Medical Waste Collection Considering Transportation and Storage Risk

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Abstract

We consider a Periodic Load-dependent Capacitated Vehicle Routing Problem (PLCVRP) encountered by medical centers and medical waste collection companies for the design of a weekly inventory routing schedule to transport medical wastes to treatment sites. In addition to minimization of transportation risk, occupational risk related to temporary storage of hazardous wastes at the health-care centers is considered. The transport risk on each arc is dependent on the weight of hazardous medical waste on the vehicle when it traverses that arc. Our paper presents a new non-linear mixed-integer programming formulation for the problem accompanied by a novel decomposition based heuristic algorithm. Experimental results confirm the efficiency of our heuristic. We present a case study to illustrate solution characteristics obtained by our solution methodology. The case study is based on medical waste management in Dolj, Romania.

Keywords: medical waste collection; hazardous materials transportation; vehicle routing; decomposition-based heuristic

1 Introduction

Collection and transport of medical waste to treatment centers is a critical operational problem that local authorities face in all cities. Of the total waste generated at hospitals, about 85% is general waste and 15% is hazardous material that can be toxic, infectious, or radioactive (World Health Organization, 2015). The majority of medical waste generators are laboratories, mortuaries, blood banks, research centers, hospitals, and nursing homes.

Medical waste contains potentially dangerous microorganisms that may infect medical center patients, staff, public, and the environment. Therefore, medical waste storage at health-care centers and the transportation of these potentially harmful materials to treatment centers, are two mutually
affected risky tasks (presented in Figure 1). The former one entails the occupational risk related to the storage and handling of hazardous medical waste while the latter one includes the public risk associated with hazardous materials transportation. The medical waste collection business involves servicing customers, depending on the customer demand and environmental regulations. Environmental rules mandate daily treatment of infectious medical waste if it is kept at room temperature, and weekly treatment if is kept at a temperature less than 5 °C (Shih and Lin, 1999). Considering this regulation, the medical waste management system has to be properly designed and capable of completing the process within a week. Therefore, a good waste management system not only depends on the treatment process, but also, on how to collect infectious waste from dispersely-located medical centers.

Collection tasks of logistics companies are usually modeled to account for minimization of the transportation costs of servicing customers in the framework of vehicle routing problems (VRPs). Three of the most well-studied extensions of VRPs related to medical waste collection are, the capacitated vehicle routing problems (CVRPs), where vehicle’s capacity is limited (Toth and Vigo, 2002), the load-dependent vehicle routing problems (LVRPs), where the transportation costs depend on the vehicle’s load while traveling on its assigned route (see, e.g., Kara et al., 2008) and periodic vehicle routing problems (PVRPs), where a set of routes is obtained for a specified time period (see, e.g., Shih and Lin, 1999).

Moreover, routing models for medical waste collection have to include the important practical constraints of both the service provider and customers. First, a comprehensive load-dependent transportation risk has to be defined, which depends on the amount of hazardous waste on the vehicle while giving service to the customers. Second, vehicle capacity restriction must be considered for sharing of pick-up operations. Third, storage capacity limitations at health-care centers have to
be modeled. Due to limitations on medical waste storage capacity at medical centers and service requests depending on the size of the medical institutions, a schedule for weekly services is needed. For instance, some large hospitals may need daily services, while small clinics may require service once a week. However, in some cases even small medical centers with limited storage capacity might need a collecting service two or three times a week (Shih and Lin, 2003).

In this paper, we introduce a periodic load-dependent capacitated vehicle routing problem (PLCVRP) for medical waste collection, which:

1. incorporates minimization of both occupational risk at health-care centers and transportation risk;
2. captures the limitations on the medical waste storage capacity at medical centers, the vehicle capacity, and the maximum allowable route length;
3. considers leaving some medical centers unserved in one or more time-periods;
4. considers the inventory dynamics of medical wastes; and
5. ensures providing service for all medical centers at least once during the time horizon.

Since PLCVRP extends the computationally challenging LVRP, it is very difficult to solve and exact methods are inadequate for solving large size instances (Fukasawa et al., 2015). To solve PLCVRP, we develop a decomposition based heuristic approach that incorporates column-generation. The efficiency of our heuristic approach is proved on numerical instances of PLCVRP. A set of small-sized instances are solved exactly using the optimization software CPLEX to assess the efficiency of our heuristic algorithm. Moreover, a real case study is proposed which incorporates the medical waste collection in Dolj, Romania.

This paper is organized as follow: In Section 2, a review of the related literature is presented. In Section 3, we introduce a mixed-integer non-linear programming formulation of PLCVRP. Section 4 describes the solution methodology for solving PLCVRP. Experimental results and a case study are proposed in Sections 5 and 6, respectively. Conclusions are given in Section 7.

2 Literature Review

We can classify the existing literature based on our problem’s characteristics into two sections: Inventory Routing Problems (IRPs), and Load-dependent Vehicle Routing Problems.

2.1 Inventory Routing Problems for Medical Waste Collection

The IRP considers collection (or delivery) of a product from (or to) different customers in a specified time horizon (Bertazzi et al., 2008). The IRPs generally minimize routing and inventory costs. For excellent surveys on routing problems, see Dror (2000). A medical waste collection problem can be
viewed as a routing problem which determines minimum-cost routes on a network. A two-phase scheduling algorithm for collecting medical waste from health-care centers is introduced by Shih and Chang (2001). The first and second phases respectively propose a standard VRP and a mixed integer programming approach to specify collection routes. Nuortio et al. (2006) propose a vehicle routing model to collect municipal solid waste. A meta-heuristic approach is applied to solve real waste collection problems. Baati et al. (2014) introduce two problems for infectious medical waste; the first problem selects the optimal medical waste treatment, and the second problem is a capacitated VRP to capture the off-site transport of hospital waste.

Shih and Lin (1999) propose a periodic vehicle routing problem to pickup medical waste from disperse hospitals. A two-phased approach composed of a standard vehicle routing problem and a mixed-integer programming method is proposed to find and assign routes to specific days of the week. Shih and Lin (2003) introduce a model to minimize transportation risk, cost, and balance of workers and vehicles transporting hazardous waste. They applied a dynamic programming method and integer linear programming approach to capture the three main mentioned objectives. Nolz et al. (2014) propose an inventory routing problem to design a collection service for medical waste. Two solution approaches are applied to optimize the visiting schedule and vehicle routing.

2.2 Load-dependent Vehicle Routing Problems

The common objective in VRPs is to minimize the vehicles’ total travel distance, but this objective can be improved by adding some terms related to vehicle load. Kara et al. (2007) propose a cumulative VRP for minimizing the energy consumption where the flow on the links changes along the tour. When the vehicle has the pick-up service, the load of the vehicle is an increasing piecewise function, and a decreasing piecewise function for the delivery service. Therefore, the vehicle’s load accumulates or diminishes along the way. This type of VRP captures the vehicle’s load dependency in the optimization of transportation risk or transportation cost. The most common application of LVRP is in fuel consumption minimization. Fuel consumption models commonly focus on vehicle, traffic, and environmental effects. Increase in vehicle load boosts the engine demand power, which results in a higher fuel consumption. Transportation costs are highly affected by vehicle payload, thus, it can be a vital part of routing decisions (Demir et al., 2014). Kara et al. (2007) and Bektaş and Laporte (2011) consider the effect of vehicle load on fuel consumption. Demir et al. (2011) conclude that, the fuel consumption of a 1000 kilograms-loaded vehicle increases by 1 gallon per 100 kilometers traveling. Fukasawa et al. (2015) introduce a branch-cut-and-price algorithm to minimize the energy consumption in the framework of a vehicle routing problem which was first proposed by Kara et al. (2007). They show that a significant improvement can be achieved by their algorithm over other methods.
Table 1: Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$V'$</td>
<td>Set of all nodes (medical centers and depot) on graph $G$</td>
</tr>
<tr>
<td>$V$</td>
<td>Set of all nodes (medical centers) on graph $G$</td>
</tr>
<tr>
<td>$A$</td>
<td>Set of links on complete graph $G$</td>
</tr>
<tr>
<td>$T$</td>
<td>Set of time-periods</td>
</tr>
<tr>
<td>$T'$</td>
<td>Set of time-periods including time 0</td>
</tr>
<tr>
<td>$K$</td>
<td>Set of identical vehicles</td>
</tr>
<tr>
<td>$Q$</td>
<td>Maximum capacity of the vehicle</td>
</tr>
<tr>
<td>$L$</td>
<td>Maximum allowed route length for each vehicle in a time period</td>
</tr>
<tr>
<td>$\bar{q}_i$</td>
<td>Medical waste capacity of storage at medical center $i$</td>
</tr>
<tr>
<td>$\rho_{ij}$</td>
<td>Hazmat accident probability per unit length for vehicles traveling on link $(i, j) \in A$</td>
</tr>
<tr>
<td>$l_{ij}$</td>
<td>Travel distance from medical center $i$ to medical center $j$</td>
</tr>
<tr>
<td>$\alpha_{ij}$</td>
<td>Consequence of hazardous medical waste accident for happening on link $(i, j) \in A$</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Occupational risk of medical waste storage at medical center $i$</td>
</tr>
<tr>
<td>$\Delta q^t_i$</td>
<td>Medical waste accumulated at medical center $i$ at the beginning of time period $t$</td>
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3 Problem Description and Formulation

We assume there are two parties having significant roles in the medical waste collection system; the company that provides collection services and, the health-care center that requires the service to transport medical waste to treatment centers. In real applications, the shipping company (contractor) agrees to provide collection services for customers on a year round basis. Thus, a weekly schedule for medical waste pick-up is inevitable. The collection service company has a limited number of vehicles available to serve its customers. The vehicle starts its travel from the depot at the beginning of the working shift and returns to unload the waste at the depot.

The planning horizon consists of five working days, Monday through Friday. It is not necessary for customers to be visited at the end of the each day, but, each customer must be served at least once. This flexibility allows the carrier to give service priority to the customers based on the vehicle capacity or based on the risk caused by serving (or not serving) the customers. There is also a limitation on the length of each route. The problem can be described on an asymmetric distance matrix with homogeneous vehicles as follows.

Let $G = (V', A)$ be a complete directed graph with $V' = \{0, 1, 2, ..., n\}$ as a set of nodes. Node 0 denotes the depot and $V = \{1, 2, ..., n\}$ denotes the set of medical centers. Let $T' = \{0, 1, 2, ..., s\}$ be the set of time periods including time 0, and $T = \{1, 2, ..., s\}$ be the set of time periods excluding period 0. Let $K = \{1, 2, ..., m\}$ denote the set of $m$ identical vehicles, each of which has capacity $Q$. For every $i \in V'$, let $\Delta q^t_i$ be the accumulated medical waste at center $i$ at the beginning of period $t$, and $\bar{q}_i$ be the maximum medical waste storage at center $i$. We suppose that for the depot ($i = 0$), $\bar{q}_0$ and $\Delta q^0_0$ are equal to zero. For every link $(i, j) \in A$, let $l_{ij}$ be the distance between node $i$ and node $j$, and $\rho_{ij}$ be the hazardous material (Hazmat) accident probability per unit length on link $(i, j)$. For each link $(i, j) \in A$, $\alpha_{ij}$ denotes the consequence of hazardous medical waste exposure to people.
and environment for an accident happening on link \((i,j)\). Let parameter \(r_i\) be the occupational risk associated with medical waste storage at medical center \(i, \forall i \in V\). The following decision variables are introduced. Binary variable \(x^t_{ijk}\) is equal to 1 if vehicle \(k\) travels from node \(i\) to node \(j\) in time period \(t\), 0 otherwise. Continuous variable \(q^t_i\) specifies the amount of accumulated medical waste at medical center \(i\) at the end of time period \(t\). Continuous variable \(y^t_{ij}\) determines the vehicle’s load while traveling on link \((i,j)\) at time period \(t\). Table 1 summarizes the general notation for the PLCVRP.

**PLCVRP:**

\[
\min \sum_{t \in T'} \sum_{i \in V'} \sum_{j \in V'} \rho_{ij} l_{ij} a_{ij} y^t_{ij} + \sum_{t \in T} \sum_{i \in V} q^t_i \theta_i
\]

subject to:

\[
\sum_{i \in V} x^t_{0ik} = 1 \quad \forall t \in T, \forall k \in K
\]

\[
\sum_{i \in V} x^t_{i0k} = 1 \quad \forall t \in T, \forall k \in K
\]

\[
\sum_{i \in V', i \neq j} x^t_{ijk} - \sum_{l \in V', l \neq j} x^t_{ilk} = 0 \quad \forall j \in V, t \in T, \forall k \in K
\]

\[
\sum_{k \in K} \sum_{i \in V', i \neq j} x^t_{ijk} \leq 1 \quad \forall t \in T, j \in V
\]

\[
\sum_{k \in K} \sum_{i \in V', i \neq j} x^t_{ijk} = 1 \quad \forall j \in V, t = s
\]

\[
\sum_{i \in V'} \sum_{j \in V'} x^t_{ijk} \leq L \quad \forall t \in T, k \in K
\]

\[
q^t_i + \Delta q^t_i = \left(1 - \sum_{k \in K} \sum_{j \in V', i \neq j} x^t_{ijk}\right) q^t_i + \Delta q^t_i \quad \forall i \in V', t \in T'
\]

\[
\sum_{j \in V', i \neq j} y^t_{ij} - \sum_{j \in V', i \neq j} y^t_{ji} \leq \left(1 - \sum_{k \in K} \sum_{j \in V', i \neq j} x^t_{ijk}\right) q^t_i \quad \forall i \in V, t \in T
\]

\[
\sum_{t \in T} \sum_{j \in V', i \neq j} y^t_{ij} - \sum_{t \in T} \sum_{j \in V', i \neq j} y^t_{ji} = \sum_{t \in T} \Delta q^t_i \quad \forall i \in V
\]

\[
y^t_{ij} \leq Q \sum_{k \in K} x^t_{ijk} \quad \forall i, j \in V', t \in T'
\]

\[
q^t_i \leq \bar{q}_i \quad \forall i \in V, \forall t \in T
\]

\[
q^t_i \leq Q \quad \forall i \in V, \forall t \in T
\]

\[
x^t_{ijk} \in \{0,1\} \quad \forall (i,j) \in A, t \in T
\]

\[
y^t_{ij} \geq 0 \quad \forall (i,j) \in A, t \in T
\]

\[
q^t_i \geq 0 \quad \forall i \in V, t \in T
\]
The objective function (1) is the sum of the transport risk caused by medical waste collection and the occupational risk caused by medical waste storage at health-care centers. Constraints (2), (3), and (4) are flow conservation constraints. Constraints (5) allow the vehicles not to visit all of the customers in one period. Constraints (6) guarantee that each customer must be covered at least once during the time horizon. Constraints (7) state that the length of the routes traveled by vehicles cannot be greater than a specific amount \((L)\). Constraints (8) update the customers’ demands dynamically at the beginning of each time period. Constraints (9) show that each vehicle can visit a customer if the vehicle’s capacity is less than or equal the customers demand. Constraints (10) guarantee that all the demands should be covered at the end of the time horizon. Constraints (11) satisfy the capacity limit for vehicles. Constraints (12) establish a limit on medical waste storage capacity at medical centers. Constraints (13) implies that the accumulated medical waste at each medical center can be less than or equal to the vehicle’s capacity. Constraints (14)–(16) define the problem’s variables.

3.1 Reformulation of The Mixed Integer Bilinear Term

To make PLCVRP tractable, we linearize the non-linear constraints (8) and (9). Let \(q^t_i\) be a bounded continuous variable, and \(x^t_ijk\) be a binary variable.

Let us define \(\eta^t_ijk = q^t_i x^t_ijk\). We note that this term is a mixed-integer bilinear term with \(x^t_ijk\) being a binary variable and \(q^t_i \in [0,Q]\) a continuous variable. We can linearize this bilinear term using the following additional constraints (Gupte et al., 2013):

\[
\begin{align*}
\eta^t_ijk & \leq Q x^t_ijk \\
\eta^t_ijk & \leq q^t_i \\
\eta^t_ijk & \geq q^t_i + Q(x^t_ijk - 1) \\
\eta^t_ijk & \geq 0
\end{align*}
\]

We can now solve PLCVRP by commercial optimization software such as CPLEX for very small size problem instances.

3.2 Upper Bound on The Length of Each Collection Route

We suppose there is a vehicle with capacity \(Q\) at the treatment center. We assume \(w_i\) is the weight of medical waste at medical center \(i\) and \(d_i\) is the shortest distance between medical center \(i\) and the treatment center. The following upper bound on the travel distance of any route was proposed by Altinkemer and Gavish (1987) for the traveling salesman problem (TSP):

**Theorem 1** (Altinkemer and Gavish 1987). Suppose that customers’ demands have any integer weights less than or equal to \(Q\). Let \(C\) be a TSP tour of graph \(G\) and let \(Q\) be the vehicle capacity. Then, there exists a cluster partition \(P\) of \(C\) of total length at most \(\frac{4\sum_{i=1}^{n} w_i d_i}{Q} + \left(1 - \frac{2}{Q}\right)|C|\).
For detailed explanation see Gaur et al. (2013). Based on Theorem 1, the maximum route length $L$, mentioned in the constraints (7), for medical waste collection can be calculated as:

$$L_t = \frac{4 \sum_{i=1}^{n} q_t^i d_i}{Q} + \left(1 - \frac{2}{Q}\right) |C|$$

$$L = \max_{t \in T} \{L_t\}$$

where $q_t^i$ is the amount of medical waste at center $i$ at the end of period $t$.

4 Solution Methodology

Although many researchers have developed exact approaches to solve variations of VRP, they are usually intractable to solve large size problems in real applications. We now discuss solution methodologies for the PLCVRP. Knowing that VRP with more than 3 customers is NP-hard (Haimovich and Rinnooy Kan, 1985), our PLCVRP formulation, which is a periodic, capacitated and load-dependent variation of the classical VRP, is also NP-hard. If we consider a simplified version of our problem with no periodic characteristic, the Energy Minimization Vehicle Routing Problem (Fukasawa et al., 2015) is the closest problem in the literature to our problem. As mentioned in Section 2.2, Fukasawa et al. (2015) proposed a branch-cut-and-price heuristic algorithm to solve the model. Thus, as we can expect, the PLCVRP requires a tailored heuristic approach to deal with large size problem instances.

4.1 Decomposition Based Heuristic Approach

Taking advantage of the methodical structure of the PLCVRP, we propose a decomposition based heuristic algorithm that incorporates the basics of the column generation approach. One possible but not logical solution method for the PLCVRP is to enumerate a comprehensive set of route alternatives for each vehicle in each time period. Complete enumeration guarantees optimality, but, in real applications, it is computationally difficult as the number of paths increases exponentially in size with the number of customers. The linear relaxation of PLCVRP generates the fractional value for most of the variables and this makes the branch and bound solution method impractical in real applications due to the exponential increase in the number of nodes. Afterwards, the column generation can be an appropriate approach. The column generation method is commonly applied for solving computationally difficult linear programming (LP) problems (Gilmore and Gomory, 1961). It is also well known for dealing with mixed integer programming problems with a large set of variables (referred to as columns). Column generation itself is compromised of two problems. The first problem, known as the restricted master problem, starts solving the linear programming model with only a subset of basic variables while the remaining variables are non-basic at optimality. Then, the second problem, known as pricing subproblem, adds new columns if they are potential to enhance the objective value of the linear relaxation of the problem. Desaulniers et al. (2006) give a
Our heuristic approach decomposes the problem into a set of single period load-dependent
capacitated vehicle routing problems (LCVRP(t)), where LCVRP(t) corresponds to period t, and
applies column generation method to solve LCVRP(t) for all t ∈ T. The applied column generation
method divides each LCVRP(t) into two parts; a smaller LCVRP(t) with reduced number of
collection route alternatives, and a subproblem related to a graph composed of the customers as its
nodes. The reduced LCVRP(t) finds a design policy from the set of feasible alternatives already
obtained by the subproblem. The subproblem tries to generate new feasible columns that boost the
present objective of the LCVRP(t).

Our heuristic algorithm has four components: (i) the restricted master problem, (ii) the pricing
subproblem, (iii) the definition of penalty function, and (iv) a supplementary subproblem. In this
section, we discuss a detailed explanation of these components.

4.2 The Restricted Master Problem (RMP)

The master problem (MP) in our study is an integer programming problem. z_r denotes a binary
decision variable for a vehicle’s route choice. Here, a route is an order of medical centers visited
by a vehicle in a time period. Variable z_r is equal to 1 if route r is selected in the solution, and 0
otherwise. The MP can find the optimal solution promptly when R includes all the feasible routes.
We call our MP as restricted master problem (RMP) because it tries to find a subset of minimum
penalty feasible routes R ⊂ R. We use an optimization solver (CPLEX) to solve the LP relaxation
of our RMP. A subproblem, which is called the pricing problem is iteratively solved to find more
routes which are capable of enhancing the objective of RMP. To present the RMP, we recall the
notation from Section 3 and introduce some new variables as needed.

Notation:

R : Set of all feasible routes.
c_r : Cost of route r.
a_{ir} : 1 if customer i is visited on route r, 0 otherwise.
î : The index corresponding to a period for which RMP is implemented
z_r : Routing variable, 1 if the route r is chosen, 0 otherwise.
μ_i : Dual variable corresponding to constraint (18).
π : Dual variable corresponding to constraint (19).
γ : Dual variable corresponding to constraint (20).

\[
\text{RMP}(î): \min \sum_{r \in R} c_r z_r \quad (17)
\]

subject to:
The objective of the RMP($\bar{t}$) is to minimize the total risk of the selected routes. When route $r$ is denoted by the ordered set of nodes $N_r$ or the ordered set of links $A_r$, the total risk of route $r$ is $c_r$, defined as follows:

$$c_r = \sum_{(i,j) \in A_r} \rho_{ij}l_{ij}\alpha_{ij}w_i + \sum_{i \in N_r} \theta_id_i^\bar{t}$$

where $w_i$ is the waste load on the vehicle after visiting all customers on route $r$ starting at the depot and ending at customer $i$. Note that $d_i^\bar{t}$ is the medical waste storage at medical center $i$ at the beginning of period $\bar{t}$. Constraints (18) guarantee that each medical center is covered by at most one route and constraint (19) implies that the total number of selected routes are equal to the total number of available vehicles. Constraint (20) forces the total left-over medical waste at unserved centers in period $\bar{t}$ to be less than the extra capacity of vehicles in all remaining periods ($t > \bar{t}$).

To be able to serve all the customers at least once during the time horizon, we assume the total generated medical waste at the healthcare centers in the final period, $\bar{t} = s$, is less than or equal to the vehicle’s capacity. Also, we suppose that each medical center must be visited exactly once in the final period. Thus, the corresponding RMP is formulated as RMP($s$).

**RMP($s$):**

$$\min \sum_{r \in R} c_r z_r$$

subject to:

$$\sum_{r \in R} a_{ir} z_r = 1 \quad \forall i \in V$$

$$\sum_{r \in R} z_r = m$$

$$0 \leq z_r \leq 1 \quad \forall r \in R$$

Dual variables $\mu_i$, $\pi$, and $\gamma$ are used in the pricing problem. We now explain the pricing sub problem, which finds the candidate routes.
4.3 The Pricing Sub Problem (PSP)

In general, the PSP is defined based on the duality concept in linear programming to obtain better alternative routes that can be applied in the RMP. The PSP’s objective function is the reduced cost of the newly defined variables (in this paper, routes) regarding the current dual variables. In each iteration of the column generation algorithm, we obtain the optimal value of the dual variables $\mu_i$, $\pi$, and $\gamma$ by solving the RMP($\bar{t}$) as a linear programming problem. Then, the corresponding PSP($\bar{t}$) tries to find an alternative route with negative reduced cost.

In each iteration, we solve an elementary shortest path problem with resource constraints (ESPPRC). The network corresponding to each ESPPRC is composed of a set of nodes \{0, 1, 2, ..., $n$, $n+1$\}, where 0 is the source node and $n+1$ is the sink node (in our study, both source and sink nodes are denoted as the depot). Links are defined between every two nodes, and their associated risks are obtained using the current dual values of the RMP($\bar{t}$) constraints. We need to modify the risk of links before solving the PSP($\bar{t}$), each time we solve RMP($\bar{t}$). We first give the formula used to modify risk of the link ($i,j$).

The risk corresponding to a vehicle carrying medical waste shipments when traveling on link ($i,j$), is calculated using equation (27). Note that we assumed all the vehicles are of the same type.

$$c_{ij} = \rho_{ij} l_{ij} \alpha_{ij} w_i + \theta_i d_i$$

To calculate the revised risk of traveling from customer $i$ to customer $j$, we need an additional notation:

$$\phi_{ij}^r = \begin{cases} 1, & \text{if a vehicle travels on link } (i,j) \text{ when using route } r, \\ 0, & \text{otherwise.} \end{cases}$$

Thus, the revised link risk $\hat{c}_{ij}^\bar{t}$, corresponding to a vehicle traveling on link ($i,j$) in period $t = \bar{t} \neq s$ is presented as:

$$\hat{c}_{ij}^\bar{t} = c_{ij} - \sum_{r \in R} \mu_i a_{ir} \phi_{ij}^r + \left( \sum_{r \in R} a_{ir} \Delta q_{ij}^r \phi_{ij}^r \right) \gamma$$

Consequently, the revised total risk associated with alternative $r$, $\bar{c}_r$, is obtained by subtracting the dual variable $\pi$ as follow:

$$\bar{c}_r = c_r - \sum_{(i,j) \in A^r} \sum_{r \in R} \mu_i a_{ir} \phi_{ij}^r + \sum_{(i,j) \in A^r} \left( \sum_{r \in R} a_{ir} \Delta q_{ij}^r \phi_{ij}^r \right) \gamma - \pi$$

with $A^r$ being the set of links constituting route $r$. Note that $\mu_i, \gamma < 0$, and $\pi$ is free in sign, thus, the revised link risk, $\hat{c}_{ij}^\bar{t}$, can take any real value.

All the routes found in the current step must meet the following three requirements: travel distance, vehicle capacity, and precedence relations. As it is desirable to visit each medical center.
only once along a route, the alternative routes must be elementary. Therefore, the sub problem for every time period, PSP(\(t\)), is an ESPPRC, which is proved to be strongly NP-hard by Dror (1994) and Cheung et al. (1999). Interestingly, throughout our experiments, we find out that a very important factor to have a quick and effective column generation approach, is to solve the sub problem efficiently. ESPPRC’s solution is the most negative reduced cost elementary shortest path on the given graph. However, our aim is to find any possible negative reduced cost on the network. We develop Nemani et al. (2010)’s proposed algorithm and suggest a new heuristic approach which solves the ESPPRC by repeatedly solving a label setting algorithm for each time-period. In our problem, the link risks \(\hat{c}_{ij}\) are unrestricted in sign, however, the label setting algorithm enables us to obtain elementary routes with negative reduced costs, although the underlying graph may contain negative cost cycles.

The label setting approach we apply to solve ESPPRC is presented in Algorithm 1. In this algorithm the predecessor nodes on every partial route are stored and any cycles are removed. In a partial route, the start location is the depot, but the finishing location can be any of medical centers (nodes). The revised link risks are updated repeatedly after solving the RMP(\(t\)). The main idea of this algorithm is to create a label, \(\{u, L_u, W_u\}\), associated with every incomplete route closing at node \(u\). In each label, the first entry represents the last medical center (node) on the partial route, the second one indicates the travel distance \(L_u\), and the third entry is the load of the vehicle after visiting the last node, \(u\) on the partial route. Therefore, a label indicates the consumptions of the resources on each partial route. The label cost shown as \(\text{Cost}(\{u, L_u, W_u\})\), is found by getting sum of the revised risk of the links on the incomplete route. Inputs for the label setting algorithm are updated demands at medical centers, \(q_i\), link lengths, \(l_{ij}\), Updated link risks \(\hat{c}_{ij}\); depot, vehicle capacity, \(Q\), and maximum allowable route length, \(L\). The output of label setting algorithm is a set of elementary routes covering a set of medical centers starting from depot, \(d(s)\), and going back to depot, \(d(e)\). These routes are complied with the vehicle capacity and maximum route length constraints, and they have negative reduced costs.

The predecessor nodes relating to each partial route are stored in \(\text{Preds}\) array to prevent visiting the nodes more than once in any extension of the partial route. The number of labels that can be generated increases exponentially even for small-size problems. However, two procedures help the algorithm to be implemented efficiently, (i) feasibility check, and (ii) dominance check. Algorithm 1 explains the steps of the feasibility, existence, dominance, and improvement checks. The feasibility check removes the labels that cause one or more of these problems; travel distance violation, vehicle capacity violation, and cycling. The dominance check exclude labels that break the dominance rule. Finally, it is important mentioning that one-cycle solving the decomposition-based heuristic involves \(|T| = s\) iterations of the column generation approach. Also, the implementation of the column generation heuristic itself incorporates solving one RMP and one PSP. We need to emphasize that all the route alternatives generated by PSPs during the \(|T|\) iterations of column generation implementations, are stored in one unique set, \(R\). This way of route storage improves the computational effort of the decomposition-based heuristic algorithm.
Algorithm 1: Label setting algorithm for solving ESPPRC in time-period $t$ for PSP($\bar{t}$)

// Initialization
Obtain the updated link risks corresponding to the recent RMP solved;
Add label $\{d(s), 0, 0\}$ to the set of unprocessed labels $\Omega$ and set $\text{cost}(\{d(s), 0, 0\}) = 0$;
Create two sets, one for storing each route found, and the other one for saving the predecessor nodes of each label;
$u \leftarrow d(s)$;

while $u \neq d(e)$ do

Find the best label $u_u = \{u, L_u, W_u\}$ such that $L_u = \min\{L_k : \{k, L_k, W_k\} \in \Omega, k \neq d(e)\}$;
if $L_u \leq L$ then

Find the set of instant neighbors of $u, \Gamma_u$;
for each $v \in \Gamma_u$ do

// Feasibility Check
isFeasible $\leftarrow$ false;
if $L_u + l_{uv} \leq L$ & $W_u + q_{\bar{t}u} \leq Q$ & $v \not\in \text{Preds}[\{u, L_u, W_u\}]$ then

isFeasible $\leftarrow$ true;
$L_v \leftarrow L_u + l_{uv}$;
$W_v \leftarrow W_u + q_{\bar{t}u}$;
end

// Existence Check
if Label $\{v, L_v, W_v\} \not\in \Omega$ then

$\text{Cost}(\{v, L_v, W_v\}) \leftarrow \infty$;
end

// Improvement Check
isImproved $\leftarrow$ false;
if $\text{Cost}(\{u, L_u, W_u\}) + \hat{c}_{uv} < \text{Cost}(\{v, L_v, W_v\})$ then

// Dominance Check
if $\hat{\Omega}\{k, L_k, W_k\} \in \Omega$ such that

$k = v$ & $L_k \leq L_v$ & $W_k \leq W_v$ & $\text{Cost}(\{k, L_k, W_k\}) \leq \text{Cost}(\{v, L_v, W_v\})$ then

Create label $(\{v, L_v, W_v\})$ and add it to the set of unprocessed labels, $\Omega$;

$\text{Cost}(\{v, L_v, W_v\}) \leftarrow \text{Cost}(\{u, L_u, W_u\}) + \hat{c}_{uv}$;
$\text{Preds}[v, L_v, W_v] \leftarrow \text{Preds}[u, L_u, W_u] \cup \{u\}$;

isImproved $\leftarrow$ true;
end
end

if isFeasible & isImproved & $v = d(e)$ & $\text{Cost}(\{v, L_v, W_v\}) - \pi < 0$ then

Add the unique path from $d(s)$ to $d(e)$ to the set of routes;
end
end

end

Remove label $\{u, L_u, W_u\}$ from the set $\Omega$;
4.4 Penalty Function Description

It is evident from Section 4 that we need to consider two important concepts in our decomposition-based heuristic: (i) feasibility of solutions obtained for each time period in the complete framework of the problem, and (ii) potential improvement in the objective value found by our heuristic algorithm. Since we decompose the PLCVRP into $|T|$ different LCVRPs, the set of solutions of these $|T|$ sub problems found by column generation approach must construct a feasible solution for PLCVRP. The constraint (20) presented in the RMP formulation guarantees the feasibility of LCVRPs’ solutions in the PLCVRP. The second concept helps the decomposition-based algorithm find optimal or near optimal solutions. To reach this goal, we consider a penalty for leaving a customer unvisited in the first period. Although this penalty is defined based on demand coverage in the first period, its definition captures the increase in total risk in all the $|T|$ time periods. To present the formulation of the penalty function, we keep the notations from Sections 3 and 4.1, and define new notations as needed.

**Parameters:**

- $\bar{z}_{jr}^j$: 1 if route $r$ is selected in the solution corresponding to period $j$, 0 otherwise.
- $c_{prj}^j$: Partial cost of serving the first period’s left over medical waste at center $i$ on route $r$ in period $j$.

$$P_{i\tau} = \sum_{j=\tau+1}^{|T|} \left\{ \prod_{t=1}^{j-1} \left( 1 - \sum_{r \in R_t} a_{ir} \bar{z}_{jr}^t \right) \right\} \times \left( \sum_{r \in R_j} a_{ir} \bar{z}_{jr}^j \right) \times \left( (j-1)q_i^\tau \theta_i + \sum_{r \in R_j} c_{prj}^j a_{ir} \bar{z}_{jr}^j \right)$$

\[ \forall i \in V, \tau = 1, 2, \ldots, |T| - 1 \] (30)

Equation (30) describes the risk penalty corresponding to medical center $i$, $P_{i\tau}$, if that center is left unserved at the end of time period $\tau$. In order to calculate these penalty functions, we should solve the PLCVRP using the aforementioned column generation method for each LCVRP to find a feasible solution. This step can be considered as initialization to find a solution or a routing schedule as $\bar{z}_r = \{ \bar{z}_j^r : j \in T \}$, which is not necessarily an optimal solution for PLCVRP. The penalty function $P_i$ is defined as sum of occupational and transport risk caused by covering the demand of medical center $i$ in any time period except period $t = 1$ based on routing schedule, $\bar{z}_r$. We add these penalty functions to the objective of RMP at the start of the decomposition-based algorithm. Thus, minimizing the penalties can help the algorithm to select a good set of medical centers to visit in the first period, and consequently, improve the routing schedules of the next periods. Finally, we can revise the RMP($\bar{t} = 1$)’s objective function (17) as the following:

$$\sum_{r \in R} c_r z_r + \sum_{i \in V} \sum_{\tau=1}^{|T|-1} b_i P_{i\tau}$$

(31)

Note that $b_i$ are weight factors for the penalty function values. These weight factors play significant
roles in prioritizing customers to be covered. While one can estimate $b_i$ by surveying experts’ opinions, we suggest to randomly generate these factors, so that the proposed decomposition-based heuristic algorithm (Algorithm 2) can generate a new solution each time it is run. We intend to compare diverse solutions obtained by running the algorithm multiple times to attain a good solution quality at the end.

<table>
<thead>
<tr>
<th>Algorithm 2: Decomposition based heuristic algorithm for solving PLCVRP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data:</strong> Number of medical centers, distance between medical centers, depot, number of vehicles, maximum route length, vehicle capacity, number of time periods, and storage capacity at medical centers</td>
</tr>
<tr>
<td><strong>Result:</strong> Schedule for medical waste collection during the time horizon</td>
</tr>
<tr>
<td>Acquire the input data; $t \leftarrow 1$; $NI \leftarrow 0$; $P_i^{NI} \leftarrow 0 \forall i \in V$;</td>
</tr>
<tr>
<td>while $NI \leq$ maximum number of iterations do</td>
</tr>
<tr>
<td>Generate a primary set of routes for the RMP($t$);</td>
</tr>
<tr>
<td>while $t \leq</td>
</tr>
<tr>
<td>Update the medical waste storage at each center;</td>
</tr>
<tr>
<td>Construct RMP($t$);</td>
</tr>
<tr>
<td>Add $P_i^{NI}$ : $i \in V$, $\tau = t,...,</td>
</tr>
<tr>
<td>do</td>
</tr>
<tr>
<td>Add new promising columns to the RMP($t$);</td>
</tr>
<tr>
<td>Solve the RMP($t$) applying the primary set of routes;</td>
</tr>
<tr>
<td>Update the RMP($t$)'s objective value and the lower bound;</td>
</tr>
<tr>
<td>Find the constraints' dual values for the RMP($t$);</td>
</tr>
<tr>
<td>Calculate the link risks applying the dual values;</td>
</tr>
<tr>
<td>Solve PSP($t$) with calculated risks;</td>
</tr>
<tr>
<td>while $ReducedCost_r &lt; 0$ for any new route $r$; $t \leftarrow t + 1$;</td>
</tr>
<tr>
<td>end</td>
</tr>
<tr>
<td>$NI \leftarrow NI + 1$; $t \leftarrow 1$;</td>
</tr>
<tr>
<td>Update $P_i^{NI}$ $\forall i \in V$, $\tau = 1,...,</td>
</tr>
<tr>
<td>end</td>
</tr>
</tbody>
</table>

### 4.5 The Framework of the Decomposition-based Algorithm

The decomposition based algorithm, applied for solving the PLCVRP, is described in Algorithm 2. Notation $NI$ and $P_i^{NI}$ are denoted as the number of iterations, and penalty function corresponding to medical center $i$ if it is left unserved at the end of period $\tau$ in iteration $NI$, respectively. The heuristic approach solves column generation method $|T|$ times for every $t \in T$, denoted as CG($t$). Each CG($t$) is composed of one RMP($t$) and one PSP($t$). Note that the initial solution which is needed to construct the RMPs can be defined as $|V|$ single-visit routes. A vehicle traveling on a single-visit route, starts from the depot and visits a medical center, then goes back to the depot. However, since we assume that every customer must be visited at least once in the time horizon, a complete route is added to the initial set of solutions to ensure the feasibility of the LCVRP. A
vehicle traveling on a complete route starts from the depot, visits all the customers, and then goes back to the depot. Recalling equation (30), an initial feasible solution for PLCVRP is also required for defining the penalty functions. We can find a good initial solution for PLCVRP by solving the CG(\(\tilde{t}\)) for \(\tilde{t} \in T\) considering \(P_{i\tau} = 0, \forall i \in V, \tau = 1,...,|T| - 1\). Then, after each time solving PLCVRP, we can apply the obtained solution to revise the penalty functions.

In order to update the medical waste storage at medical centers, we should add the left-over (or un-served) waste of the previous time period at each center to its current period’s medical waste. To satisfy this updating procedure, we compute

\[
q^t_i = \left(1 - \sum_{r \in R_t} a_{ir} z^{t-1}_r\right) q^{t-1}_i + \Delta q^t_i \quad \forall t \in T
\]

where \(1 - \sum_{r \in R_t} a_{ir} z^{t-1}_r\) indicates whether the medical center \(i\) is visited in period \(t - 1\) or not. We emphasize that all the route alternatives generated during one cycle of the decomposition based algorithm are saved in set \(R\). After implementing CG(\(\tilde{t}\)) for \(\tilde{t} \in T\), the infeasible routes with respect to the updated demand will be removed from set \(R\). The remaining routes in \(R\) will be added to the set of route alternatives in the next period, \(R_{\tilde{t}+1}\), before implementation of CG(\(\tilde{t} + 1\)). This process continues until \(\tilde{t} = |T| - 1\). This simple but beneficial procedure of saving route alternatives helps to improve the computational effort of our heuristic approach. The stopping criterion of our algorithm is based on a specified number of iterations for PLCVRP.

5 Computational Analysis

Our computational analysis is: (i) to investigate the quality of the heuristic approach for solving the PLCVRP, and (ii) to analyze the solutions provided by imposing different resource limitations. We test our algorithm on a set of network instances, with chosen number of identical vehicles, vehicle capacity (\(Q\)) and maximum travel length (\(L\)). The customer demands are randomly generated for all time periods. We apply a typical weighted-sum method to develop a single objective composed of the normalized occupational risk objective and the normalized transport risk objective with equal weights; for more details see Kim and de Weck (2005). Our heuristic algorithm was implemented in Java using CPLEX 12.6.1 on a 2.40 GHz PC with 32.0 GB memory. Whenever the CPLEX MIP solver was used for comparison purposes, the value of the integer tolerance parameter was set to \(10^{-3}\). We describe the data set we applied in our analysis following by an explanation of our observations.

5.1 The Computational Performance of Proposed Decomposition Based Algorithm

We consider different test instances to carry out our numerical experiments and investigate our goals. The VRPs usually have been solved for a complete graph, but we consider incomplete graphs for some of the test problems. The reason behind this assumption is that there is a maximum route
| Test | |A| |V| Q(kg) | |K| L(miles) |
|---|---|---|---|---|---|---|
| 1 | 20 | 5 | 1500 | 1 | 100 |
| 2 | 73 | 10 | 1500 | 2 | 100 |
| 3 | 77 | 15 | 1800 | 2 | 150 |
| 4 | 154 | 25 | 2000 | 3 | 200 |

Table 2: Test Instances

<table>
<thead>
<tr>
<th>Test</th>
<th>Exact Risk</th>
<th>Exact Run Time</th>
<th>Heuristic Risk</th>
<th>Heuristic Run Time</th>
<th>Exact vs. Heuristic Min Gap</th>
<th>Exact vs. Heuristic Mean Gap</th>
<th>Exact vs. Heuristic Max Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>502.9</td>
<td>0 sec</td>
<td>502.9</td>
<td>0 sec</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>2</td>
<td>2461.2</td>
<td>11 min 33 sec</td>
<td>2508.0</td>
<td>23 sec</td>
<td>1.87%</td>
<td>5.98%</td>
<td>7.41%</td>
</tr>
<tr>
<td>3</td>
<td>9401.7</td>
<td>19 hr 17 min</td>
<td>9826.9</td>
<td>18 min 21 sec</td>
<td>4.25%</td>
<td>5.24%</td>
<td>5.87%</td>
</tr>
<tr>
<td>4</td>
<td>6270.4</td>
<td>24 hr 10 min</td>
<td>6474.1</td>
<td>9 min 12 sec</td>
<td>3.15%</td>
<td>6.06%</td>
<td>9.57%</td>
</tr>
</tbody>
</table>

Table 3: Computational Performance of the Proposed Heuristic Algorithm

length limitation in the PLCVRP. Thus, we take advantage of this travel distance constraint, and eliminate the links from the graph if their lengths are more than the maximum allowable travel distance. The computational effort of CPLEX to solve the PLCVRP improves if we consider the underlying graph with the same number of nodes and fewer number of links.

To verify the viability of the decomposition based approach, we compare solutions obtained by the heuristic algorithm with the solutions obtained from the exact algorithm. The real-life instances are intractable due to the high complexity of the underlying problem. So, we created small and medium size examples. These instances have the similar characteristics of the real networks with |V| number of nodes (medical centers), and |A| number of links. We generated 10 test instances with a 3-day planning horizon. Note that we consider \( \rho_{ij} = \theta_i = 10^{-6} \forall i, j \in |V| \) in all our experiments based on the available data on hazardous materials transportation in the literature (Taslimi et al., 2017). We solved all the test instances using the MIP formulation, but it was intractable for large-sized instances. Therefore, we terminate the algorithm based on a maximum available time of 24 hours for test problems with 40, and 50 number of medical centers. CPLEX generated optimal solutions for instances with 5 to 25 number of customers, while there were optimality gaps for the larger instances. In Table 4, the Optimality Gap is defined by comparing the best integer and best bound found by CPLEX within 24 hours. The optimality gap denoted by Gap* in Table 4, shows the difference between the best bound found by CPLEX within 24 hours and the objective value found by heuristic. Table 3 demonstrates the results for instances with 5 to 25 medical centers shown in Table 2. Due to the presence of a randomness factor in the penalty function, we solved the heuristic approach 30 times for every instance of Table 3.

To better present the efficiency of our heuristic approach, we report the minimum, mean, and maximum optimality gaps obtained for each test instance. The minimum optimality gap varies from 0% to 3.15% and shows the impact of applying randomness in selecting the customers in our
The mean gap varies from 0% to 6.06%, and shows the effectiveness of the heuristic. CPLEX run-time exponentially grows with the number of medical centers and the number of time-periods. Although the problem becomes more complicated by increasing the number of medical centers, Table 3 shows that the run time of our heuristic for all test problems is significantly less than those of the exact method. For example, when there are 25 medical centers to be served, the solution can be obtained by the heuristic algorithm within 9 minutes and 12 seconds, whereas the exact method (CPLEX) requires 24 hours and 10 minutes to obtain the solution. Thus, the decomposition-based heuristic approach is capable of finding high quality solutions in notably shorter run time. To be able to improve the optimality gap obtained by CPLEX for large size instances, we applied the solution found by our heuristic algorithm as a MIP-Start in CPLEX. Although providing CPLEX with a good feasible initial solution helped to improve its starting best integer, the lower bound improvement became worse. In Table 4, we present the results for 6 large-sized instances with 40 and 50 number of medical centers to better judge the efficiency of each solution method. Evidently, the exact method (CPLEX) is intractable for large size problem instances. The branch and bound algorithm is inefficient in reducing the optimality gap after a certain amount of time because, the size of the tree and number of nodes grow exponentially in PLCVRP. The bottleneck in the results obtained by CPLEX is due to the lack of tangible improvement in the lower bound. Table 4 shows the results found by CPLEX after 24 hours running the program for every instance. Comparison of the final solution obtained by heuristic algorithm with its corresponding best bound and best integer found by CPLEX, demonstrates the efficiency of our proposed heuristic approach. Comparison of two columns, Optimality Gap and Gap*, in Table 4 indicates that the solutions obtained by heuristic algorithm strikingly reduce the optimality gap. Moreover, the decomposition based algorithm is able to find the near-optimal solutions or probably optimal solutions in an acceptable time interval. As one can see from Table 4, computational time of the heuristic approach is highly dependent on the size of the graph and the value of parameters.

5.2 The Analysis of Experiments with Different Resource Limitations

We now show how the number of available vehicles, vehicle capacity, and maximum allowable route length affect the routing decision in the context of the PLCVRP. We choose an instance with 15
Table 5: Results Obtained by the Decomposition Based Heuristic for Different Parameter Setting

| Instance | $|K|$ | Vehicle Capacity | Max Route Length | Total Risk | Run-time |
|----------|-----|-----------------|-----------------|------------|----------|
| 11-a     | 2   | $Q$             | $L$             | 113.5      | 5 min 30 sec |
| 11-b     | 2   | $Q$             | $\frac{1}{2}L$ | 114.8      | 2 min 25 sec |
| 11-c     | 4   | $Q$             | $\frac{1}{2}L$ | 93.3       | 4 min 46 sec |
| 11-d     | 4   | $\frac{1}{2}Q$ | $\frac{1}{2}L$ | 95.2       | 1 min 15 sec |

medical centers, 77 links, and a 3-day planning horizon from the instance pool. We see from Table 5 that four different combinations of resource availability are considered. The solutions demonstrate the minimum total risk obtained after implementing 10 iterations of the heuristic algorithm for each of the test instances. The comparison of instance 11-a and 11-b reveals that if we set a tighter limit on the maximum distance a vehicle can travel, the total risk on the network will be increased. This likely happens due to the decrease in flexibility of visiting farther medical centers which have less amount of hazardous medical wastes. Moreover, the comparison of instance 11-b with instances 11-c and 11-d indicates that increase in the number of available identical vehicles can result in a decrease of total risk. The solution obtained for instance 3 provides valuable managerial information. If we suppose that each vehicle travel distance during a time-period is at most equal to $L$, then, changing $L$ to $L/2$ is alike to doubling the total number of available vehicles. We can simply assume that the drivers may work in 2 shifts with maximum travel distance limit of $L/2$. This strategy leads to a remarkable reduction in total risk of medical waste collection for instance 3.

6 An Illustrative Case Study: Dolj, Romania

We now illustrate our method on a case study that determines weekly routing schedules for medical waste collection in the county of Dolj, Romania. The locations of 10 hospitals and the treatment center are as specified on the map in Figure 2. The locations are shown by colored pins and their corresponding unit numbers as presented in Table 6. In our case study, we use the real data set previously presented by Bulucea et al. (2008) for the assessment of the biomedical waste situation in the hospitals of Dolj. The case study presents the analysis results of medical waste (hazardous and non-hazardous) generated in 10 hospitals of Dolj County. The focus of concern is on the portion of the waste stream termed hazardous such as, pathological waste, chemical waste, genotoxic waste, and radioactive waste. In the following, we can see that PLCVRP proposes a different route alternative which imposes a remarkable reduction in risk of medical waste transportations.

6.1 The Data Set

Table 6 shows the survey results of average daily medical waste generated and waste handling corresponding to a month of observation. Since we were able to find the exact location of 10 out of 11 hospitals on the Google map, we consider this case study for 10 hospitals. We chose a medical waste
treatment center in Dolj as the depot for our PLCVRP (Basel Convention, 2011). The shortest path between any pair of nodes represents their corresponding link on the graph consisted of 11 nodes (10 hospitals and depot). Using street address as of nodes, shortest path lengths were obtained using google map (See Appendix A). We defined a weekly schedule with 5 working days (or periods) from Monday to Friday. The daily medical waste accumulated at each hospital is randomly generated from a uniform distribution with the mean equal to the average daily amount of medical waste presented in Table 6. The portion of generated medical waste at each hospital that is hazardous waste is found by dividing the hazardous waste by the total non-hazardous and hazardous generated medical waste. In order to make the one-time pick-up possible for all the vehicles, we assume that the maximum storage capacity at hospitals is equal to the vehicle’s capacity. Moreover, the maximum allowable travel distance on a route for a vehicle is considered to be 300 miles. The case study involves routing two identical medical waste collection vehicles in 5 days. For our testing, we assume that the consequence of a hazardous material accident is proportional to the average population density in the county of Dolj (230 /sq. miles), and generate reasonable estimates of the parameters including $\rho_{ij}$, and $\theta_i$.

6.2 Results

The decomposition based heuristic is used to solve the PLCVRP. Just as Section 5, we apply Java and CPLEX 12.6.1 on a 2.4 GHz computer to implement our algorithm. We investigate the characteristics of the PLCVRP’s solutions with respect to four different objective functions: (i)
transport and occupational risk, (ii) transport risk, (iii) occupational risk, and (iv) transportation cost. Tables 7 to 10 summarize the implications of these four alternative schemes. In Table 7, the medical waste collection routes corresponding to vehicles in each day are represented. Evidently, each of four aforementioned schemes has a different routing schedule during the planning horizon. Our goal of considering the last scheme, the transportation cost, is to indicate how the route schedules would change in terms of defining different objective functions for our PLCVRP. Transportation cost in this study is described as follow:

\[
\sum_{i \in V'} \sum_{j \in V'} f w_{ij} l_{ij}
\]

where, \(f\) is the fuel cost per kilogram per mile and \(w_{ij}\) denotes the vehicle’s load in kg on link \((i, j)\).

Similar to the model of Kara et al. (2008), the transportation cost is a function of vehicle’s load. As we can see from Table 9, when the objective of PLCVRP is occupational risk, all the medical centers should be served in every single day of a week. This implies that medical centers have no tendency to store the medical wastes even for a single period. Another interesting observation is increase in the daily number of unvisited medical centers when the objective is the transport risk. Since our planning horizon is finite, we force the PLCVRP to find the route schedules such that all the medical centers in the last day of time horizon are visited. This assumption results in having more work load on the last day of the week. To address this issue we solve the PLCVRP for a longer planning horizon and extract our desired solution for a shorter time horizon. Since PLCVRP imposes a storage limit on the amount of accumulated medical waste at hospitals, the solution obtained for a longer time horizon includes the necessity to serve all the customers after some time periods.

From Table 8, one can conclude that the solutions impose a good work balance on each vehicle, such that, any of two drivers who has to pick up more amount of medical waste from health-care centers travels a shorter distance compared to the other vehicle. In this situation, the drivers of two

Table 6: Average Daily Medical Waste Generated in Hospitals of Dolj District

<table>
<thead>
<tr>
<th>No.</th>
<th>Beds (kg/24 h)</th>
<th>Average Beds (kg/24 h)</th>
<th>Hazardous Waste (kg/24 h)</th>
<th>Non-Hazardous Waste (kg/24 h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1452</td>
<td>1452</td>
<td>443.00</td>
<td>765.00</td>
</tr>
<tr>
<td>2</td>
<td>539</td>
<td>461</td>
<td>71.50</td>
<td>321.50</td>
</tr>
<tr>
<td>3</td>
<td>480</td>
<td>361</td>
<td>336.00</td>
<td>10.00</td>
</tr>
<tr>
<td>4</td>
<td>370</td>
<td>365</td>
<td>32.95</td>
<td>244.00</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>190</td>
<td>53.00</td>
<td>8.90</td>
</tr>
<tr>
<td>6</td>
<td>285</td>
<td>223</td>
<td>26.50</td>
<td>88.50</td>
</tr>
<tr>
<td>7</td>
<td>500</td>
<td>457</td>
<td>4.50</td>
<td>13.25</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
<td>99</td>
<td>10.00</td>
<td>2.00</td>
</tr>
<tr>
<td>9</td>
<td>162</td>
<td>160</td>
<td>8.90</td>
<td>144.00</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
<td>92</td>
<td>7.77</td>
<td>71.00</td>
</tr>
</tbody>
</table>
Table 7: Implications of different routing schemes

<table>
<thead>
<tr>
<th>Objective of PLCVRP</th>
<th>Vehicle No.</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transport Risk</td>
<td>1</td>
<td>(0.10,7,2,1,0)</td>
<td>(0.9,4,1,0)</td>
<td>(0.9,4,1,0)</td>
<td>(0.10,7,4,2,1,0)</td>
<td>(0.10,6,9,4,1,0)</td>
</tr>
<tr>
<td>&amp; Occupational Risk</td>
<td>2</td>
<td>(0.6,9,3,0)</td>
<td>(0.10,7,2,0)</td>
<td>(0.10,3,2,0)</td>
<td>(0.9,6,5,3,0)</td>
<td>(0.5,7,8,2,3,0)</td>
</tr>
<tr>
<td>Transport Risk</td>
<td>1</td>
<td>(0.7,9,2,1,0)</td>
<td>(0.4,1,0)</td>
<td>(0.10,2,3,0)</td>
<td>(0.6,9,7,4,2,3,0)</td>
<td>(0.9,5,6,4,2,3,0)</td>
</tr>
<tr>
<td>2</td>
<td>(0.10,4,0)</td>
<td>(0.9,7,3,2,0)</td>
<td>(0.6,4,1,0)</td>
<td>(0.10,5,1,0)</td>
<td>(0.10,7,8,1,0)</td>
<td>(0.13,8,0)</td>
</tr>
<tr>
<td>Occupational Risk</td>
<td>1</td>
<td>(0.1,4,8,0)</td>
<td>(0.1,4,8,0)</td>
<td>(0.4,3,8,0)</td>
<td>(0.1,3,8,0)</td>
<td>(0.1,3,8,0)</td>
</tr>
<tr>
<td>2</td>
<td>(0.2,3,7,5,9,10,0)</td>
<td>(0.2,3,7,5,9,10,0)</td>
<td>(0.2,1,7,5,9,10,0)</td>
<td>(0.2,4,7,5,6,9,10,0)</td>
<td>(0.2,4,7,5,6,9,10,0)</td>
<td>(0.1,3,8,0)</td>
</tr>
<tr>
<td>Transportation Cost</td>
<td>1</td>
<td>(0.9,7,1,0)</td>
<td>(0.9,4,0)</td>
<td>(0.8,4,0)</td>
<td>(0.7,10,1,0)</td>
<td>(0.8,10,7,4,1,3,0)</td>
</tr>
<tr>
<td>2</td>
<td>(0.10,2,3,0)</td>
<td>(0.8,3,2,1,0)</td>
<td>(0.7,1,2,3,0)</td>
<td>(0.8,5,4,2,3,0)</td>
<td>(0.5,6,9,2,0)</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Vehicles’ load (travel distance) for two routing schemes

<table>
<thead>
<tr>
<th>Objective of PLCVRP</th>
<th>Vehicle No.</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transport Risk</td>
<td>1</td>
<td>776 (178)</td>
<td>1744 (141)</td>
<td>1326 (141)</td>
<td>1470 (195)</td>
<td>1798 (287)</td>
</tr>
<tr>
<td>&amp; Occupational Risk</td>
<td>2</td>
<td>616 (181)</td>
<td>882 (175)</td>
<td>1092 (164)</td>
<td>1334 (198)</td>
<td>904 (276)</td>
</tr>
<tr>
<td>Transportation Cost</td>
<td>1</td>
<td>652 (129)</td>
<td>692 (133)</td>
<td>610 (90)</td>
<td>1474 (173)</td>
<td>1732 (270)</td>
</tr>
<tr>
<td>2</td>
<td>581 (164)</td>
<td>1652 (84)</td>
<td>1764 (65)</td>
<td>1323 (263)</td>
<td>1462 (199)</td>
<td></td>
</tr>
</tbody>
</table>

A valuable observation regarding Table 10 is that solving the PLCVRP that aims to minimize both transport and occupational risk on the network, helps the decision makers to come up with a better routing schedule in terms of the imposed risk of hazardous medical waste. A comparison of the solution found by minimizing the transportation cost with the solution obtained by minimizing the total risk demonstrates a 26.25% reduction. Moreover, the run time in Table 10 shows that the PLCVRP is computationally more difficult to solve compared with the single-objective PLCVRP. Although the routing schedule with the minimum total risk is not necessarily a schedule with minimum transportation cost, the remarkable reduction in total risk can convince decision makers to implement the obtained schedule. Although, solving the PLCVRP expends more efforts and entails complicated analysis, the route schedule and the risk value look to be plausible according to our observations.

Table 9: Unvisited medical centers for three routing schemes

<table>
<thead>
<tr>
<th>Objective of PLCVRP</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transport Risk &amp; Occupational Risk</td>
<td>4,5,8</td>
<td>3,5,6,8</td>
<td>5,6,7,8</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>Transport Risk</td>
<td>3,5,6,8</td>
<td>5,6,7,8</td>
<td>5,6,7,8,9</td>
<td>5,8</td>
<td>-</td>
</tr>
<tr>
<td>Occupational Risk</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 10: Risk values for all routing schemes

<table>
<thead>
<tr>
<th>Objective of PLCVRP</th>
<th>Risk Value</th>
<th>Run Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transport Risk</td>
<td>101.71</td>
<td>2 min 20 sec</td>
</tr>
<tr>
<td>Occupational Risk</td>
<td>4.42</td>
<td>2 min 25 sec</td>
</tr>
<tr>
<td>Transport Risk &amp; Occupational Risk</td>
<td>110.92</td>
<td>5 min 12 sec</td>
</tr>
<tr>
<td>Transportation Cost</td>
<td>150.12</td>
<td>5 min 17 sec</td>
</tr>
</tbody>
</table>

7 Conclusion and Future Research

We introduce a periodic load-dependent capacitated vehicle routing problem to find the least risky routing schedule for medical waste collection. We propose a new decomposition based heuristic approach to solve the PLCVRP, where each decomposed sub-problem itself is solved by a column generation approach. A dynamic programming algorithm is embedded in the column generation algorithm to reinforce the performance of the heuristic. Computational results on capability of the decomposition based heuristic confirms its efficiency and tractability. We apply our PLCVRP and the heuristic approach to a case study to verify the importance of this study in real applications. We consider our proposed model and algorithm a first step toward solving other types of multi-period inventory routing problems. A suggested refinement of our model is to consider stochasticity in medical waste generation, because changes in demand may result in a different optimal solution. The load dependency assumption in PLCVRP can probably spread out the application of our model in other areas of transportation such as, hazardous materials transportation and green transportation. Future research suggestions include collection of real data from health-care centers and medical waste shipping companies. Future work can also consider multi criteria decision making techniques and Pareto optimal solutions to find the ideal route schedules with respect to minimization of both risk and cost.

References


A Appendix

<table>
<thead>
<tr>
<th>Unit No.</th>
<th>Unit Name</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Treatment Center</td>
<td>S.C. SIGMAFLEX S.R.L., DJ, Craiova, str. Braida Novac, BL. 7</td>
</tr>
<tr>
<td>1</td>
<td>Emergency Clinical Hospital of Craiova</td>
<td>Spitalul Clinic Judetean de Urgenta Strada Tabaci 1 Craiova 200642 Romania</td>
</tr>
<tr>
<td>2</td>
<td>Municipal Clinical Hospital of Craiova</td>
<td>Spitalul Clinic Municipal Filantropia Strada Filantropiei 1 Craiova 200143 Romania</td>
</tr>
<tr>
<td>3</td>
<td>Infectious Diseases Clinical Hospital of Craiova</td>
<td>Spitalul de Paromedizologie Leumna de Sus 207129 Romania</td>
</tr>
<tr>
<td>4</td>
<td>LungPhysiology Hospital of Leumna</td>
<td>Spitalul de Paromedizologie Leumna de Sus 207129 Romania</td>
</tr>
<tr>
<td>5</td>
<td>Municipal Hospital of Calafat</td>
<td>Spitalul Municipal Calafat Strada Traian 5 Calafat 205200 Romania</td>
</tr>
<tr>
<td>6</td>
<td>Psychiatry Hospital of Poiana Mare</td>
<td>Spitalul de psihiatrie DJ553 Poiana Mare 207470 Romania</td>
</tr>
<tr>
<td>7</td>
<td>Urban Hospital of Segarcea</td>
<td>Spitalul orasenesc Strada Dealului Segarcea Romania</td>
</tr>
<tr>
<td>8</td>
<td>Urban Hospital of Filiasi</td>
<td>Filiasi City Hospital Bulevardul Racoteanu 216 Filiasi 205300 Romania</td>
</tr>
<tr>
<td>9</td>
<td>Urban Hospital of Bailesti</td>
<td>Spital Strada Depozitelor Bailesti Romania</td>
</tr>
<tr>
<td>10</td>
<td>Hospital of Dabuleni</td>
<td>Spitalul Orasenesc Asezamintele Brancovenesti Dabuleni DN54A Dabuleni Romania</td>
</tr>
</tbody>
</table>

Table 11: Locations of 10 hospitals and a treatment center in the city of Dolj, Romania

B Appendix

Detailed route-schedules in the map in Figures 3–7.
<table>
<thead>
<tr>
<th>Unit</th>
<th>Vehicle's load</th>
<th>Unit's hazmat load</th>
<th>Travel distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>776</td>
<td>183</td>
<td>174</td>
</tr>
<tr>
<td>2</td>
<td>276</td>
<td>32.4</td>
<td>171</td>
</tr>
<tr>
<td>3</td>
<td>616</td>
<td>313.3</td>
<td>177</td>
</tr>
<tr>
<td>4</td>
<td>652</td>
<td>183.3</td>
<td>125</td>
</tr>
<tr>
<td>5</td>
<td>159</td>
<td>37.2</td>
<td>87</td>
</tr>
<tr>
<td>6</td>
<td>152</td>
<td>5</td>
<td>98</td>
</tr>
<tr>
<td>7</td>
<td>96</td>
<td>5</td>
<td>142</td>
</tr>
<tr>
<td>8</td>
<td>293</td>
<td>7.7</td>
<td>118</td>
</tr>
<tr>
<td>9</td>
<td>134</td>
<td>7.8</td>
<td>60</td>
</tr>
<tr>
<td>10</td>
<td>78</td>
<td>7.8</td>
<td>80</td>
</tr>
<tr>
<td>11</td>
<td>152</td>
<td>5</td>
<td>142</td>
</tr>
</tbody>
</table>

(a) Routes with minimum total risk

(b) Routes with minimum cost

Figure 3: Route schedule for medical waste collection on Monday in Dolj, Romania.
Figure 4: Route schedule for medical waste collection on Tuesday in Dolj, Romania.
Figure 5: Route schedule for medical waste collection on Wednesday in Dolj, Romania.
Figure 6: Route schedule for medical waste collection on Thursday in Dolj, Romania.
Figure 7: Route schedule for medical waste collection on Friday in Dolj, Romania.