Incorporating Driver Behaviors in Network Design Problems: Challenges and Opportunities

Longsheng Sun[1], Mark H. Karwan[1], and Changhyun Kwon*[2]

[1]Department of Industrial and Systems Engineering, University at Buffalo (SUNY), Buffalo, New York, USA

[2]Department of Industrial and Management Systems Engineering, University of South Florida, Tampa, Florida, USA

August 25, 2015

Abstract

The goal of a network design problem (NDP) is to make optimal decisions to achieve a certain objective such as minimizing total travel time or maximizing tolls collected in the network. A critical component to NDP is how travelers make their route choices. Researchers in transportation have adopted human decision theories to describe more accurate route choice behaviors. In this paper, we review the NDP with various route choice models: the random utility model (RUM), Random Regret-Minimization (RRM) model, bounded rationality (BR), cumulative prospect theory (CPT), the fuzzy logic model (FLM) and dynamic learning models (DLM). Moreover, we identify challenges in applying behavioral route choice models to NDP and opportunities for future research.

Keywords: network design; behavior route choice; random utility; random regret; bounded rationality; cumulative prospect theory; fuzzy logic; dynamic learning; SILK theory

*Corresponding Author: `chkwon@usf.edu`, +1-813-974-5588

$$\min_{y} \quad \text{total travel time } (l)$$

$$l = \text{traffic pattern determined by a traffic assignment module given } y$$

Figure 1: NDP Structure

1 Introduction

In the network design problem (NDP) in which a central authority determines the network conditions, it is essential to predict and model the behavior of network users. In the context of vehicular networks, the primary interest in NDP is which routes network users will choose. Figure 1 shows the typical structure of NDP. The objective of the shown NDP is to minimize the total travel time of all travelers. However, other objectives such as maximizing total tolls collected can also be used. The decision variables, y, can be tolls enforced on the links for a congestion pricing problem, new streets to be built for discrete NDP, and the capacities of existing streets for continuous NDP among others. Given y, the flow pattern l can be decided by a traffic assignment module which models how travelers make route choices and reach equilibrium. NDP optimizes a certain objective function by making decisions, y, based on the travelers' traffic pattern.

Many NDPs include route choice models that assume drivers to be rational and homogeneous with perfect information. Travelers are assumed to have a deterministic generalised cost for each route. Then based on Wardrop's principles, a deterministic user equilibrium (DUE, Sheffi (1985)) is achieved if travelers can only obtain a higher cost by switching routes. Most NDPs use a DUE to model the traffic pattern.

However, Zhu and Levinson (2010) empirically study commuters' routes and find that most people do not choose the shortest path. Nakayama et al. (2001) compare different route choice rules of no switching, random switching, rational and experience based rules using simulation. Their results show that drivers are neither homogeneous nor fully rational by the observed attributes. Thus in order to design policies that are effective in reality, it is essential to incorporate travelers' route choice behaviors to NDPs.

For behavioral route choice models and user equilibrium (UE) analysis, there are many studies and reviews. However, the NDPs incorporating behavioral route choices lack reviews and attention. In this paper, we extend the work of Zhang (2011) by considering behavioral route choice models with a particular focus on their applications in NDPs. We also discuss challenges in practical applications and implications on computability of NDPs. The contributions of the paper can be summarized as follows:

- We conduct an updated review of the major route choice models considering travelers' behaviors (Section 2);
- We review the NDP with user equilibrium based on different route choice models (Section 3);
- We compare the models from various aspects to denote their benefits and challenges (Section 4); and
- We make suggestions on applications of route choice models to NDP and further research opportunities on this topic (Section 4.6 and Section 5).

2 Review of Route Choice Models

Route choice modeling deals with which path travelers choose for their certain trip plan. It is essential to model route choice behavior since it is the basis for any traffic planning problem. However, it can be hard due to uncertain network conditions (for example, how congested certain road segments are), drivers' perceptions of route characteristics and the preferences to risk (risk taking, neutral, averse, or having an indifference band).

In this section, we review the current literature in modeling route choice involving travelers' behaviors. Various models have been proposed in order to address this problem. The most important ones are the random utility model (RUM), Random Regret-Minimization (RRM), bounded rationality (BR), cumulative prospect theory (CPT), fuzzy logic model (FLM) and Dynamic Learning Models (DLM). Route choice models can focus on risk or uncertainty. For risk, the outcomes are measurable or known but uncertainty can include situations in which the outcomes are unknown or unmeasurable (Hensher et al., 2015). Most of the models except DLM focus on modelling risky route choices with few exceptions. For DLM, we discuss the models based on risk or uncertainty separately.

2.1 Random Utility and Discrete Choice Models

The conventional random utility model (RUM) assumes travelers still maximize their utility and have perfect information of the utility function. A random error term is associated with the utility function to capture perception error of travelers and attributes that are unobservable to analysts. The utility of using the k-th path between an OD pair w, U_k^w is given by

$$U_k^w = -\theta^w c_k^w + \xi_k^w, \qquad \forall\ k, w,$$

where c_k^w is the generalised cost of all the observed attributes, θ^w is a positive parameter, and ξ_k^w is a random term. Considering only one attribute, c_k^w is usually travel time. If the random terms ξ_k^w are independently and identically distributed Gumbel variates, according to Sheffi (1985), the choice probability for path k is

$$P_k^w = \frac{e^{-\theta^w c_k^w}}{\sum\limits_l e^{-\theta^w c_l^w}} \ .$$

If the random error term of each utility is normally distributed, then it becomes a multinomial probit model. However, the calculation of the probit choice probability is not straightforward in the presence of more than two alternatives. The choice probability can be estimated by using either an analytical approximation or a Monte Carlo simulation (Sheffi, 1985).

Since there is a high degree of similarity among alternatives generated by RUM, the independence of alternatives is questioned by researchers. Many route choice models are then proposed in the literature; for example, the Logit structure models such as C-Logit (Cascetta et al., 1996) and Path Size Logit (Ben-Akiva and Bierlaire, 1999). The Logit structure models introduce a correction term within the deterministic part of the utility function to approximate the correlation among alternative routes.

Generalised Extreme Value (GEV) models are also proposed to address the independence issue. These kinds of models account for similarities by allowing the random components of alternatives to be correlated. Examples include Paired Combinatorial Logit (Chu, 1989; Koppelman and Wen, 2000), Cross Nested Logit (Vovsha, 1997), Generalized Nested Logic (Wen and Koppelman, 2001) among others. Readers can refer to Prato (2009) for a more detailed introduction. Besides Logit structure and GEV models, Castillo et al. (2008) propose the multinomial weibit model by considering a Weibull perceived travel cost.

With the development of RUM, there is growing literature on attribute non-attendence, which focuses on ways to identify the role of observed attributes associated with a pre-specified set of alternatives (Hensher, 2014). Additionally, by including attributes to denote risk preference (Li et al., 2012), belief (Hensher et al., 2013b), and even learning and habit, RUM has the ability to consider more realistic travelers' behaviors. The assumption of perfect knowledge of the utility function and rationality can be relaxed.

2.2 Random Regret-Minimization Model

Chorus et al. (2008) were the first to explore the random regret-minimization (RRM) model in the transportation field based on regret theory (Loomes and Sugden, 1982). While regret theory focuses on risky choices, RRM is initially developed for riskless decision under multiple attributes and then extended to risky choices. Instead of maximizing utility, RRM minimizes the regret for choosing a route. The formulation of RRM involves comparing the considered route i with any other route j. For an attribute m, the level of regret is defined as

$$R_{i \leftrightarrow j}^{m} = \max\Big\{0, \beta_m \cdot (x_{jm} - x_{im})\Big\},$$

where β_m is the slope of the regret function for attribute m. Then the systematic regret for the considered candidate can be written as follows:

$$R_i = \max_{j \neq i} \Big\{ \sum_{m=1,\cdots,M} \max\{0, \beta_m \cdot (x_{jm} - x_{im})\} \Big\} + \xi_i, \tag{1}$$

where ξ_i is the perception error and unobserved heterogeneity in regret. We can see that RRM has a similar structure to RUM with the utility defined differently. Thus the discrete choice models can be used in RRM as well. For example, for the multinomial logit model under a Gumbel distributed error term, the choice probability for route i is

$$P_i = \frac{e^{-R_i}}{\sum_{j=1,\cdots,J} e^{-R_j}}.$$

The regret function (1) uses two max operations and is non-smooth. This requires a designed procedure for model estimation and creates difficulty for application by practitioners. Chorus (2010) considers a new RRM model using Logsum to approximate function (1) by

$$R_i = \sum_{j \neq i} \sum_{m=1,\cdots,M} \ln[1 + e^{\beta_m \cdot (x_{jm} - x_{im})}] + \xi_i.$$

Although RRM has a similar structure with RUM, there are fundamental differences. One major difference is the violation of independence from irrelevant alternatives (IIA). The performance of a certain route is dependent on the quality of the others. Another difference is in regards to the utility function. RRM predicts a very specific kind of semi-compensatory choice behavior (compromise effect). Chorus (2010) highlights that RRM gives extra preference to alternatives with an in-between performance on all attributes.

After Chorus proposed the RRM model, and due to a similar framework with a different interpretation compared to RUM, researchers made efforts to incorporate RRM and RUM models together (Chorus et al., 2013; Leong and Hensher, 2015; Hess et al., 2012; Hess and Stathopoulos, 2013; Chorus, 2014). Additionally, Chorus (2012b) studies a regret based route choice model by proposing a modified expected utility (MEU) incorporating both expected utility and regret. Ben-Elia et al. (2013b) use MEU in their dynamic learning model for informed and non-informed route-choice. For more details regarding RRM, an empirical study can be found in Chorus and Bierlaire (2013) and a review of RRM in Chorus (2012a).

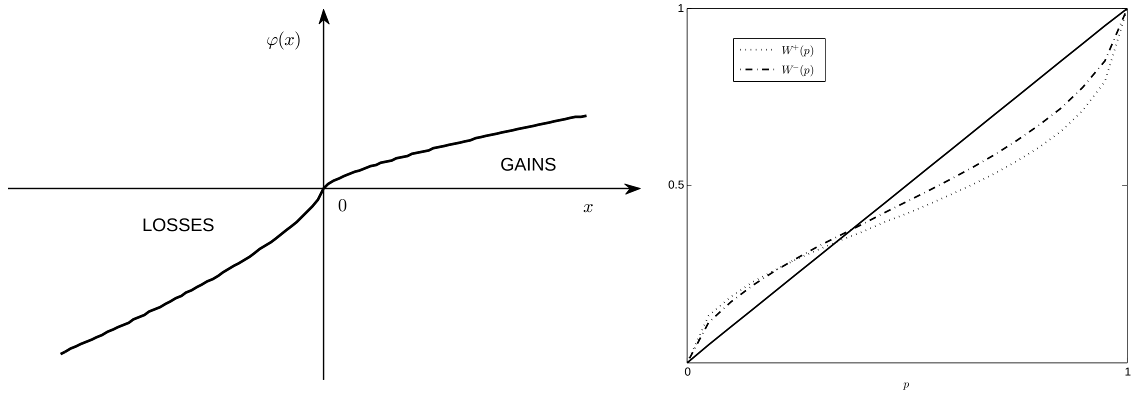
2.3 Bounded Rationality Models

The bounded rationality (BR) models replace utility maximization of RUM with an alternative-satisfying behavior paradigm (Mahmassani and Chang, 1987). Instead of trying to optimize their route utility, drivers are indifferent with near optimal routes. According to Lou et al. (2010), the drivers with bounded rationality can be defined as those who (1) always choose routes with no cycles and (2) do not necessarily switch to the minimum cost routes when the difference between the travel costs on their current routes and the cheapest one is no greater than a threshold value.

Psychology and economics provide wide-ranging evidence that bounded rationality is important and include bounded rationality into various models for greater accuracy. Simon (1982) introduces bounded rationality to economic analysis. Conlisk (1996) surveys the economic literature and provides the reasons for incorporating bounded rationality into economic models. In the transportation field, Mahmassani and Chang (1987) first study bounded rationality user equilibrium (BRUE) in departure time decision problems. Commuters are assumed to behave as if they had an indifference band (IB) of tolerable schedule delay which can be defined as $[\mathrm{IB}_{\min}, \mathrm{IB}_{\max}]$. They investigate the existence and uniqueness of BRUE under an idealized commuting system with only one OD pair affected by a single bottleneck. Then they show how to extend BR to multiple origins, bottlenecks and multiple origin-destination cases. Simulation experiments are later proposed to test the bounded rationality assumption by (Hu and Mahmassani, 1997; Mahmassani and Liu, 1999) using the simulation tool DYNASMART (Jayakrishnan et al., 1994).

2.4 Prospect and Cumulative Prospect Theory

Before we introduce prospect theory (PT) and cumulative prospect theory (CPT), we first discuss expected utility theory. Expected utility theory (Von Neumann, 1944) states that a decision is made based on the expected utility value when involving uncertainty. It assumes drivers behave as if assigning probabilities to random travel costs and choose a route that maximizes the expected outcome. The expected utility model assumes equal preference for different uncertainty outcomes. However, this is criticized by behavioral scientists (Kahneman and Tversky, 1979;



(a) Value Function

(b) Weight Function

Figure 2: Preference Functions (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992)

Tversky and Kahneman, 1992) and also by transportation researchers (Avineri and Prashker, 2004). They find the expected utility model has two violations: the extreme under-weighting of high probabilities which is known as the Allais paradox (Allais, 1979) and the inflation of small probabilities.

In order to deal with violations of the expected utility model, prospect theory (PT) (Kahneman and Tversky, 1979) and cumulative prospect theory (CPT) (Tversky and Kahneman, 1992) are proposed. PT is first analyzed by lottery evaluations. The lotteries are first mapped to gains and losses using a reference point. Then an S-shaped value function $\varphi(x)$ and inverse S-shaped weighting functions $W^+(p)$, $W^-(p)$ (as shown in Figure 2) are used to evaluate the lotteries. x is the mapped outcome and p is the corresponding occurring probability. Finally the perceived value of utility V can be calculated.

In transportation, Avineri and Prashker (2004) first examine the violations of the expected utility model and introduce prospect theory into the route choice model. They achieve this by conducting a survey using questionnaires inspired by Tversky and Kahneman (1992) to examine the choices of responders' alternative routes from home to work. Senbil and Kitamura (2004) apply prospect theory to departure time choice of morning commuters. Schwanen and Ettema (2009) apply CPT to model how employed parents make choices in unreliable transport networks to pick up their child(ren) from a nursery. Gao et al. (2010) compare CPT and the expected utility model and find CPT is more accurate in modeling travelers' route choice behavior. Nonlinear models (Li et al., 2012; Hensher et al., 2011; Hensher and Li, 2012; Hensher et al., 2013a) incorporating risk attitude and preferences as attributes based on prospect theory are also proposed. Ramos et al. (2014) compare prospect theory, utility theory and regret theory to investigate travelers' behavior with travel time uncertainty. Additionally, a review of CPT theory can be found in Li and Hensher (2011).

Due to the difference in applications, researchers have argued the need for parameter estimation in travel route choice settings (Avineri and Bovy, 2008; Schwanen and Ettema, 2009; Xu et al., 2011b; Jou and Chen, 2013). Another issue for the application of CPT is choosing the reference point (RP) value since CPT is sensitive to the RP value (Avineri and Prashker, 2004). Researchers have considered a fixed RP value (Avineri and Prashker, 2004), a predefined set of RP values (Connors and Sumalee, 2009), determining an RP value from surveys (Jou and Chen, 2013), heterogeneity in RP values (de Moraes Ramos et al., 2013) and optimization model (Xu et al., 2011b) to obtain RP values.

2.5 Fuzzy Logic Models

Incorporating fuzzy sets (Zadeh, 1965) into the route choice model is another way to deal with uncertainty. There are two different kinds of fuzzy route choice models: fuzzy rule based models and fuzzy cost based models (Henn and Ottomanelli, 2006).

For the fuzzy rule based models, Teodorović and Kikuchi (1990) were first to address the route choice problem using fuzzy inference techniques. Similar models are proposed in (Lotan and Koutsopoulos, 1993; Teodorović et al., 1998; Vythoulkas and Koutsopoulos, 2003; Murat and Uludag, 2008). Such models are based on linguistic rules using fuzzy sets (if the travel time on route 1 is very short and on route 2 is intermediate, then the driver will certainty choose route 1). But actually, the proposed models are only valid for one particular network and do not give any representation of how information (and, conversely, uncertainty) is carried out by drivers (Henn and Ottomanelli, 2006).

For fuzzy cost based models, more arithmetic approaches are proposed. Some route choice models (Henn, 2000, 2003; Henn and Ottomanelli, 2006) are based on a comparison of fuzzy numbers representing the costs of the routes. These costs are represented and compared using possibility theory, which is based on a max-axiom instead of an additive axiom of the expected theory. Quattrone and Vitetta (2011) specify and calibrate FLM to be applied in real-size systems and compare it with RUM. Other models use fuzzy logic to model the perceived link cost. Wang and Liao (1999) consider the node-arc incidence as fuzzy while Chang and Chen (2000) formulate link cost using a triangular fuzzy member function.

2.6 Dynamic Learning Models

The Dynamic learning model (DLM) is a *positive* approach to how travelers actually make route decisions, instead of assuming the travelers are rational, have perfect information and use utility maximization as in the *normative* way. DLMs study the behavioral aspects of travelers mainly regarding deviations from utility maximization, information acquisition and learning in both day-to-day events and within day activities. We introduce the DLMs based on whether the route-choice is made under risk or uncertainty.

For DLM under risk, it assumes travelers have some information on the variability of the travel time. Most of these models study how information affects travelers' route choices. Avineri and Prashker (2005) explore travelers' risk attitudes towards variability using a feedback mechanism on route choice decisions by comparing static and dynamic models. Ben-Elia et al. (2008) study the risk taking behavior of travelers by investigating the effects of both experience feedback and provided travel time variability description. Gao et al. (2008) study the within-day adaptation to revealed network conditions *en route* as opposed to the day-to-day adjustment process of route choices. Lu et al. (2011) examine how *en route* real time and *ex post* foregone payoffs information affect travelers' time savings and risk behavior. Additionally, the effect of accuracy of the provided information (Ben-Elia et al., 2013a) and regret (Ben-Elia et al., 2013b) are also explored.

For DLM under uncertainty, travelers know nothing regarding the variability of the travel time. The information on variability is usually gained through experience. Horowitz (1984), Mahmassani and Liu (1999) consider the perceived cost as a function of the weighted average of past travel times. Jha et al. (1998) consider using Bayesian learning models to update current perceived travel time by using information from the previous travel time periods. Erev and

Barron (2005) propose a reinforced learning model that addresses deviations from utility maximization, learning curves and the effect of information on uncertainty avoidance. Hensher et al. (2013b) consider how to incorporate belief into ex-ante assessment of support for alternative road pricing schemes. Tang and Gao (2015) present an interesting application of instance-based learning models for route-choice using a synthetic data set. They show that inclusion of forgetting and learning in the dynamic specification improves the model. Zhang (2006a, 2007) proposes search, information, learning, and knowledge (SILK) theory and develops agent-based models to incorporate spatial knowledge, Bayesian learning, route search and human heuristic decision rules of traveler's individual behaviors.

3 Review of NDP with Route Choice Models

The goal of NDP is to make an optimal decision in order to achieve a certain objective in the network while accounting for the route choice behavior of network users. The network users are usually modelled by user equilibrium with different route choice models. Although route choice models have been studied widely, the applications of them in network design models are not well developed and reviewed. In this section, we review different user equilibrium models and provide literature and applications in NDP. First however, we list some notation that will be used in Table 1. Note here the path cost perceived by travelers consists of only travel time, however, more attributes can be considered.

3.1 NDP with RUM

After significant initial research on random utility in the transportation area, stochastic user equilibrium (SUE) is proposed to address the perceived cost uncertainty in Waldrop's first principle. Prashker and Bekhor (2004) review the SUE based on various random utility models: logit based SUE (Fisk, 1980), C-logit based SUE (Zhou et al., 2012), path size logit based SUE (Chen et al., 2012), cross nested logit based SUE (Prashker and Bekhor, 2000) and general nested logic based SUE (Bekhor and Prashker, 2001). Additionally, Kitthamkesorn and Chen (2013) propose a weibit SUE considering a Weibull perceived cost uncertainty distribution. Dynamic

$\mathcal{N}$:	set of nodes,
$\mathcal{A}$:	set of links,
$\mathcal{W}$:	set of origin-destination (OD) pairs,
q^w:	travel demand between OD pair $w \in \mathcal{W}$,
$\mathbf{q}$:	column vector of all the OD travel demands, $\mathbf{q} = (q^w, w \in \mathcal{W})^{\mathbf{T}}$,
P^w:	set of all paths between OD pair $w \in \mathcal{W}$,
δ^w_{ijk}:	$\delta^w_{ijk} = 1$ if path $k \in P^w$ between OD pair $w \in \mathcal{W}$ traverse link $(i, j) \in \mathcal{A}$, $\delta^w_{ijk} = 0$ otherwise,
Δ^w:	link/path incidence matrix associated with OD pair $w \in \mathcal{W}$
Δ:	link/path incidence matrix for the entire network, $\Delta = (\Delta^w, w \in \mathcal{W})$
f_{kw}:	traffic flow on path $k \in P^w$ between OD pair $w \in \mathcal{W}$,
$\mathbf{f}^w$:	column vector of traffic flows on all the paths between OD pair $w \in \mathcal{W}$,
$\mathbf{f}$:	column vector of traffic flows on all the paths in the network,
Λ:	OD pair/path incidence matrix, $\Lambda = (\sigma_{wk} : w \in \mathcal{W}, k \in P^w)$, where σ_{wk} equals 1 if path $k \in P^w$ and 0, otherwise,
$\mathit{\Theta}$:	node link incidence matrix,
$\boldsymbol{D}^w$:	demand vector satisfying $D^w_{o(w)} = q^w$, $D^w_{d(w)} = -q^w$ and $D^w_i = 0$ for all other $i \in \mathcal{N}$, $o(w)$ and $d(w)$ are the origin and destination node for OD pair w,
z^w_{ij}:	flow on link $(i, j) \in \mathcal{A}$ for OD pair w,
$\boldsymbol{z}^w$:	column vector of all link flows for OD pair w
v_{ij}:	traffic flow on link $(i, j) \in \mathcal{A}$,
$\mathbf{v}$:	column vector of all the link traffic flows, $\mathbf{v} = (v_{ij}, (i, j) \in \mathcal{A})^{\mathbf{T}}$,
$t_{ij}(v_{ij})$:	travel time on link $(i, j) \in \mathcal{A}$, and it is a nonnegative, monotonically increasing and continuously differentiable function,
$c^w_k(\mathbf{v})$:	travel cost on path $k \in P^w$, and $c^w_k = \sum_{(i,j)\in\mathcal{A}} t_{ij}(v_{ij})\delta^w_{ijk}$,
$\mathbf{c}^w(\mathbf{v})$:	column vector of travel cost on all the paths connecting OD pair $w \in \mathcal{W}$,
τ_{ij}:	toll collected on link $(i, j) \in \mathcal{A}$,
$\boldsymbol{\tau}$:	vector form of toll collected for all links $(i, j) \in \mathcal{A}$.

Table 1: Notation for the Network Design Problem

user equilibrium (Bliemer et al., 2014) is also studied in Han (2003) and Lim and Heydecker (2005).

RUM is the most applied method for NDP among all the behavioral route choice models. Davis (1994) considers the continuous NDP with an SUE based on the logit model, which leads to a differentiable and large-scale, but tractable version of NDP. Lim et al. (2005) suggest a heuristic solution algorithm for continuous NDP by directly using the derivative deduced from the logit route choice model. Uchida et al. (2007) study NDP for multi-model networks with probit-based SUE flow.

For congestion toll pricing problem, Yang (1999) demonstrates that the classical principle of marginal-cost pricing is still applicable in a network under SUE and is a good alternative to drive an SUE flow pattern toward system optimum. Chen et al. (2004) study a toll design problem based on stochastic route choice behavior for multiple user groups when only certain links are tollable. Liu et al. (2013) address the toll design problem with logit-based SUE with first-best pricing. Let the feasible path flow set and feasible link flow set be defined as

$$\Omega_f = \{\mathbf{f} : \mathbf{q} = \Lambda \mathbf{f},\ \mathbf{f} \geq 0\},$$
$$\Omega_v = \{\mathbf{v} : \mathbf{q} = \Lambda \mathbf{f},\ \mathbf{v} = \Delta \mathbf{f},\ \mathbf{f} \geq 0\}.$$

Then under logit-based user equilibrium, the feasible set of alternative tolls can be formulated as a variational inequality (VI) problem:

$$\sum_{w \in \mathcal{W}} \sum_{k \in P^w} \left(\bar{c}_k^w + \frac{1}{\theta} \ln f_{kw}^* \right) (f_{kw} - f_{kw}^*) \geq 0, \forall \mathbf{f} \in \Omega_{\mathbf{f}}, \tag{2}$$

where $\bar{c}_k^w$ is the tolled path cost. According to Hearn and Ramana (1998), problem (2) can be formulated as a set of constraints. Then the congestion toll pricing problem with SUE can be solved with a two stage process. We first calculate the system optimal flow, then using these equivalent constraints based on the system optimal flow, it is possible to decide how to toll the network using different objectives such as minimizing the total tolls collected:

$$\min_{\tau_{ij}} \sum_{(i,j) \in \mathcal{A}} \tau_{ij} v_{ij}. \tag{3}$$

Liu et al. (2013) also propose a toll design model using C-logit based user equilibrium. Similar models based on (2) and (3) can be formulated and solved using the two stage process.

3.2 NDP with RRM

Due to the similar choice probability definition with RUM, the application of RRM for traffic assignment has the same structure. Bekhor et al. (2012) study the static user equilibrium problem based on RRM. The difference of RRM based UE is that the RRM utility function is inseparable by links due to regret being obtained by comparing paths. They formulate the RRM based UE as the following VI problem:

$$(f_{kw} - f_{kw}^*)(f_{kw}^* - \Pr(R_{kw}^*) \cdot q^w) \geq 0, \quad \forall f \in \Omega, \tag{4}$$

where R_{kw} is the regret for path k of OD pair w and $\Pr(R_{kw})$ is the route choice probability. Due to the definition of the regret function, the VI for RRM can have multiple solutions. Bekhor et al. (2012) explore the RRM based UE by generating a set of routes before solving the assignment module. For regret theory based route choice model, Chorus (2012b) studies how regret aversion affects the traffic equilibria when travel times are risky.

The application of RRM to NDP is not yet explored. However, the formulation of NDP with RRM will be similar to RUM as well. Using certain objectives of interest, such as the total travel time and applying VI (4) as UE to model travelers' behaviors, we can have a RRM based NDP. Due to the inseparable cost on links and the complexity of the regret function, solving the RRM based NDP could be a challenge.

3.3 NDP with BR

Since early work applying BR in transportation (Mahmassani and Chang, 1987), researchers in the transportation area have applied BR to model more accurate traveling behaviors. Szeto and Lo (2006) formulate a route choice bounded rationality dynamic user equilibrium problem as a discrete time nonlinear complementarity problem. Guo and Liu (2011) consider the irreversible network change using BRUE due to necessity in the existence of multiple network equilibria. Wu et al. (2013) propose a day-to-day dynamic evolution model with consideration of BR within an urban railway network. Travelers will not change their path if the difference between the previous day's perceived cost and the actual cost does not exceed an acceptable value. Han et al. (2014) analyze the continuous-time simultaneous route-and-departure choice dynamic user equilibrium model that incorporates the boundedness of travelers' rationality. Di et al. (2014) study the Braess paradox (Braess et al., 2005) in the setting of BRUE.

BR has also been applied to network design problems. Lou et al. (2010) use BR to model travelers' route choice behavior in the congestion pricing problem. They first propose two models for formulating the user equilibrium using BR: flow-based and link-based. One flaw for the flow formulation is that it allows cycles in the resulting network. Here we only list the link-based model:

$$t_{ij}(v_{ij}) + \pi_i^w - \pi_j^w - \varepsilon_{ij}^w = 0 \qquad \forall (i,j) \in \mathcal{A}, w \in \mathcal{W}, \tag{5}$$

$$z_{ij}^w(-\varepsilon_{ij}^w + \mu_i^w - \mu_j^w) \geq 0 \qquad \forall (i,j) \in \mathcal{A}, w \in \mathcal{W}, \tag{6}$$

$$-\mu_{d(w)}^w \leq \bar{\varepsilon}^w - \mu_{o(w)}^w \qquad \forall w \in \mathcal{W}, \tag{7}$$

$$\mathit{\Theta} \cdot \boldsymbol{z}^w = \boldsymbol{D}^w \qquad \forall w \in \mathcal{W}, \tag{8}$$

$$v_{ij} = \sum_{w \in \mathcal{W}} z_{ij}^w \qquad \forall (i,j) \in \mathcal{A}, \tag{9}$$

$$z_{ij}^w, \varepsilon_{ij}^w \geq 0, \pi_i^w, \mu_i^w \text{ free}, \qquad \forall (i,j) \in \mathcal{A}, w \in \mathcal{W}, \tag{10}$$

where $\bar{\varepsilon}^w$ is the indifference threshold value, $\pi^w \in R^{|N|}$, $\varepsilon^w \in R^{|\mathcal{A}|}$ and $\mu^w \in R^{|N|}$ are vectors of node potentials associated with travel time, travel times in excess of the minimum, and node potentials associated with the excess travel times respectively. Constraints (5)–(7) restrict the path utility within a certain band and constraints (8)–(10) make the flow feasible. Due to the non-convexity and non-uniqueness of the BRUE set, a common method for the toll network design problem is to use a robust objective such as minimizing the maximum total cost. Lou et al. (2010) consider a robust toll network design problem with the lower level problem as a BRUE set. The problem can be formulated as a min-max problem:

$$\begin{aligned}
&\min_{\boldsymbol{\tau}} \max_{(v,z,\varepsilon,\pi,\mu)} \sum_{(i,j)\in\mathcal{A}} t_{ij}(v_{ij}) v_{ij}, \\
\text{s.t.} \quad & t_{ij}(v_{ij}) + \tau_{ij} - \varepsilon_{ij}^w + \pi_i^w - \pi_j^w = 0 \quad \forall (i,j) \in \mathcal{A}, w \in \mathcal{W}, \\
& \boldsymbol{0} \leq \boldsymbol{\tau} \leq \boldsymbol{\tau}_{\max}, \\
& (6) - (10),
\end{aligned}$$

where $\boldsymbol{\tau}_{\max}$ is the maximum toll vector that can be possibly charged to each link. Additionally, a BRUE set with the restriction that the best performance BRUE can achieve is the same as when DUE is considered.

Guo (2013) develops a toll sequence method to guide users to system optimal equilibrium under boundedly rational drivers. Di et al. (2013) propose a methodology for constructing the BRUE set. They formulate the BRUE as a nonlinear complementarity problem which is equivalent to the path flow formulation defined by Lou et al. (2010). With a method chosen for obtaining the BRUE set, we can then apply it with a certain objective to formulate and solve NDP.

3.4 NDP with CPT

Avineri (2006) examines the possibility of applying prospect theory for modeling stochastic network equilibria and presents an investigation of the effect of reference point values on such equilibria. The paper considers a two path illustrative example by applying CPT with discrete utility outcomes. Connors and Sumalee (2009) extend CPT to the continuous case and consider a network equilibrium model using CPT as travelers' perception of travel time. Existence and uniqueness of the user equilibrium is proven with the assumption that the link cost function, value function, probability weighting function and cumulative density function of the outcomes are continuous and strictly monotonic. They use a different weight function

$$W(p) = \exp\Big(- [-\log(p)]^{\zeta}\Big) \tag{11}$$

to guarantee monotonicity for all parameter settings $0 < \zeta < 1$. For a route k of OD pair w, its utility U_k^w is a random variable. Suppose its CDF is $F_k^w(u)$ and u_0^w is the reference point for all routes between OD pair w, then the perceived cost value of this route can be calculated by the following equations:

$$c_k^w = \int_{u_0^w}^{\infty} -\frac{\mathrm{d}W(1-F_k^w(u))}{\mathrm{d}u}\psi_k^w(u)\mathrm{d}u + \int_{-\infty}^{u_0^w} \frac{\mathrm{d}W(F_k^w(u))}{\mathrm{d}u}\psi_k^w(u)\mathrm{d}u,\ \forall k \in P^w, w \in \mathcal{W}, \tag{12}$$

$$\psi_k^w(u) = \begin{cases} (u_0^w - u)^\alpha, & u \leq u_0^w \\ -\eta(u - u_0^w)^\beta, & u > u_0^w \end{cases} \qquad \forall k \in P^w, w \in \mathcal{W}. \tag{13}$$

Using equations (11)–(13), we obtain the vector form of the perceived cost $\mathbf{c}(\mathbf{f})$. Then the user equilibrium under CPT can be formulated as a VI as follows:

$$\mathbf{c}(\mathbf{f}^*)(\mathbf{f} - \mathbf{f}^*) \geq \mathbf{0}, \forall \mathbf{f} \in \Omega_f. \tag{14}$$

Tian et al. (2012) study the dynamic user equilibrium model using CPT to formulate the travelers' risk evaluation of arrival time. Note that the value function they consider is asymmetric since it is disutility if travelers arrive early or late. They formulate the model as a VI in discrete time and show that a commuter's risk preference has a big influence on flow distribution.

Xu et al. (2011a) extend the prospect-based user equilibrium of Connors and Sumalee (2009) with endogenous reference points. The reference point values are decided by an on-time arrival probability, which is based on the method in Xu et al. (2011b). They consider different demand portions with different on-time arrival probabilities and compare the flow results with standard UE and multiple endogenous reference points. Furthermore, they are the first to apply a CPT based user equilibrium to NDP. They consider two congestion pricing models. The first one is that travelers view tolls as a sure loss and consider the outcome as a gain only when the travel time they save can pay for the toll. Instead of using equation (13), the value function for this model can be written as follows:

$$\psi_k^w(u) = \begin{cases} (u_0^w - \tau_k^w - u)^\alpha, & u \leq u_0^w - \tau_k^w, \\ -\lambda(u + \tau_k^w - u_0^w)^\beta, & u > u_0^w - \tau_k^w, \end{cases}$$

where τ_k^w are the tolls enforced along the route k for OD pair w. Another view is that the travelers treat tolls the same as travel time. Then the network design problems can seek a link toll pattern that either maximizes the total travel prospect or minimizes the total expected travel time. .

3.5 NDP with FLM

For user equilibrium using FLM, due to the lack of behavioral interpretation and UE models, we focus only on the cost based models. Henn (2000) models predicted cost for each path by a fuzzy subset and uses various comparison indices from possibility theory to represent the different possible natures (pessimistic/optimistic, risk-taking/risk-averting) of drivers for the traffic assignment problem. Ridwan (2004) utilizes a fuzzy preference based model of route choice in the traffic assignment problem. Henn and Ottomanelli (2006) consider using both expected utility and fuzzy logic to model both road condition uncertainty and perceived cost and preference uncertainty. These models adopt possibility theory to compare fuzzy quantities. Given the universe X and two subsets A and B, the possibility measure satisfies the following:

$$\mathrm{Poss}(X) = 1, \quad \mathrm{Poss}(\phi) = 0,$$
$$\mathrm{Poss}(A \cup B) = \max\{\mathrm{Poss}(A); \mathrm{Poss}(B)\}.$$

In addition, the possibility measure has a dual necessity measure defined as $\mathrm{Nec}(A) = 1 - \mathrm{Poss}(\bar{A})$ where $\bar{A}$ is the complement of A. In possibility theory, there are four indices that can compare two fuzzy sets M and N:

$$I_1(M) = \mathrm{Poss}(M \leq N), \quad I_2(M) = \mathrm{Poss}(M < N),$$
$$I_3(M) = \mathrm{Nec}(M \leq N), \quad I_4(M) = \mathrm{Nec}(M < N).$$

Henn (2000) denotes the indices by human interpretations. I_1 and I_2 are based on possibility measures and consider the cost positively. On the other hand, I_3 and I_4 use necessity measures and consider things negatively: they diminish their value when costs are encountered. I_2 is interested in gains whereas I_3 compares costs more conservatively. Then based on certain indices, the probability that a driver chooses route k for OD pair w based on indices $i \in \{1, 2, 3, 4\}$ can be defined as

$$P_k^w | \, i = \frac{I_i(c_k^w)}{\sum_{j \in P^w} I_i(c_j^w)}. \tag{15}$$

Using the calculated probability from (15), we are able to divide the flow of travelers accordingly. Ramazani et al. (2011) show how to construct the fuzzy function and apply a fuzzy logic model to the traffic assignment problem by quantifying possibility theory. The user equilibrium is modeled as a VI using possibility theory indices $I_3(M)$. The obtained formulation uses a link travel time of $t_{ij}^M + t_{ij}^U$, which is the summation of mode and upper limit travel time for a triangular member link cost function.

There are other papers which model user equilibrium using fuzzy logic functions. Wang and Liao (1999) consider the node-arc incidence as fuzzy to solve a user equilibrium problem in traffic assignment. Chang and Chen (2000) use the variational inequality approach to formulate a link-based fuzzy user-optimal route choice problem embedding link interactions. In formulating the problem, a single point estimation for the perceived cost is usually used. For example, Chang and Chen (2000) uses

$$\bar{t}_{ij} = \frac{t_{ij}^L + 4t_{ij}^M + t_{ij}^U}{6}$$

as the perceived travel time where t_{ij}^L, t_{ij}^M and t_{ij}^U are the lower limit, mode and upper limit for a triangular function of the link cost. This estimation is similar to CPT but with only three different outcomes and weights.

For FLM, there is a lack of applications to NDP. It is possible to apply the VI formulation of Chang and Chen (2000) or Ramazani et al. (2011) with certain objectives to formulate a NDP. More research needs to be done to validate fuzzy cost-based FLM for further applications to NDP.

3.6 NDP with DLM

For NDP with DLM, Gao (2005) proposes a dynamic time user equilibrium incorporating information, which is particularly useful in analysing effectiveness of advanced traveler information system (ATIS). Zhang (2007) introduces SILK-UE to model the travelers' traffic pattern within an agent based simulation framework. Although the models discussed in Section 2.6 are all possible to model user equilibrium, the SILK-UE is the one which has been explored in current literature. In this section, we focus on the traffic assignment problem and propose possible applications of SILK theory that are solved to simulate traveler behavior. However, we note that other DLMs (for example, reinforced learning) are applicable as well.

The SILK-UE is reached on a network when all users with limited spatial knowledge stop searching for alternatives because for each user the perceived search cost exceeds the expected gain from additional search. Mathematically, assume that an individual's perception capabilities allow the separation of specific route attributes such as travel time into $\mathcal{I}$ categories, and assume that travel time T_i has been observed m_i times between an OD pair in previous experiences. The vector $\boldsymbol{\mathcal{K}}(m_1, \cdots, m_i, \cdots, m_{\mathcal{I}})$ represents an individual's network knowledge about routes connecting this OD pair. Let the vector $\boldsymbol{\mathcal{B}}(b_1, \cdots, b_i, \cdots, b_{\mathcal{I}})$ represent an individual's subjective beliefs, where b_i is the subjective belief probability that an additional route search would produce an alternative route with attribute T_i. By assuming Bayesian learning with Dirichlet prior and posterior distributions, one can compute subjective probabilities as $b_i = m_i/\mathcal{M}$, where $\mathcal{M}$ is the total number of searches conducted. Assuming an individual's travel time on the route currently used is T, then the subjective search gain in regard to travel time saving per trip from an additional search is

$$g = \sum_{i(T_i<T)} b_i(T - T_i).$$

An individual stops route search after n rounds when search cost is higher than gain. Then individuals will make decisions under uncertainty following a set of designed rules, which can be derived by survey. For example, the traveler will change routes if the total time savings is 39% or higher.

Zhang (2011) implemented DUE, SUE and SILK-UE on the Twin Cities road network, which has 7976 nodes, 20194 links and about 600000 travelers during peak hours. He showed that DUE underestimates the level of congestion on the most congested links (e.g. freeway, bottlenecks). Additionally, SILK-UE performs slightly better in this network than DUE and SUE. Xiong and Zhang (2013) apply SILK theory to a departure time problem by modifying the search for alternative routes to departure times.

While SILK-UE models can be a good method for analyzing traffic assignment problems due to its way of modeling how the route choice is made in the actual process, it is not straightforward how to apply SILK-UE to NDP. Since it is an agent based simulation model, we would have to obtain the design decision values beforehand so that the model could be run. It is nearly impossible to enumerate all the possible values. However, we can apply optimization via simulation (OvS) techniques to obtain a good solution. A recent review of OvS can be found in Hong and Nelson (2009). The OvS problems can be mainly categorized as: (1) Ranking-and-Selection (R&S) problems when the number of feasible solutions is small (often less than 100); (2) Continuous OvS (COvS) problems when the feasible region is continuous and convex and (3) Discrete OvS (DOvS) problems when the feasible solutions are discrete values. For congestion pricing, we can apply the COvS since the feasible toll values region is $\mathbf{0} \leq \boldsymbol{\tau} \leq \boldsymbol{\tau}_{\max}$. The continuous NDP is also similar. For discrete NDP, the DOvS can be applied.

Another possible method to apply SILK theory to NDP is surrogate-based optimization (Queipo et al., 2005). Surrogate-based optimization is proposed for engineering design when expensive analysis is needed. It first approximates the objective function by using a sampled set of design points, which are usually obtained by a design of experiment. Then it uses a search algorithm to find a new design point and refine the approximated function. This process continues until certain criteria are fulfilled. Chen et al. (2014) consider a surrogate-based optimization method to approximate the network performance to toll charges and solve the congestion pricing problem. They utilize the open source simulation software DynusT (Chiu et al., 2010) to model the dynamic traffic assignment and evaluate the objective using a mesoscopic vehicle simulation tool. We can apply SILK-UE instead to model the traffic pattern to solve the congestion toll pricing problem using surrogate-based optimization method.

4 Discussion

After reviewing the literature on different route choice and NDP models, we now compare them in the following aspects: behavioral assumptions, UE uniqueness, data requirements, computational effort and case studies. The behavioral assumptions which we are generally interested in are: are travelers fully rational, do they search for maximum utility, are their perceptions of link costs accurate and are they fully aware of the network structure and conditions.

4.1 Behavioral Assumptions

The route choice problem is a human decision process and modeling drivers' behaviors is very important. Here we consider the behavioral aspects that are captured by the models we have reviewed.

Conventional RUM assumes that travelers still maximize their utilities but a random error is associated with the utility function to denote the perception error of the travelers and the unobservable attributes to the analyst. The probability of choosing one route between an OD pair can be calculated based on the probability that this route has the highest utility. With the development of RUM, researchers study the role of information processing (Hensher, 2014), risk preference (Li et al., 2012) and belief (Hensher et al., 2013b) to capture travelers' behaviors more precisely.

RRM minimizes regret for the route. Due to its specific utility function, it captures the comprise effect of the travelers, especially in the multi-attributes setting. Additionally, the random error captures the perception error of the travelers and the observation error (unobserved attributes, for example) of the analysts.

BR considers bounded rationality of the travelers. The behavioral assumptions under BR have been studied in psychology and economics and are shown to be efficient in modeling human decision behavior (Conlisk, 1996). Due to the complexity of the environment, travelers are satisfied with the route if its utility falls into a certain bound. However, no cost perception uncertainty is considered and drivers are assumed to know the entire network structure.

CPT models human risk preference to different utility outcomes. Similar to the findings in Tversky and Kahneman (1992), travelers are risk averse when confronted with the prospect of gains, risk seeking when confronted with the prospect of losses, and more sensitive to losses than gains. CPT is another way of modeling travelers' rationality. But still, drivers are fully aware of how cost uncertainty is distributed and have perfect knowledge of the network. They maximize the route utility by weighting different outcomes.

FLM uses possibility theory, fuzzy functions or fuzzy rules to model the choosing of routes. For fuzzy rule based models, travelers utilize fuzzy rules to compare routes. Possibility based FLM uses different indices which model travelers' risk preferences. Aggregating link costs based on fuzzy member functions such as Chang and Chen (2000) is the same as putting different weights on outcomes. These all model travelers as not being fully rational. Fuzzy member functions capture cost uncertainty. Based on the cost obtained, travelers then try to maximize their utilities.

DLM models the actual way finding process of travelers as it is in a *positive* way. They focus on the travelers' behavior aspects of deviations from utility maximization, learning and information acquisition.

4.2 User Equilibrium

The uniqueness of the UE is important for forecasting and evaluating policies. To show UE uniqueness, it is sufficient to show that the objective is strictly convex and the feasible region is convex (Sheffi, 1985). For DUE, since the Hessian of the objective is positive definite, the objective function is strictly convex. The constraints' linearity ensures the convexity of the feasible region.

For CPT, uniqueness is established if the link travel cost, the value and decision weight functions are strictly monotonic, which guarantee that the objective function is strictly convex. The SILK-UE flow pattern is also unique given an initial system state based on deterministic search, learning and decision rules (Zhang, 2006b). However, for BRUE, due to bounded rationality, drivers can choose multiple routes between an OD pair. The BRUE set is a composite of nonlinear constraints and violates the convexity of the feasible region. This results in the non-uniqueness of the user equilibrium, which creates more challenges to solve the NDP. For the VI form of RRM, the regret function is nonconvex, thus it can have multiple UEs. However, UE is unique if solved by pre-generating the path set as in Bekhor et al. (2012).

4.3 Data Requirements

For RUM considering only travel time, we require the uncertainty values' standard deviation to decide the θ^w value (inverse proportional to the standard deviation) in the route utility. It is the least demanding model regarding data requirements. However, when considering multiple attributes, more data is needed to estimate the weights for different attributes. For RRM, the data requirement is similar to RUM due to the similarity of the framework.

BR models need to decide the band for different OD pairs. In the literature such as (Lou et al., 2010), threshold values are usually calculated as a certain percentage of the travel time in the traffic assignment problem using DUE. However, this assumes travelers are homogeneous. In order to model more accurately considering heterogeneity, different band values should be considered for different individuals, which should be obtained using real data.

For FLM, the rule based models need experiments to design the fuzzy rules. For example, Teodorović et al. (1998) collect results on 26 drivers within a 30-day period to extract the possibility of choosing two alternative routes. For cost based models, fuzzy member functions regarding cost need to be validated. For a triangular fuzzy member function, the lower limit, mode value and the upper limit should be estimated based on data. For example, the lower limit can be the free flow time computed based on off-peak traffic volumes while the upper limit should be based on highly congested cases.

For CPT and SILK theory, more effort in data preparation and analysis is required. For CPT, various parameters need to be estimated. Although multiple papers use estimated values from Tversky and Kahneman (1992), it is better to estimate the parameter values empirically to be more accurate based on the specific network and transportation background. Additionally, data is also required for choosing reference points. This information can be, for example, the smallest, average, and largest times needed for commuters, on-time arrival probabilities and the portion of demand for different reference points. These two issues have been studied in the literature as shown in Section 2.4 and are relatively complex to handle.

For SILK theory, the estimation of cost requires the sequence of route searches to extract the underlying patterns, which can be obtained by actually observing certain travelers or collecting questionnaires from them. Additionally, data is needed to obtain decision rules of finding alternatives and the rules for changing routes. Zhang (2007) collects data from 82 student drivers at the University of Minnesota, Twin Cities regarding their socio-economic and demographic characteristics, typical travel patterns and routes considered, and chosen for three different OD pairs: home to university, home to a randomly-selected destination A, and A to another randomly selected destination B. The procedure of collecting and analyzing data can be very difficult.

4.4 Computational Efforts of NDP

Here we consider the computation required for NDP by considering different route choice models. NDP is very hard to solve and the solution algorithm is usually heuristic. Take the congestion toll pricing problem for example. With RUM, due to the similar closed form structure of SUE to DUE, it can be solved using a two stage process. For NDP based on FLM, it is very hard to adopt the rule based FLM due to the lack of quantitative UE. Solving of NDP under cost based FLM is similar to RUM by using UE in Ramazani et al. (2011) or Chang and Chen (2000). These two models are relative easy to solve.

For NDP with RRM, the underlying VI formulation is very hard to solve exactly since we need to enumerate all the possible paths and calculate the regret value based on a fairly difficult function. Even with a limited number of paths, the problem remains hard due to the complexity of the regret function. In order to solve the problem, designed algorithms are needed. Moreover, for combined models considering RRM and RUM, how to apply and solve it in the NDP setting needs exploration.

For NDP based on BR, the non-uniqueness and non-convexity of the UE set increase computational difficulty. A non-convexity illustration of BRUE can be found in Lou et al. (2010). There are multiple UEs in the BRUE set as shown in Section 2.3. Thus a way of choosing a certain UE among the BRUE set should be designed. A common strategy is to be conservative and consider the worst case. This is like adding another level to the problem which makes it hard to solve. In order to address this problem, more efficient bi-level algorithms or heuristics should be developed.

For NDP with CPT, even though it has a unique user equilibrium under certain weight and value functions, the calculation of route utility is based on numerical integration. Thus for each calculation of the flow pattern, a numerical integration is needed. This is required for both the LP relaxation if mixed-integer-programming is involved and any iteration of heuristic methods. So NDP based on CPT is hard to solve. Since each step is computationally expensive, we would prefer an algorithm which converges in a fewer number of steps. Additionally, due to humans' lack of differentiating utilities if they are very close, it is possible to discretize the utility value function and weight functions to reduce the computation needed for numerical integration.

As the SILK-UE is a simulation model, incorporating SLIK theory into NDP is very computationally demanding. We may employ search methods for network design variables, which will require solving a SILK-UE problem in each objective function evaluation. To lessen the computational burden, it is important to reduce the search space. We introduced two classes of search methods based on optimization-via-simulation techniques in Section 3.6: OvS-based methods and surrogate-based methods. For the OvS-based methods, at each iteration, the gradient and the objective value are obtained by running SILK theory based simulation models. For the surrogate-based methods, multiple initial toll plans should be evaluated and more should be tested in the following iterations.

4.5 Case studies

For RUM and SILK theory, Zhang (2011) compares DUE, SUE and SILK-UE for convergence speed for the Twin Cities road network with 7976 nodes and 20194 links. Other studies regarding NDP with RUM usually use small networks. For example, Yang (1999) applies SUE to an illustrative network and Chen et al. (2004) apply SUE to the Sioux Falls network which consists of 76 links and 552 OD pairs. Liu et al. (2013) apply SUE to a toll pricing problem on a designed network with 7 nodes, 11 links and 4 OD pairs.

Regarding RRM based UE, Bekhor et al. (2012) apply it to a small grid network and also a real network of Winnipeg, Manitoba, Canada. The Winnipeg network consists of 948 nodes, 2535 links and 4345 OD pairs. For FLM, Ramazani et al. (2011) apply FLM based UE for the traffic assignment problem to the city of Mashhas with 935 nodes, 2538 links and 7157 OD pairs. However, most works use small networks. For instance, Wang and Liao (1999) apply FLM to an illustrative network with 5 links.

The applications of CPT and BR have been applied to only small networks. Connors and Sumalee (2009) apply CPT based UE to a four link network. Xu et al. (2011a) apply the CPT based congestion pricing problem to Nguyen and Dupui's network with 13 nodes and 19 links.

Feature	RUM	RRM	BR	CPT	FLM	DLM
Fully rational	No	No	No	No	No	No
Utility maximization	Yes	Yes	No	Yes	Yes	No
Link cost uncertainty	Yes	Yes	No	Yes	Yes	Yes
Perfect knowledge	Yes	Yes	Yes	Yes	No	No
UE uniqueness	Yes	No	No	Yes	Yes	Yes
Data requirement	medium	medium	medium	high	low	high
Computational effort of NDP	low	high	high	high	low	high
Large network application of UE	Yes	Yes	No	No	Yes	Yes
Large network application of NDP	No	No	No	No	No	No

Table 2: Comparison of features of studied models

Lou et al. (2010) apply congestion pricing with BRUE and Han et al. (2014) apply dynamic BRUE to the Sioux Falls network. This is due to the high computational requirements by BRUE and CPT based UE.

4.6 Summary and Opportunities

After discussing the reviewed models from different aspects, we make a summary of comparisons in Table 2.

Please note that the summary is based on the current literature. With the development of these models, travelers' behaviors can be captured more precisely. From the summary table, opportunities of further research could be identified. For example, for RUM, it is possible to accommodate learning and habit to capture the learning and information acquisition behaviors of travelers. For BR, the current literature does not consider link cost uncertainty. It is highly possible to combine it into BR and also reasonable since variability is a very important consideration for travelers. One possible way to consider link cost uncertainty in BR is to only make a route feasible when the probability that the cost of that route falls within a band is higher than a certain threshold. For CPT, it aggregates the utility outcomes by value and weight functions to model the risk preferences of drivers. However, will drivers always seek the optimal route among the resulting route utilities? It is possible to consider the satisfying paradigm as BR suggests. Additionally, the reference point value for a traveler is always unique. Would it be possible to consider a reference band instead of a single point? The modeling of traveler route choice behaviors has many opportunities and is attracting more research.

For NDP with different route choice models, the computation of NDP with BR and CPT is very demanding, which requires further research into efficient algorithms. Aside from RUM, applications of the reviewed behavioral route choice models to NDP are limited to congestion toll pricing problems. For further research, other kinds of NDP such as continuous or discrete NDP could be considered. Furthermore, the UEs considered are mostly static UEs. Dynamic UEs (Bliemer et al., 2014) should be considered to better model the traffic assignment module with route choice models. Additionally, case studies of NDP are usually restricted to small networks. It is interesting to consider and compare real network studies of NDP with different behavioral route choice models.

5 Conclusion

In this paper, we first review route choice models considering drivers' behaviors. We then review the application of these behavioral route choice models with emphasis on NDP. The user equilibrium is first studied and then embedded into the NDP. The NDPs based on different behavioral assumptions are compared by behavioral assumptions, UE uniqueness, data requirement, computational efforts and case studies.

This paper not only reviews the current literature, but also points out challenges and opportunities in applying more accurate behavioral route choice models to NDP. We have discussed many opportunities in Section 4.6. The challenge remains as to how some of the more behaviourally appealing methods might be incorporated within a complex network given data availability. Zhu and Levinson (2010) empirically study how travelers make route choices and state that the current behavioral models still fit poorly. Thus more research is still needed on the behavioral route choice models. Furthermore, with the development of routing aid devices and information communication technologies, how they can affect the route choice behaviors and user equilibria will be worth exploring.

The NDPs are usually modeled as bilevel problems or mathematical programming problems with equilibrium constraints, which are both hard to solve. The incorporation of travelers' behavioral route choice models can be even harder. So further research needs to be done by either advancing algorithms for solving general NDP or methods to simplify the computation of incorporating different behavioral models. Additionally, except RUM, applications of the reviewed behavioral route choice models to NDP have been limited to congestion toll pricing problems. Other kinds of NDP should also be considered. Many opportunities exist in this area and further research should be done to address these problems.

References

Allais, M. 1979. The so-called allais paradox and rational decisions under uncertainty. *Expected utility hypotheses and the Allais paradox*. Springer, 437–681.

Avineri, E. 2006. The effect of reference point on stochastic network equilibrium. *Transportation Science* **40**(4) 409–420.

Avineri, E., P. H. Bovy. 2008. Identification of parameters for a prospect theory model for travel choice analysis. *Transportation Research Record: Journal of the Transportation Research Board* **2082**(1) 141–147.

Avineri, E., J. N. Prashker. 2004. Violations of expected utility theory in route-choice stated preferences: Certainty effect and inflation of small probabilities. *Transportation Research Record: Journal of the Transportation Research Board* **1894**(1) 222–229.

Avineri, E., J. N. Prashker. 2005. Sensitivity to travel time variability: travelers learning perspective. *Transportation Research Part C: Emerging Technologies* **13**(2) 157–183.

Bekhor, S., J. Prashker. 2001. Stochastic user equilibrium formulation for generalized nested logit model. *Transportation Research Record: Journal of the Transportation Research Board* **1752**(1) 84–90.

Bekhor, S., C. Chorus, T. Toledo. 2012. Stochastic user equilibrium for route choice model based on random regret minimization. *Transportation Research Record: Journal of the Transportation Research Board* **2284**(1) 100–108.

Ben-Akiva, M., M. Bierlaire. 1999. Discrete choice methods and their applications to short term travel decisions. *Handbook of Transportation Science*. Springer, 5–33.

Ben-Elia, E., R. Di Pace, G. N. Bifulco, Y. Shiftan. 2013a. The impact of travel informations accuracy on route-choice. *Transportation Research Part C: Emerging Technologies* **26** 146–159.

Ben-Elia, E., I. Erev, Y. Shiftan. 2008. The combined effect of information and experience on drivers route-choice behavior. *Transportation* **35**(2) 165–177.

Ben-Elia, E., R. Ishaq, Y. Shiftan. 2013b. "If only I had taken the other road...": Regret, risk and reinforced learning in informed route-choice. *Transportation* **40**(2) 269–293.

Bliemer, M. C., M. P. Raadsen, E.-S. Smits, B. Zhou, M. G. Bell. 2014. Quasi-dynamic traffic assignment with residual point queues incorporating a first order node model. *Transportation Research Part B: Methodological* **68** 363–384.

Braess, D., A. Nagurney, T. Wakolbinger. 2005. On a paradox of traffic planning. *Transportation Science* **39**(4) 446–450.

Cascetta, E., A. Nuzzolo, F. Russo, A. Vitetta. 1996. A modified logit route choice model overcoming path overlapping problems: specification and some calibration results for interurban networks. *Proceedings of the 13th International Symposium on Transportation and Traffic Theory*. Pergamon Lyon, France, 697–711.

Castillo, E., J. M. Menéndez, P. Jiménez, A. Rivas. 2008. Closed form expressions for choice probabilities in the Weibull case. *Transportation Research Part B: Methodological* **42**(4) 373–380.

Chang, M.-S., H.-K. Chen. 2000. A fuzzy user-optimal route choice problem using a link-based fuzzy variational inequality formulation. *Fuzzy Sets and Systems* **114**(2) 339–345.

Chen, A., S. Pravinvongvuth, X. Xu, S. Ryu, P. Chootinan. 2012. Examining the scaling effect and overlapping problem in logit-based stochastic user equilibrium models. *Transportation Research Part A: Policy and Practice* **46**(8) 1343–1358.

Chen, M., D. H. Bernstein, L. N. Spasovic. 2004. Toll-design problem with stochastic route choice. *Environment and Planning B* **31** 731–742.

Chen, X. M., L. Zhang, X. He, C. Xiong, Z. Li. 2014. Surrogate-based optimization of expensive-to-evaluate objective for optimal highway toll charges in transportation network. *Computer-Aided Civil and Infrastructure Engineering* **29**(5) 359–381.

Chiu, Y.-C., L. Zhou, H. Song. 2010. Development and calibration of the anisotropic mesoscopic simulation model for uninterrupted flow facilities. *Transportation Research Part B: Methodological* **44**(1) 152–174.

Chorus, C. 2012a. Random regret minimization: an overview of model properties and empirical evidence. *Transport Reviews* **32**(1) 75–92.

Chorus, C. G. 2012b. Regret theory-based route choices and traffic equilibria. *Transportmetrica* **8**(4) 291–305.

Chorus, C. G. 2014. A generalized random regret minimization model. *Transportation Research Part B: Methodological* **68** 224–238.

Chorus, C. G., T. A. Arentze, H. J. Timmermans. 2008. A random regret-minimization model of travel choice. *Transportation Research Part B: Methodological* **42**(1) 1–18.

Chorus, C. G., M. Bierlaire. 2013. An empirical comparison of travel choice models that capture preferences for compromise alternatives. *Transportation* **40**(3) 549–562.

Chorus, C. G., J. M. Rose, D. A. Hensher. 2013. Regret minimization or utility maximization: it depends on the attribute. *Environment and Planning B: Planning and Design* **40**(1) 154–169.

Chorus, C. 2010. A new model of random regret minimization. *European Journal of Transport and Infrastructure Research* **10**(2).

Chu, C. 1989. A paired combinatorial logit model for travel demand analysis. *Proceedings of the Fifth World Conference on Transportation Research*.

Conlisk, J. 1996. Why bounded rationality? *Journal of Economic Literature* **34**(2) 669–700.

Connors, R. D., A. Sumalee. 2009. A network equilibrium model with travellers' perception of stochastic travel times. *Transportation Research Part B: Methodological* **43**(6) 614–624.

Davis, G. A. 1994. Exact local solution of the continuous network design problem via stochastic user equilibrium assignment. *Transportation Research Part B: Methodological* **28**(1) 61–75.

de Moraes Ramos, G., W. Daamen, S. Hoogendoorn. 2013. Modelling travellers' heterogeneous route choice behaviour as prospect maximizers. *Journal of Choice Modelling* **6** 17–33.

Di, X., X. He, X. Guo, H. X. Liu. 2014. Braess paradox under the boundedly rational user equilibria. *Transportation Research Part B: Methodological* **67** 86–108.

Di, X., H. X. Liu, J.-S. Pang, X. J. Ban. 2013. Boundedly rational user equilibria (BRUE): Mathematical formulation and solution sets. *Transportation Research Part B: Methodological* **57** 300–313.

Erev, I., G. Barron. 2005. On adaptation, maximization, and reinforcement learning among cognitive strategies. *Psychological Review* **112**(4) 912.

Fisk, C. 1980. Some developments in equilibrium traffic assignment. *Transportation Research Part B: Methodological* **14**(3) 243–255.

Gao, S. 2005. Optimal adaptive routing and traffic assignment in stochastic time-dependent networks. Ph.D. thesis, Massachusetts Institute of Technology.

Gao, S., E. Frejinger, M. Ben-Akiva. 2008. Adaptive route choice models in stochastic time-dependent networks. *Transportation Research Record: Journal of the Transportation Research Board* **2085**(1) 136–143.

Gao, S., E. Frejinger, M. Ben-Akiva. 2010. Adaptive route choices in risky traffic networks: A prospect theory approach. *Transportation Research Part C: Emerging Technologies* **18**(5) 727–740.

Guo, X. 2013. Toll sequence operation to realize target flow pattern under bounded rationality. *Transportation Research Part B: Methodological* **56** 203–216.

Guo, X., H. X. Liu. 2011. Bounded rationality and irreversible network change. *Transportation Research Part B: Methodological* **45**(10) 1606–1618.

Han, K., W. Szeto, T. L. Friesz. 2014. Formulation, existence, and computation of simultaneous route-and-departure choice bounded rationality dynamic user equilibrium with fixed or endogenous user tolerance. *arXiv preprint arXiv:1402.1211* .

Han, S. 2003. Dynamic traffic modelling and dynamic stochastic user equilibrium assignment for general road networks. *Transportation Research Part B: Methodological* **37**(3) 225–249.

Hearn, D. W., M. V. Ramana. 1998. *Solving congestion toll pricing models*. Springer.

Henn, V. 2000. Fuzzy route choice model for traffic assignment. *Fuzzy Sets and Systems* **116**(1) 77–101.

Henn, V. 2003. Route choice making under uncertainty: a fuzzy logic based approach. *Fuzzy Sets Based Heuristics for Optimization*. Springer, 277–292.

Henn, V., M. Ottomanelli. 2006. Handling uncertainty in route choice models: From probabilistic to possibilistic approaches. *European Journal of Operational Research* **175**(3) 1526–1538.

Hensher, D. 2014. Attribute processing as a behavioural strategy in choice making. *Handbook of Choice Modelling*. Edward Elgar, 268–289.

Hensher, D. A., W. H. Greene, Z. Li. 2011. Embedding risk attitude and decision weights in non-linear logit to accommodate time variability in the value of expected travel time savings. *Transportation Research Part B: Methodological* **45**(7) 954–972.

Hensher, D. A., Z. Li. 2012. Valuing travel time variability within a rank-dependent utility framework and an investigation of unobserved taste heterogeneity. *Journal of Transport Economics and Policy (JTEP)* **46**(2) 293–312.

Hensher, D. A., Z. Li, C. Ho. 2015. The role of source preference and subjective probability in valuing expected travel time savings. *Travel Behaviour and Society* **2**(1) 42–54.

Hensher, D. A., Z. Li, J. M. Rose. 2013a. Accommodating risk in the valuation of expected travel time savings. *Journal of Advanced Transportation* **47**(2) 206–224.

Hensher, D. A., J. M. Rose, A. T. Collins. 2013b. Understanding buy-in for risky prospects: incorporating degree of belief into the ex-ante assessment of support for alternative road pricing schemes. *Journal of Transport Economics and Policy (JTEP)* **47**(3) 453–473.

Hess, S., A. Stathopoulos. 2013. A mixed random utilityrandom regret model linking the choice of decision rule to latent character traits. *Journal of Choice Modelling* **9** 27–38.

Hess, S., A. Stathopoulos, A. Daly. 2012. Allowing for heterogeneous decision rules in discrete choice models: an approach and four case studies. *Transportation* **39**(3) 565–591.

Hong, L. J., B. L. Nelson. 2009. A brief introduction to optimization via simulation. *Winter Simulation Conference*. Winter Simulation Conference, 75–85.

Horowitz, J. L. 1984. The stability of stochastic equilibrium in a two-link transportation network. *Transportation Research Part B: Methodological* **18**(1) 13–28.

Hu, T.-Y., H. S. Mahmassani. 1997. Day-to-day evolution of network flows under real-time information and reactive signal control. *Transportation Research Part C: Emerging Technologies* **5**(1) 51–69.

Jayakrishnan, R., H. S. Mahmassani, T.-Y. Hu. 1994. An evaluation tool for advanced traffic information and management systems in urban networks. *Transportation Research Part C: Emerging Technologies* **2**(3) 129–147.

Jha, M., S. Madanat, S. Peeta. 1998. Perception updating and day-to-day travel choice dynamics in traffic networks with information provision. *Transportation Research Part C: Emerging Technologies* **6**(3) 189–212.

Jou, R.-C., K.-H. Chen. 2013. An application of cumulative prospect theory to freeway drivers' route choice behaviours. *Transportation Research Part A: Policy and Practice* **49** 123–131.

Kahneman, D., A. Tversky. 1979. Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society* 263–291.

Kitthamkesorn, S., A. Chen. 2013. A path-size weibit stochastic user equilibrium model. *Procedia-Social and Behavioral Sciences* **80** 608–632.

Koppelman, F. S., C.-H. Wen. 2000. The paired combinatorial logit model: properties, estimation and application. *Transportation Research Part B: Methodological* **34**(2) 75–89.

Leong, W., D. A. Hensher. 2015. Contrasts of relative advantage maximisation with random utility maximisation and regret minimisation. *Journal of Transport Economics and Policy (JTEP)* **49**(1) 167–186.

Li, Z., D. Hensher. 2011. Prospect theoretic contributions in understanding traveller behaviour: a review and some comments. *Transport Reviews* **31**(1) 97–115.

Li, Z., A. Tirachini, D. A. Hensher. 2012. Embedding risk attitudes in a scheduling model: application to the study of commuting departure time. *Transportation Science* **46**(2) 170–188.

Lim, Y., B. Heydecker. 2005. Dynamic departure time and stochastic user equilibrium assignment. *Transportation Research Part B: Methodological* **39**(2) 97–118.

Lim, Y., B. G. Heydecker, S. Lee. 2005. A continuous network design model in stochastic user equilibrium based on sensitivity analysis. *Journal of Advanced Transportation* **39**(1) 63–79.

Liu, Z., S. Wang, Q. Meng. 2013. Toll pricing framework under logit-based stochastic user equilibrium constraints. *Journal of Advanced Transportation* .

Loomes, G., R. Sugden. 1982. Regret theory: An alternative theory of rational choice under uncertainty. *The Economic Journal* 805–824.

Lotan, T., H. N. Koutsopoulos. 1993. Models for route choice behavior in the presence of information using concepts from fuzzy set theory and approximate reasoning. *Transportation* **20**(2) 129–155.

Lou, Y., Y. Yin, S. Lawphongpanich. 2010. Robust congestion pricing under boundedly rational user equilibrium. *Transportation Research Part B: Methodological* **44**(1) 15–28.

Lu, X., S. Gao, E. Ben-Elia. 2011. Information impacts on route choice and learning behavior in a congested network. *Transportation Research Record: Journal of the Transportation Research Board* **2243**(1) 89–98.

Mahmassani, H. S., G.-L. Chang. 1987. On boundedly rational user equilibrium in transportation systems. *Transportation Science* **21**(2) 89–99.

Mahmassani, H. S., Y.-H. Liu. 1999. Dynamics of commuting decision behaviour under advanced traveller information systems. *Transportation Research Part C: Emerging Technologies* **7**(2) 91–107.

Murat, Y. S., N. Uludag. 2008. Route choice modelling in urban transportation networks using fuzzy logic and logistic regression methods. *Journal of Scientific and Industrial Research* **67**(1) 19.

Nakayama, S., R. Kitamura, S. Fujii. 2001. Drivers' route choice rules and network behavior: Do drivers become rational and homogeneous through learning? *Transportation Research Record: Journal of the Transportation Research Board* **1752**(1) 62–68.

Prashker, J. N., S. Bekhor. 2000. Congestion, stochastic, and similarity effects in stochastic: User-equilibrium models. *Transportation Research Record: Journal of the Transportation Research Board* **1733**(1) 80–87.

Prashker, J. N., S. Bekhor. 2004. Route choice models used in the stochastic user equilibrium problem: a review. *Transport Reviews* **24**(4) 437–463.

Prato, C. G. 2009. Route choice modeling: past, present and future research directions. *Journal of Choice Modelling* **2**(1) 65–100.

Quattrone, A., A. Vitetta. 2011. Random and fuzzy utility models for road route choice. *Transportation Research Part E: Logistics and Transportation Review* **47**(6) 1126–1139.

Queipo, N. V., R. T. Haftka, W. Shyy, T. Goel, R. Vaidyanathan, P. Kevin Tucker. 2005. Surrogate-based analysis and optimization. *Progress in Aerospace Sciences* **41**(1) 1–28.

Ramazani, H., Y. Shafahi, S. Seyedabrishami. 2011. A fuzzy traffic assignment algorithm based on driver perceived travel time of network links. *Scientia Iranica* **18**(2) 190–197.

Ramos, G. M., W. Daamen, S. Hoogendoorn. 2014. A state-of-the-art review: Developments in utility theory, prospect theory and regret theory to investigate travellers' behaviour in situations involving travel time uncertainty. *Transport Reviews* **34**(1) 46–67.

Ridwan, M. 2004. Fuzzy preference based traffic assignment problem. *Transportation Research Part C: Emerging Technologies* **12**(3) 209–233.

Schwanen, T., D. Ettema. 2009. Coping with unreliable transportation when collecting children: examining parents' behavior with cumulative prospect theory. *Transportation Research Part A: Policy and Practice* **43**(5) 511–525.

Senbil, M., R. Kitamura. 2004. Reference points in commuter departure time choice: a prospect theoretic test of alternative decision frames. *Intelligent Transportation Systems*, vol. 8. Taylor & Francis, 19–31.

Sheffi, Y. 1985. *Urban transportation networks: equilibrium analysis with mathematical programming methods*.

Simon, H. A. 1982. *Models of bounded rationality: Empirically grounded economic reason*, vol. 3. MIT press.

Szeto, W., H. K. Lo. 2006. Dynamic traffic assignment: properties and extensions. *Transportmetrica* **2**(1) 31–52.

Tang, Y., S. Gao. 2015. An exploratory study of instance-based learning for network route choice. *Transportation Research Board 94th Annual Meeting*. 15-5503.

Teodorović, D., S. Kikuchi. 1990. Transportation route choice model using fuzzy inference technique. *Uncertainty Modeling and Analysis, 1990. Proceedings., First International Symposium on*. IEEE, 140–145.

Teodorović, D., S. Vukanović, K. Obradović. 1998. Modeling route choice with advanced traveler information by fuzzy logic. *Transportation Planning and Technology* **22**(1) 1–25.

Tian, L.-J., H.-J. Huang, Z.-Y. Gao. 2012. A cumulative perceived value-based dynamic user equilibrium model considering the travelers' risk evaluation on arrival time. *Networks and Spatial Economics* **12**(4) 589–608.

Tversky, A., D. Kahneman. 1992. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty* **5**(4) 297–323.

Uchida, K., A. Sumalee, D. Watling, R. Connors. 2007. A study on network design problems for multi-modal networks by probit-based stochastic user equilibrium. *Networks and Spatial Economics* **7**(3) 213–240.

Von Neumann, J. 1944. Morgenstern, o.(1944) theory of games and economic behavior. *Princeton Uuniversity Press, NJ* .

Vovsha, P. 1997. Application of cross-nested logit model to mode choice in Tel Aviv, Israel, metropolitan area. *Transportation Research Record: Journal of the Transportation Research Board* **1607**(1) 6–15.

Vythoulkas, P. C., H. N. Koutsopoulos. 2003. Modeling discrete choice behavior using concepts from fuzzy set theory, approximate reasoning and neural networks. *Transportation Research Part C: Emerging Technologies* **11**(1) 51–73.

Wang, H.-F., H.-L. Liao. 1999. User equilibrium in traffic assignment problem with fuzzy N–A incidence matrix. *Fuzzy Sets and Systems* **107**(3) 245–253.

Wen, C.-H., F. S. Koppelman. 2001. The generalized nested logit model. *Transportation Research Part B: Methodological* **35**(7) 627–641.

Wu, J., H. Sun, D. Z. Wang, M. Zhong, L. Han, Z. Gao. 2013. Bounded-rationality based day-to-day evolution model for travel behavior analysis of urban railway network. *Transportation Research Part C: Emerging Technologies* **31** 73–82.

Xiong, C., L. Zhang. 2013. Positive model of departure time choice under road pricing and uncertainty. *Transportation Research Record: Journal of the Transportation Research Board* **2345**(1) 117–125.

Xu, H., Y. Lou, Y. Yin, J. Zhou. 2011a. A prospect-based user equilibrium model with endogenous reference points and its application in congestion pricing. *Transportation Research Part B: Methodological* **45**(2) 311–328.

Xu, H., J. Zhou, W. Xu. 2011b. A decision-making rule for modeling travelers' route choice behavior based on cumulative prospect theory. *Transportation Research Part C: Emerging Technologies* **19**(2) 218–228.

Yang, H. 1999. System optimum, stochastic user equilibrium, and optimal link tolls. *Transportation Science* **33**(4) 354–360.

Zadeh, L. A. 1965. Fuzzy sets. *Information and Control* **8**(3) 338–353.

Zhang, L. 2007. Developing a positive approach to travel demand forecasting: Theory and model. *Traffic and Transportation Theory* **17** 136–150.

Zhang, L. 2006a. Agent-based behavioral model of spatial learning and route choice. *Transportation Research Board 85th Annual Meeting*. 06-2577.

Zhang, L. 2006b. Search, information, learning, and knowledge in travel decision-making. Ph.D. thesis, University of Minnesota.

Zhang, L. 2011. Behavioral foundation of route choice and traffic assignment. *Transportation Research Record: Journal of the Transportation Research Board* **2254**(1) 1–10.

Zhou, Z., A. Chen, S. Bekhor. 2012. C-logit stochastic user equilibrium model: formulations and solution algorithm. *Transportmetrica* **8**(1) 17–41.

Zhu, S., D. Levinson. 2010. Do people use the shortest path? An empirical test of Wardrop's first principle. *91th Annual Meeting of the Transportation Research Board, Washington*, vol. 8.