jou and Chen (2013) empirically study CPT. SILK theory is developed based on survey studies. The BR is used for irreversible network change but is not empirically applied for BRUE and NDP.

4.6 Summary and Opportunities

After discussing the reviewed models from different aspects, we make a summary of comparisons in Table 2.

Please note that the summary is based on the current literature. From the summary table, opportunities of further research could be identified. For example, for BR, the current literature does not consider link cost uncertainty. It is highly possible to combine it into BR and also reasonable since variability is a very important consideration for travelers. One possible way to consider link cost uncertainty in BR is to only make a route feasible when the probability that the cost of that route falls within a band is higher than a certain threshold. For CPT, it aggregates the utility outcomes by value and weight functions to model the risk preferences of drivers. However, will drivers always seek the optimal route among the resulting route utilities? It is possible to consider the satisfying paradigm as BR suggests. Additionally, the reference point value for a traveler is always unique. Would it be possible to consider a reference band instead
of a single point? The modeling of traveler route choice behaviors has many opportunities and is attracting more research.

For NDP with different route choice models, the computation of NDP with BR and CPT is very demanding, which requires further research into efficient algorithms. Aside from RUM, applications of the reviewed behavioral route choice models to NDP are limited to congestion toll pricing problems. For further research, other kinds of NDP such as continuous NDP could be considered. Additionally, case studies of NDP are usually restricted to small networks. It is interesting to consider and compare real network studies of NDP with different behavioral route choice models.

5 Conclusion

In this paper, we first review route choice models considering drivers' behaviors. RUM considers drivers' perception uncertainty of the route cost. BR and CPT provide two different methods of modeling drivers' rationality. BR assumes that drivers are satisfied with the route if its utility is within a certain band. CPT models the drivers' risk preference of different route utilities. FLM adopts fuzzy set theory to model drivers' route decision making. SILK theory provides a positive (non-normative) approach to model the drivers' route finding and choice making process.

We then review the application of these behavioral route choice models with emphasis on NDP. The user equilibrium is first studied and then embedded into the NDP. The NDPs based on different behavioral assumptions are compared by behavioral assumptions, UE uniqueness, data requirement, computational efforts and case studies.

This paper not only reviews the current literature, but also points out challenges and opportunities in applying more accurate behavioral route choice models to NDP. The NDPs are usually modeled as bilevel problems or mathematical programming with equilibrium constraints, which are hard to solve. The incorporation of travelers' behavioral route choice models can be even harder. So further research needs to be done by either advancing algorithms for solving general NDP or methods to simplify the computation of incorporating different behavioral models. Additionally, except RUM, applications of the reviewed behavioral route choice models to
NDP have been limited to congestion toll pricing problems. Other kinds of NDP should also be considered. Thus many opportunities exist in this area and further research should be done to address these problems.

References


