

# Medical Waste Collection Considering Transportation and Storage Risk

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## Abstract

We consider a Periodic Load-dependent Capacitated Vehicle Routing Problem (PLCVRP) encountered by healthcare centers and medical waste collection companies for the design of a weekly inventory routing schedule to transport medical wastes to treatment sites. In addition to minimization of transportation risk, occupational risk related to temporary storage of hazardous wastes at the healthcare centers is considered. The transport risk on each arc is dependent on the weight of hazardous medical waste on the vehicle when it traverses that arc. We devise a decomposition based heuristic algorithm to solve this problem. We analyze the characteristics of the PLCVRP's solutions with respect to four different criteria: (i) transport and occupational risk, (ii) transport risk, (iii) occupational risk, and (iv) transportation cost. Solving different versions of PLCVRP reveals that minimizing both transport and occupational risk on the network can aid decision makers to develop a better routing schedule in terms of the imposed risk of hazardous medical waste. Experimental results confirm the efficiency of our heuristic. We present a case study to illustrate solution attributes obtained by our solution methodology. The case study is based on medical waste management in Dolj, Romania.

**Keywords:** medical waste collection; hazardous materials transportation; vehicle routing; decomposition-based heuristic

## 1 Introduction

Collection and transport of medical waste to treatment centers is a critical operational problem that local authorities face in all cities. Of the total waste generated at hospitals, about 85% is general waste and 15% is hazardous material that can be toxic, infectious, or radioactive (World Health Organization, 2015). The majority of medical waste generators are laboratories, mortuaries, blood banks, research centers, hospitals, and nursing homes.

Medical waste contains potentially dangerous microorganisms that may infect medical center patients, staff, public, and the environment. Therefore, medical waste storage at healthcare centers

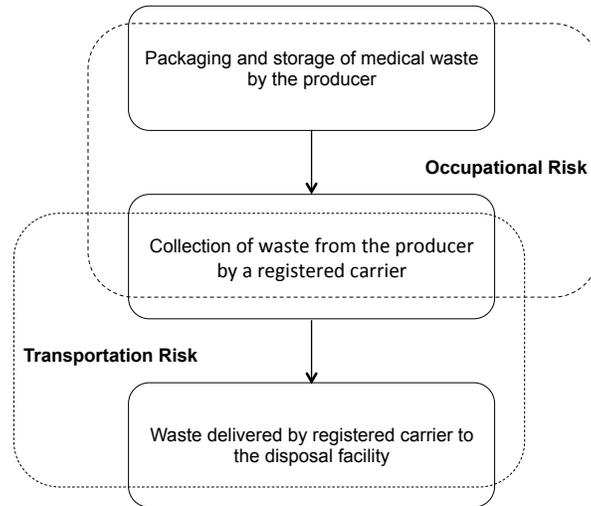


Figure 1: Risk of medical waste management

and the transportation of these potentially harmful materials to treatment centers, are two mutually affected risky tasks (presented in Figure 1). The former one entails the occupational risk related to the storage and handling of hazardous medical waste while the latter one includes the public risk associated with hazardous materials transportation. The medical waste collection business involves servicing customers, depending on the customer demand and environmental regulations. Environmental rules mandate daily treatment of infectious medical waste if it is kept at room temperature, and weekly treatment if it is kept at a temperature less than  $5^{\circ}\text{C}$  (Shih and Lin, 1999). Considering this regulation, the medical waste management system has to be properly designed and capable of completing the process within a week. Therefore, a good waste management system not only depends on the treatment process, but also, on how to collect infectious waste from dispersely-located medical centers.

Collection tasks of logistics companies are usually modeled to account for minimization of the transportation costs of servicing customers in the framework of vehicle routing problems (VRPs). Four of the most well-studied extensions of VRPs related to medical waste collection are (i) the capacitated vehicle routing problems (CVRPs), where vehicle’s capacity is limited (Toth and Vigo, 2002); (ii) the load-dependent vehicle routing problems (LVRPs), where the transportation costs depend on the vehicle’s load while traveling on its assigned route (see, e.g., Kara et al., 2008); (iii) periodic vehicle routing problems (PVRPs), where a set of routes is obtained for a specified time period (see, e.g., Shih and Lin, 1999); and (iv) a green inventory routing problem where pollution cost is included for travel along the assigned route (see, e.g., Cheng et al., 2017).

Routing models for medical waste collection have to include the important practical constraints of both the service provider and customers. First, a comprehensive load-dependent transportation risk has to be defined, which depends on the amount of hazardous waste on the vehicle while

giving service to the customers. Second, a vehicle capacity restriction must be considered for sharing of pick-up operations. Third, storage capacity limitations at healthcare centers have to be modeled. Due to limitations on medical waste storage capacity at medical centers and service requests depending on the size of the medical institutions, a schedule for weekly services is needed. For instance, some large hospitals may need daily services, while small clinics may require service once a week. However, in some cases even small medical centers with limited storage capacity might need a collecting service two or three times a week (Shih and Lin, 2003).

In this paper, we introduce a periodic load-dependent capacitated vehicle routing problem (PLCVRP) for medical waste collection, which:

1. incorporates minimization of both occupational risk at healthcare centers and transportation risk;
2. captures the limitations on the medical waste storage capacity at medical centers, the vehicle capacity, and the maximum allowable route length;
3. considers leaving some medical centers unserved in one or more time-periods;
4. considers the inventory dynamics of medical wastes; and
5. ensures providing service for all medical centers at least once during the time horizon.

The PLCVRP problem we consider is an inventory-routing problem (IRP) with load-dependent link travel cost, arising in medical-waste collection applications. From a technical perspective the challenge lies in combining the IRP with the load-dependent capacitated VRP (LCVRP), both computationally challenging problems. An IRP considers both the inventory holding cost and the transportation cost in a multi-day planning horizon, and chooses customers to visit on each day considering the impact on inventory dynamics. In contrast, LCVRP in the current literature considers only a single day for customers visited (e.g. Fukasawa et al., 2015). Our PLCVRP problem is a multi-period problem, for which we encounter a new variant of LCVRP as a single-period sub-problem of the whole PLCVRP. This characteristics of PLCVRP requires innovative computational approaches. As we explain in Section 2, existing solution methodologies applied for multi period vehicle routing problems, cannot be applied to the proposed model in this paper without drastic modifications. The main reason is, in majority of similar researches, the problem was solved in two phases; first, finding shortest paths and second, assigning these paths to time periods. But in PLCVRP, some medical centers can remain unserved in a time period and this assumption makes solutions of future periods highly dependent on current period, therefore, solving our problem in two phases is impractical. As a result, we propose a new efficient algorithm to solve PLCVRP. We develop a decomposition based heuristic approach that incorporates column-generation. The efficiency of our heuristic approach is empirically verified on numerical instances of PLCVRP. A set of small-sized instances for an arc-based formulation of PLCVRP is solved exactly using an optimization software, CPLEX, to assess the efficiency of our heuristic algorithm. Then, a set of

large instances are solved to investigate the efficiency of the heuristic decomposition method. Also, a case study is proposed to incorporate the medical waste collection in Dolj, Romania.

This paper is organized as follow: In Section 2, a review of the related literature is presented. In Section 3, we describe the problem and introduce its key notation, parameters and decision variables. Section 4 describes our solution methodology. Experimental results and a case study are proposed in Sections 5 and 6, respectively. Conclusions are given in Section 7.

## 2 Literature Review

We can classify the existing literature based on our problem’s characteristics into two sections: Inventory Routing Problems (IRPs), and Load-dependent Vehicle Routing Problems.

### 2.1 Inventory Routing Problems for Medical Waste Collection

The IRP considers collection (or delivery) of a product from (or to) different customers in a specified time horizon (Bertazzi et al., 2008). IRPs generally minimize routing and inventory costs. For an excellent survey on routing problems, see Dror (2000). A medical waste collection problem can be viewed as a routing problem which determines minimum-cost routes on a network. Studies of VRPs for medical waste collection include Shih and Chang (2001), Nuortio et al. (2006), and Baati et al. (2014).

Shih and Lin (1999) propose a periodic vehicle routing problem to pickup medical waste from disperse hospitals. A two-phased approach composed of a standard vehicle routing problem and a mixed-integer programming method is proposed to find and assign routes to specific days of the week. Shih and Lin (2003) introduce a model to minimize transportation risk, cost, and balance of workers and vehicles transporting hazardous waste. They applied a dynamic programming method and integer linear programming approach to capture the three main mentioned objectives. Markov et al. (2020) consider an IRP for waste collection considering stochastic demands. Timajchi et al. (2019) study an IRP for hazardous pharmaceutical items considering en-route accident risk. Malladi and Sowlati (2018) provide a recent review of the IRP literature focusing on sustainability aspects, including transportation of hazardous materials and medical wastes. We refer readers to the references therein for more related IRP problems.

Relevant to our problem, Nolz et al. (2014) propose an inventory routing problem to design a collection service for medical waste. Two solution approaches are applied to optimize the visiting schedule and vehicle routing. What distinguishes our work from earlier models for medical waste collection is the consideration of transportation risk that is a function of the load of hazardous waste being carried by the vehicle along with storage risk for hazardous waste at hospitals in one integrated framework that allows multiple days for waste collection. Unlike most of inventory routing problems, a medical center can remain unvisited at the end of each time period and this characteristic of our problem makes existing solution methodologies impractical. As mentioned earlier in this section, one possible approach to solve PLCVRP could be first, to find all feasible

shortest paths and then assign these routes to time periods (days of the week). But, by relaxing the assumption of visiting all medical centers in each period, PLCVRP turns into a challenging problem to solve. Our remedy to tackle this challenge is to decompose PLCVRP into multiple LCVRPs and apply a customized column generation approach to solve each LCVRP. A penalty function is designed in our decomposition-based heuristic approach so that the selection of medical centers to pick up on a particular period is guided towards overall feasibility and better objective value. These conditions mirror closely the conditions that are likely to be encountered in practice.

## 2.2 Load-dependent Vehicle Routing Problems

The common objective in VRPs is to minimize the vehicles' total travel distance, but this objective can be improved by adding some terms related to vehicle load. Kara et al. (2007) propose a cumulative VRP for minimizing the energy consumption where the flow on the links changes along the tour. When the vehicle has the pick-up service, the load of the vehicle is an increasing piecewise function, and a decreasing piecewise function for the delivery service. Therefore, the vehicle's load accumulates or diminishes along the way. This type of VRP captures the vehicle's load dependency in the optimization of transportation risk or transportation cost. The most common application of LVRP is in fuel consumption minimization. Fuel consumption models commonly focus on vehicle, traffic, and environmental effects. Increase in vehicle load boosts the engine demand power, which results in a higher fuel consumption. Transportation costs are highly affected by vehicle payload, thus, it can be a vital part of routing decisions (Demir et al., 2014). Kara et al. (2007) and Bektaş and Laporte (2011) consider the effect of vehicle load on fuel consumption. Demir et al. (2011) conclude that, the fuel consumption of a 1000 kilograms-loaded vehicle increases by 1 gallon per 100 kilometers traveling. Fukasawa et al. (2015) introduce a branch-cut-and-price algorithm to minimize the energy consumption in the framework of a vehicle routing problem which was first proposed by Kara et al. (2007). They show that a significant improvement can be achieved by their algorithm over other methods. Another closely related piece of work is the green inventory routing problem studied by Cheng et al. (2017) which incorporates pollution costs in the cost function. We use principles from the LVRP models to develop our algorithms. The distinguishing feature of our work is that we have a requirement that all customer demands have to be met at the end of the time horizon. Due to this requirement, nonlinear terms appear in the constraints as well as in the objective function (Constraints (5) and Objective (2) respectively), while nonlinear terms appear only in the objective function in the model of Cheng et al. (2017).

## 3 Problem Description and Notation

Two parties have significant roles in a medical waste collection system: the company that provides collection services and the healthcare centers that require transportation of medical waste to treatment centers. In practice, a shipping company (contractor) usually agrees to provide collection services for customers for a long term (e.g. one year) and establishes a periodic (e.g. weekly) col-

lection schedule for medical waste pick-up. The collection service company has a limited number of vehicles available to serve its customers. Vehicles start their travel from the depot at the beginning of a time period (day) and return to unload collected waste at the depot at the end of the day. A vehicles visits a medical center if it has enough capacity to collect all the waste stored in that center. In other words, partial pick-up is not allowed.

It is not necessary for customers to be visited at the end of each day, but each customer must be served on the last day of the periodic collection schedule, to guarantee that there is no medical waste left at any medical center at the end of the planning horizon. This flexibility allows the carrier to give service priority to the customers based on the vehicle capacity or based on the risk caused by serving (or not serving) the customers. There is also a limitation on the total travel distance corresponding to each route. In this paper, number of vehicles is specified large enough to guarantee the feasibility of the problem. Another possible approach can be adding a fixed cost objective term to the model and finding an optimal number of required vehicles considering associated cost.

Let  $G = (\mathcal{V}', \mathcal{A})$  be a complete directed graph with  $\mathcal{V}' = \{0, 1, 2, \dots, n\}$  as a set of nodes. Node 0 denotes the depot and  $\mathcal{V} = \{1, 2, \dots, n\}$  denotes the set of medical centers. Let  $\mathcal{T}' = \{0, 1, 2, \dots, T\}$  be the set of time periods including time 0, and  $\mathcal{T} = \{1, 2, \dots, T\}$  be the set of time periods excluding period 0. Note that time period 0 is added to allow medical waste storage at the beginning of period 1. There are  $m$  identical vehicles available, each of which has capacity  $C$ . For every  $i \in \mathcal{V}'$ , let  $\Delta q_i^t$  ( $\Delta q_i^t > 0 \forall i \in \mathcal{V}$ ) be the medical waste produced at center  $i$  during period  $t$ , and  $Q_i$  be the maximum medical waste storage at center  $i$ . We suppose that for the depot ( $i = 0$ ),  $Q_0$  and  $\Delta q_0^t$  are equal to zero. For every link  $(i, j) \in \mathcal{A}$ , let  $l_{ij}$  be the distance between node  $i$  and node  $j$ , and  $\rho_{ij}$  be the hazardous waste accident probability per unit length on link  $(i, j)$ . For each link  $(i, j) \in \mathcal{A}$ ,  $\alpha_{ij}$  denotes the consequence of hazardous waste exposure to people and environment for an accident happening on link  $(i, j)$  for each unit of medical waste. Let parameter  $\theta_i$  be the occupational risk associated with medical waste storage at medical center  $i$ ,  $\forall i \in \mathcal{V}$  for each unit of storage. Let  $\mathcal{R}_t$  be a set of all feasible paths in period  $t$  and let  $c_r^t$  be the total risk of traveling on path  $r$  in period  $t$ . We note that a feasible path is one that covers every customer at most once while not violating the vehicle's capacity and travel time constraints at the end of the time period. We let  $l_r$  denote the total distance a vehicle travels on path  $r$ . Table 1 summarizes the notation.

Given the sets, the parameters, and the variables defined in Table 1, we can write the total risk associated with path  $r$  in period  $t$  as

$$c_r^t = \sum_{i \in \mathcal{V}'} \sum_{j \in \mathcal{V}'} \rho_{ij} l_{ij} \alpha_{ij} \delta_{ij}^r y_i^t + \sum_{i \in \mathcal{V}} q_i^t \theta_i a_{ir}^t. \quad (1)$$

## 4 Decomposition Based Heuristic Approach

We decompose the problem into a set of single period load-dependent capacitated vehicle routing problems (LCVRP( $t$ )), where LCVRP( $t$ ) corresponds to period  $t$ , and applies a column generation method to solve LCVRP( $t$ ) for all  $t \in \mathcal{T}$ . The applied column generation method divides each

Table 1: Notation

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<b>Sets</b>	
$\mathcal{V}'$	Set of all nodes (medical centers and depot) on graph $G$
$\mathcal{V}$	Set of all nodes (medical centers) on graph $G$
$\mathcal{A}$	Set of links on complete graph $G$
$\mathcal{V}_r$	Set of nodes (medical centers) contained in path $r$
$\mathcal{A}_r$	Set of links contained in path $r$
$\mathcal{T}$	Set of time-periods: $\{1, 2, \dots, T\}$
$\mathcal{T}'$	Set of time-periods including time 0: $\{0, 1, 2, \dots, T\}$
$\mathcal{R}_t$	Set of all feasible paths in period $t \in \mathcal{T}$
<b>Parameters</b>	
$C$	Maximum capacity of the vehicle
$L$	Maximum allowable total travel distance for each vehicle in a time period
$Q_i$	Medical waste capacity of storage at medical center $i$
$\rho_{ij}$	Hazmat accident probability per unit length for vehicles traveling on link $(i, j) \in \mathcal{A}$
$l_{ij}$	Travel distance from medical center $i$ to medical center $j$
$\alpha_{ij}$	Consequence of hazardous medical waste accident for happening on link $(i, j) \in \mathcal{A}$ for each unit medical waste transported
$\theta_i$	Occupational risk of medical waste storage at medical center $i$ for each unit storage
$\Delta q_i^t$	Medical waste produced at medical center $i$ during time period $t$
$l_r$	A vehicle's total travel distance if traveling on path $r$
$m$	Number of vehicles
$a_{ir}^t$	Binary coefficient that takes value 1 if medical center $i \in \mathcal{V}$ belongs to path $r \in \mathcal{R}_t$ in period $t \in \mathcal{T}$ ; 0 otherwise
$\delta_{ij}^r$	Binary coefficient that takes value 1 if medical center $j \in \mathcal{V}$ is visited after medical center $i \in \mathcal{V}$ on path $r$ ; 0 otherwise
<b>Variables</b>	
$c_r^t$	Total risk of path $r \in \mathcal{R}_t$ in period $t \in \mathcal{T}$
$z_r^t$	Binary variable that takes value 1 if path $r \in \mathcal{R}_t$ is assigned to a vehicle in period $t \in \mathcal{T}$ , else 0
$y_i^t$	Vehicle's load after visiting medical center $i \in \mathcal{V}$ in period $t \in \mathcal{T}$
$q_i^t$	Medical waste storage at medical center $i \in \mathcal{V}$ at the beginning of period $t$

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LCVRP( $t$ ) into two parts; a smaller LCVRP( $t$ ) with reduced number of collection route alternatives, and a pricing problem related to a graph composed of the customers as its nodes. The reduced LCVRP( $t$ ) finds a design policy from the set of feasible alternatives already obtained by the pricing problem. The pricing problem tries to generate new feasible columns that boost the present objective of the LCVRP( $t$ ).

Our (heuristic) algorithm has four components: (i) the restricted master problem, (ii) the pricing problem, (iii) the introduction and specification of a penalty function, and (iv) a supplementary pricing problem. We now provide detailed explanations of these components.

#### 4.1 The Restricted Master Problem (RMP)

The master problem (MP) in our study is an integer programming problem.  $z_r$  denotes a binary decision variable for a vehicle's route choice. Here, a route is an order of medical centers visited by a vehicle in a time period. The time period index is dropped in the notation previously introduced, for ease of presentation. Variable  $z_r$  is equal to 1 if route  $r$  is selected in the solution, and 0 otherwise. The MP can find an optimal solution if  $\bar{\mathcal{R}}$  includes all the feasible routes and it is solved as an integer program. We call our MP as a restricted master problem (RMP) because it uses only a subset of minimum penalty feasible routes  $\mathcal{R} \subset \bar{\mathcal{R}}$  when generating its solution. Furthermore, we do not solve the RMP to integrality. Instead, we use an optimization solver (CPLEX) to solve the LP relaxation of RMP. The dual variable obtained from solving the LP relaxation of the RMP are used to formulate a RA *pricing problem* that generates additional which are capable of enhancing the objective of the LP relaxation of the RMP. To present the RMP, we recall the notation from Section 3 and introduce some new variables.

##### Notation:

$\bar{\mathcal{R}}$  : Set of all feasible routes.

$\mathcal{R}$  : Set of feasible routes currently added to the problem.

$c_r^\tau$  : Cost of route  $r$ .

$a_{ir}$  : 1 if customer  $i$  is visited on route  $r$ , 0 otherwise.

$\tau$  : The index corresponding to a period for which RMP is implemented

$z_r$  : Routing variable, 1 if the route  $r$  is chosen, 0 otherwise.

$\mu_i$  : Dual variable corresponding to constraint (3).

$\pi$  : Dual variable corresponding to constraint (4).

$\gamma$  : Dual variable corresponding to constraint (5).

RMP( $\tau$ ):

$$\text{minimize } \sum_{r \in \mathcal{R}} c_r^\tau z_r \tag{2}$$

$$\text{subject to } \sum_{r \in \mathcal{R}} a_{ir} z_r \leq 1 \quad \forall i \in \mathcal{V} \tag{3}$$

$$\sum_{r \in \mathcal{R}} z_r = m \quad (4)$$

$$\sum_{i \in \mathcal{V}} \left( 1 - \sum_{r \in \mathcal{R}} a_{ir} z_r \right) q_i^\tau \leq mC(T - \tau + 1) - \sum_{i \in \mathcal{V}} \sum_{t=\tau+1}^T \Delta q_i^t \quad (5)$$

$$0 \leq z_r \leq 1 \quad \forall r \in \mathcal{R} \quad (6)$$

Note that the quantity  $q_i^\tau$  can be precomputed for each  $i \in \mathcal{V}$  before solving  $\text{RMP}(\tau)$ .

The objective of the  $\text{RMP}(\tau)$  is to minimize the total risk of the selected routes. When route  $r$  is denoted by the ordered set of nodes  $\mathcal{N}_r$  or the ordered set of links  $\mathcal{A}_r$ , the total risk of route  $r$  is  $c_r^\tau$ , defined as follows:

$$\begin{aligned} c_r^\tau &= \sum_{(i,j) \in \mathcal{A}_r} c_{ij}^{\tau\tau} \\ &= \sum_{(i,j) \in \mathcal{A}_r} \left[ \rho_{ij} l_{ij} \alpha_{ij} w_{ir} + \theta_i q_i^\tau \right] \end{aligned}$$

where  $w_{ir}$  is the waste load on the vehicle after visiting all customers on route  $r$  starting at the depot and ending at customer  $i$ . Note that  $q_i^\tau$  is the medical waste storage at medical center  $i$  at the beginning of period  $\tau$ . Constraints (3) guarantee that each medical center is covered by at most one route and constraint (4) implies that the total number of selected routes are equal to the total number of available vehicles. Constraint (5) forces the total left-over medical waste at unserved centers in period  $\tau$  to be less than the extra capacity of vehicles in all remaining periods ( $t > \tau$ ). Note that the aim of constraint (5) is to be able to find a feasible multi-period schedule. In theory, it is possible that the right-hand-side of (5) is not tight enough, implying that no feasible solutions exist in a future time period. Our numerical experiences, however, indicate that (5) works well in practice. If no feasible solution is found in the future time periods, we can replace  $mCT$  in (5) by  $\epsilon mCT$  with some constant  $\epsilon \in (0, 1)$  and restart the process from time period 1.

To be able to serve all the customers at least once during the time horizon, we assume the total generated medical waste at the healthcare centers in the final period,  $\tau = T$ , is less than or equal to the vehicle's capacity. Also, we suppose that each medical center must be visited exactly once in the final period. Thus, the corresponding RMP is formulated as  $\text{RMP}(T)$ .

$\text{RMP}(T)$ :

$$\text{minimize } \sum_{r \in \mathcal{R}} c_r^T z_r \quad (7)$$

$$\text{subject to } \sum_{r \in \mathcal{R}} a_{ir} z_r = 1 \quad \forall i \in \mathcal{V} \quad (8)$$

$$\sum_{r \in \mathcal{R}} z_r = m \quad (9)$$

$$0 \leq z_r \leq 1 \quad \forall r \in \mathcal{R} \quad (10)$$

Dual variables  $\mu_i$ ,  $\pi$ , and  $\gamma$  for  $\text{RMP}(\tau)$  are used in the pricing problem. We now explain the pricing sub problem, which finds additional candidate routes.

## 4.2 The Pricing Sub Problem (PSP)

The PSP's objective function is the reduced cost of the newly defined variables (routes) using the values of the current set of dual variables from the LP relaxation of the RMP. In each iteration of the column generation algorithm, we obtain the optimal value of the dual variables  $\mu_i$ ,  $\pi$ , and  $\gamma$  by solving the  $\text{RMP}(\tau)$  as a linear programming problem. Then, the corresponding  $\text{PSP}(\tau)$  tries to find an alternative route (column or variable) with negative reduced cost.

To ensure feasibility, a route generated by PSP must meet the following three requirements: travel distance, vehicle capacity, and precedence relations. As it is desirable to visit each healthcare center only once along a route, the alternative routes must be *elementary*. Therefore, the sub problem for every time period,  $\text{PSP}(\tau)$ , is an Elementary Shortest Path Problem with Resource Constraints (ESPPRC), which is proved to be strongly NP-hard by Dror (1994) and Cheung et al. (1999). The network corresponding to each ESPPRC is composed of a set of nodes  $\{0, 1, 2, \dots, n, n+1\}$ , where 0 is the source node and  $n+1$  is the sink node (both source and sink nodes are denoted as the depot). Links are defined between every two nodes, and their associated risks are obtained using the current dual values of the  $\text{RMP}(\tau)$  constraints. We need to modify the risk of links before solving the  $\text{PSP}(\tau)$ . The risk corresponding to a vehicle carrying medical waste shipments when traveling on link  $(i, j)$ , is calculated using equation (11). We assume that all the vehicles are of the same type. We let

$$c_{ij}^{r\tau} = \rho_{ij} l_{ij} \alpha_{ij} w_{ir} + \theta_i q_i^\tau \quad (11)$$

Thus, the revised link risk  $\hat{c}_{ij}^\tau$ , corresponding to a vehicle traveling on link  $(i, j)$  in period  $\tau < T$  is as follows:

$$\hat{c}_{ij}^{r\tau} = c_{ij}^{r\tau} - \sum_{r' \in \mathcal{R}} \mu_i a_{ir'} \delta_{ij}^{r'} + \left( \sum_{r' \in \mathcal{R}} a_{ir'} q_i^\tau \delta_{ij}^{r'} \right) \gamma \quad (12)$$

Consequently, the revised total risk associated with alternative  $r$ ,  $\hat{c}_r^\tau$ , is obtained by subtracting the dual variable  $\pi$  as follows:

$$\hat{c}_r^\tau = c_r^\tau - \sum_{(i,j) \in \mathcal{A}_r} \sum_{r' \in \mathcal{R}} \mu_i a_{ir'} \delta_{ij}^{r'} + \sum_{(i,j) \in \mathcal{A}_r} \left( \sum_{r' \in \mathcal{R}} a_{ir'} q_i^\tau \delta_{ij}^{r'} \right) \gamma - \pi \quad (13)$$

with  $\mathcal{A}_r$  being the set of links constituting route  $r$ . Note that  $\mu_i, \gamma < 0$ , and  $\pi$  is free in sign; thus, the revised path risk,  $\hat{c}_r^\tau$ , can take any real value.

Interestingly, throughout our experiments, we find out that a very important factor to have a quick and effective column generation approach is to solve the sub problem efficiently. An optimal solution to ESPPRC yields the largest negative reduced cost elementary shortest path. To proceed

with the column generation process it suffices, however, to find *any* route which has a negative reduced cost. Finding any route with a negative reduced cost is of course much less time consuming than finding the route that has the maximum negative reduced cost. Motivated by this, we develop on Nemani et al. (2010)'s proposed algorithm and suggest a new heuristic approach which solves the ESPPRC by repeatedly solving a label setting algorithm for each time-period. In our problem, the link risks  $\widehat{c}_{ij}^{\tau}$  are unrestricted in sign, however, the label setting algorithm enables us to obtain elementary routes with negative reduced costs, even though the underlying graph may contain negative cost cycles.

The label setting approach we apply to solve ESPPRC is presented in Algorithm 1. In this algorithm the predecessor nodes on every partial route  $\bar{r}$  are stored and any cycles are removed. In a partial route  $\bar{r}$ , the start location is the depot, but the finishing location can be any of the healthcare centers (nodes). The revised link risks are updated repeatedly after solving the RMP( $\tau$ ). The main idea of this algorithm is to create a label,  $\{u, L_u, W_u\}$ , associated with every incomplete route closing at node  $u$ . In each label, the first entry represents the last healthcare center (node) on the partial route, the second entry indicates the travel distance  $L_u$ , and the third entry is the load of the vehicle after visiting the last node  $u$  on the partial route  $\bar{r}$ . Therefore, a label indicates the consumption of the resources on each partial route. The label cost, calculated as  $\text{Cost}(\{u, L_u, W_u\})$ , is found by obtaining the sum of the revised risk for links on the incomplete route. Inputs for the label setting algorithm are updated demands at healthcare centers,  $q_i^{\tau}$ , link lengths,  $l_{ij}$ , updated link risks  $\widehat{c}_{ij}^{\tau}$ , depot, vehicle capacity,  $C$ , and maximum allowable route travel distance,  $L$ . The output of the label setting algorithm is a set of elementary routes covering a set of healthcare centers starting from depot,  $d(s)$ , and returning back to depot. These routes comply with the vehicle capacity and maximum route length constraints, and they have negative reduced costs.

The predecessor nodes relating to each partial route are stored in an `Preds` array to prevent visiting nodes more than once in any extension of the partial route. The number of labels that can be generated increases exponentially even for small-size problems. However, two procedures help the algorithm to be implemented efficiently, (i) feasibility check, and (ii) dominance check. Algorithm 1 explains the steps of the feasibility, existence, dominance, and improvement checks. The feasibility check removes the labels that cause one or more of these problems; travel distance violation, vehicle capacity violation, and cycling. The dominance check excludes labels that break the dominance rule. Finally, it is important to mention that a single iteration of the decomposition-based heuristic involves  $T = |\mathcal{T}|$  iterations of the column generation approach. Route alternatives generated by PSPs during  $T$  iterations of the column generation implementation are stored in one unique set,  $R$ . This route storage method improves the computational effort of the decomposition-based heuristic algorithm.

### 4.3 Penalty Function Description

It is evident from Section 4 that we need to consider two important guiding principles in our decomposition-based heuristic: (i) feasibility of solutions obtained for each time period in the

**Algorithm 1:** Label setting algorithm for solving ESPPRC in time-period  $t$  for PSP( $\tau$ )

```
// Initialization
Obtain the updated link risks corresponding to the recent RMP solved;
Add label  $\{d(s), 0, 0\}$  to the set of unprocessed labels  $\Omega$  and set  $\text{cost}(\{d(s), 0, 0\})=0$ ;
Create two sets, one for storing each route found, and the other one for saving the predecessor
nodes of each label;
 $u \leftarrow d(s)$ ;

while  $u \neq d(e)$  do
    Find the best label  $u_u = \{u, L_u, W_u\}$  such that  $L_u = \min\{L_k : \{k, L_k, W_k\} \in \Omega, k \neq d(e)\}$ ;
    if  $L_u \leq L$  then
        Find the set of instant neighbors of  $u, \Gamma_u$ ;
        for each  $v \in \Gamma_u$  do
            // Feasibility Check
            isFeasible  $\leftarrow$  false;
            if  $L_u + l_{uv} \leq L$  &  $W_u + q_u^\tau \leq C$  &  $v \notin \text{Preds}[\{u, L_u, W_u\}]$  then
                isFeasible  $\leftarrow$  true;
                 $L_v \leftarrow L_u + l_{uv}$ ;
                 $W_v \leftarrow W_u + q_u^\tau$ ;
            end

            // Existence Check
            if Label  $\{v, L_v, W_v\} \notin \Omega$  then
                 $\text{Cost}(\{v, L_v, W_v\}) \leftarrow \infty$ ;
            end

            // Improvement Check
            isImproved  $\leftarrow$  false;
            if  $\text{Cost}(\{u, L_u, W_u\}) + \hat{c}_{uv}^\tau < \text{Cost}(\{v, L_v, W_v\})$  then
                // Dominance Check
                if  $\nexists \{k, L_k, W_k\} \in \Omega$  such that
                     $k = v$  &  $L_k \leq L_v$  &  $W_k \leq W_v$  &  $\text{Cost}(\{k, L_k, W_k\}) < \text{Cost}(\{v, L_v, W_v\})$  then
                        Create label  $(\{v, L_v, W_v\})$  and add it to the set of unprocessed labels,  $\Omega$ ;
                         $\text{Cost}(\{v, L_v, W_v\}) \leftarrow \text{Cost}(\{u, L_u, W_u\}) + \hat{c}_{uv}^\tau$ ;
                         $\text{Preds}[\{v, L_v, W_v\}] \leftarrow \text{Preds}[\{u, L_u, W_u\}] \cup \{u\}$ ;
                        isImproved  $\leftarrow$  true;
                    end
                end
            end

            if isFeasible & isImproved &  $v = d(e)$  &  $\text{Cost}(\{v, L_v, W_v\}) - \pi < 0$  then
                Add the unique path from  $d(s)$  to  $d(e)$  to the set of routes;
            end
        end
    end
    Remove label  $\{u, L_u, W_u\}$  from the set  $\Omega$ ;
end
```

complete framework of the problem, and (ii) potential improvement in the objective value found by our heuristic algorithm. Since we decompose our problem into  $T$  different LCVRPs, the set of solutions of these  $T$  sub problems found by the column generation approach must taken together construct a feasible solution for the original problem, which we label as PLCVRP. Constraint (5) in the RMP formulation helps to guide the feasibility of the collective achieve LCVRP solutions to achieve the PLCVRP solution. The second guiding principle helps the decomposition-based algorithm find optimal or near optimal solutions. To guide us towards this goal we assign a penalty for leaving a customer unvisited in the first period. Although this penalty is defined based on how healthcare centers are covered in the first period, its definition captures the increase in total risk in all the  $T$  time periods. To present the formulation of the penalty function, we define the following additional notation.

**Parameters:**

$\bar{z}_r^t$  : 1 if route  $r$  is selected in the solution corresponding to period  $t$ , 0 otherwise.

$\phi_i^{rt}$  : Partial cost (transport risk) of transporting the medical waste at center  $i$  to the next center on route  $r$  for the first time in period  $t$ . This quantity can be computed as follows:

$$\phi_i^{rt} = \sum_{(i,j) \in \mathcal{A}_r} \rho_{ij} l_{ij} \alpha_{ij} \delta_{ij}^r q_i^t.$$

We define the risk penalty corresponding to healthcare center  $i$ ,  $P_{it}$ , if it is left unserved until the end of time period  $t$  as follows:

$$P_{it} = \sum_{\omega=t+1}^T \left[ \left\{ \prod_{t'=1}^{\omega-1} \left( 1 - \sum_{r \in \mathcal{R}_{t'}} a_{ir} \bar{z}_r^{t'} \right) \right\} \times \left( \sum_{r \in \mathcal{R}_\omega} a_{ir} \bar{z}_r^\omega \right) \times \left( (\omega - 1) q_i^t \theta_i + \sum_{r \in \mathcal{R}_\omega} \phi_i^{r\omega} a_{ir} \bar{z}_r^\omega \right) \right] \quad (14)$$

for each  $i \in \mathcal{V}$  and  $t = 1, 2, \dots, T - 1$ . Note that  $\omega$  and  $t'$  are dummy indices for denoting time periods. In order to calculate these penalty functions, at first, we need to solve the PLCVRP using the aforementioned column generation method without incorporating a penalty function. This step can be considered as initialization to find a solution or a routing schedule as  $\bar{z}_r = \{\bar{z}_r^j : j \in \mathcal{T}\}$ , which is not necessarily an optimal solution for PLCVRP. Then, the penalty function  $P_{it}$  is defined as the sum of occupational and transport risk corresponding to leaving medical center  $i$  unserved till the end of period  $t$  considering the routing schedule,  $\bar{z}_r$ . We add these penalty functions to the objective of RMP ( $\tau$ ) at the beginning of the decomposition-based algorithm. Thus, minimizing the total penalty can help the algorithm to select a suitable set of healthcare centers to visit in each time period  $\tau < T$ , and consequently, improve the routing schedules of future periods. Finally, we replace the RMP( $\tau$ )'s objective function (2) with the following formula:

$$\sum_{r \in \mathcal{R}} c_r^\tau z_r + \sum_{i \in \mathcal{V}} \sum_{t=\tau}^{T-1} b_i P_{it} \quad (15)$$

Note that  $b_i$  are weight factors for the penalty function values which are randomly generated from

Bernoulli distribution with parameter  $p = 0.5$ . These weight factors play a significant role in prioritizing customers to be covered. While one can estimate  $b_i$  by surveying expert opinions, we suggest randomly generate these factors so that the proposed decomposition-based heuristic algorithm (Algorithm 2) can generate a new solution each time it is run. We compare solutions obtained by running the algorithm multiple times to attain a good quality solution.

<p><b>Algorithm 2:</b> Decomposition based heuristic algorithm for solving PLCVRP</p> <p><b>Data:</b> Number of medical centers, distance between medical centers, depot, number of vehicles, maximum route length, vehicle capacity, number of time periods, and storage capacity at medical centers</p> <p><b>Result:</b> Schedule for medical waste collection during the time horizon</p> <p>Acquire the input data;  <math>\tau \leftarrow 1</math>; <math>NI \leftarrow 0</math>; <math>P_i^{NI} \leftarrow 0 \forall i \in \mathcal{V}</math>;</p> <p><b>while</b> <math>NI \leq</math> maximum number of iterations <b>do</b></p> <p style="padding-left: 2em;">Generate a primary set of routes for the RMP(<math>\tau</math>);</p> <p style="padding-left: 2em;"><b>while</b> <math>\tau \leq T</math> <b>do</b></p> <p style="padding-left: 4em;">Update the medical waste storage at each center;</p> <p style="padding-left: 4em;">Construct RMP(<math>\tau</math>);</p> <p style="padding-left: 4em;">Add <math>P_{it}^{NI} : i \in \mathcal{V}, t = \tau, \dots, T - 1</math> to the objective function of RMP(<math>\tau</math>) as in (15);</p> <p style="padding-left: 4em;"><b>do</b></p> <p style="padding-left: 6em;">Add new promising columns to the RMP(<math>\tau</math>);</p> <p style="padding-left: 6em;">Solve the RMP(<math>\tau</math>) applying the primary set of routes;</p> <p style="padding-left: 6em;">Update the RMP(<math>\tau</math>)'s objective value and the lower bound;</p> <p style="padding-left: 6em;">Find the constraints' dual values for the RMP(<math>\tau</math>);</p> <p style="padding-left: 6em;">Calculate the link risks applying the dual values;</p> <p style="padding-left: 6em;">Solve PSP(<math>\tau</math>) with calculated risks;</p> <p style="padding-left: 4em;"><b>while</b> <math>ReducedCost_r &lt; 0</math> for any new route <math>r</math>;</p> <p style="padding-left: 4em;">Solve RMP(<math>\tau</math>) with binarity constraints;</p> <p style="padding-left: 4em;"><math>\tau \leftarrow \tau + 1</math>;</p> <p style="padding-left: 2em;"><b>end</b></p> <p style="padding-left: 2em;"><math>NI \leftarrow NI + 1</math>;</p> <p style="padding-left: 2em;"><math>\tau \leftarrow 1</math>;</p> <p style="padding-left: 2em;">Update <math>P_{i\tau}^{NI} \forall i \in \mathcal{V}, \tau = 1, \dots, T - 1</math>;</p> <p><b>end</b></p>
--

#### 4.4 The Framework of the Decomposition-based Algorithm

The decomposition based algorithm, applied for solving the PLCVRP, is described in Algorithm 2. Notation  $NI$  and  $P_{i\tau}^{NI}$  are denoted as the number of iterations, and penalty function corresponding to healthcare center  $i$  if it is left unserved at the end of period  $\tau$  in iteration  $NI$ , respectively. The heuristic approach invokes the column generation method  $T$  times for every  $\tau \in \mathcal{T}$ , denoted as  $CG(\tau)$ . Each  $CG(\tau)$  is composed of one RMP( $\tau$ ) and one PSP( $\tau$ ). Note that the initial solution which is needed to construct the RMPs can be defined as  $|\mathcal{V}|$  *single-visit* routes. A vehicle traveling on a *single-visit* route, starts from the depot and visits a healthcare center, then goes back to the depot. However, since we assume that every customer must be visited at least once in the time

horizon, a *complete* route is added to the initial set of solutions to ensure the feasibility of the LCVRP. A vehicle traveling on a *complete* route starts from the depot, visits all the customers, and then goes back to the depot. Recalling equation (14), an initial feasible solution for PLCVRP is also required for defining the penalty functions. We can find a good initial solution for PLCVRP by solving the CG( $\tau$ ) for  $\tau \in \mathcal{T}$  considering  $P_{i\tau} = 0, \forall i \in \mathcal{V}, \tau = 1, \dots, T - 1$ . The solutions to PLCVRP at various stages can be used to revise the penalty functions. When PSP( $\tau$ ) does generate no more route, we solve RMP( $\tau$ ) as an integer program after replacing (6) by binarity conditions to obtain a solution.

In order to update the medical waste storage at healthcare centers, we should add the left-over (or un-served) waste of the previous time period at each center to its current period's medical waste. To satisfy this updating procedure, we compute

$$q_i^t = \left(1 - \sum_{r \in \mathcal{R}_t} a_{ir} \bar{z}_r^{t-1}\right) q_i^{t-1} + \Delta q_i^t \quad \forall t \in \mathcal{T} \quad (16)$$

where  $(1 - \sum_{r \in \mathcal{R}_t} a_{ir} \bar{z}_r^{t-1})$  indicates whether the healthcare center  $i$  is visited in period  $t - 1$  or not. We emphasize that all the route alternatives generated during one cycle of the decomposition based algorithm are saved in set  $\mathcal{R}$ . After implementing CG( $\tau$ ) for  $\tau \in \mathcal{T}$ , the infeasible routes with respect to the updated demand will be removed from set  $\mathcal{R}$ . The remaining routes in  $\mathcal{R}$  will be added to the set of route alternatives in the next period,  $\mathcal{R}_{\tau+1}$ , before implementation of CG( $\tau+1$ ). This process continues until  $\tau = T - 1$ . This simple procedure of saving route alternatives helps improve the computational effort of our heuristic approach. The stopping criterion of our algorithm is based on a specified number of iterations for PLCVRP.

## 5 Computational Analysis

The goals of our computational analysis are to: (i) to investigate the quality of the heuristic approach for solving the PLCVRP, and (ii) to analyze the solutions provided by imposing different resource limitations. We test our algorithm on a set of network instances, with chosen number of identical vehicles, vehicle capacity ( $C$ ) and maximum travel length ( $L$ ). The customer demands are randomly generated from a Uniform distribution ( $U(4, 40)$ ) for all time periods. We apply a typical weighted-sum method to develop a single objective composed of the normalized occupational risk objective and the normalized transport risk objective with equal weights; for more details see Kim and de Weck (2005). Our heuristic algorithm was implemented in Java using CPLEX 12.6.1 on a 2.40 GHz PC with 32.0 GB memory. Whenever the CPLEX MIP solver was used for comparison purposes, the value of the integer tolerance parameter was set to  $10^{-3}$ . We describe the data set we applied in our analysis followed by an explanation of our observations.

Table 2: Test Instances

Test	$ \mathcal{A} $	$ \mathcal{V} $	$C(\text{kg})$	$ \mathcal{K} $	$L(\text{miles})$
1	20	5	1500	1	100
2	73	10	1500	2	100
3	77	15	1800	2	150
4	154	25	2000	3	200

## 5.1 Computational Performance of Decomposition Based Algorithm

We consider different test instances to carry out our numerical experiments and investigate our goals. VRPs usually have been solved for complete graphs, but we consider incomplete graphs for some of the test problems. The reason behind this assumption is that there is a maximum route length limitation in the PLCVRP. Thus, we take advantage of this travel distance constraint, and eliminate the links from the graph if their lengths are more than the maximum allowable travel distance. The computational effort of CPLEX to solve the PLCVRP improves if we consider the underlying graph with the same number of nodes and fewer number of links.

To verify the viability of the decomposition based approach, we compare solutions obtained by the heuristic algorithm with the solutions obtained from the exact algorithm. Comparisons on real-life instances are intractable due to the high complexity of the underlying problem. So, we created small and medium size examples. Note that  $|\mathcal{K}|$  represents the number of vehicles. These instances have similar characteristics of the real networks with  $|\mathcal{V}|$  number of nodes (healthcare centers), and  $|\mathcal{A}|$  number of links. We generated 10 test instances with a 3-day planning horizon. Note that we consider  $\rho_{ij} = \theta_i = 10^{-6} \forall (i, j) \in \mathcal{V}$  in all our experiments based on the available data on hazardous materials transportation in the literature (Taslimi et al., 2017).

We developed an arc based MIP formulation for PLCVRP for the sole purpose of obtaining bounds in small problem instances using the CPLEX solver on this MIP formulation. Details of the arc based MIP formulation are omitted for the sake of brevity. CPLEX generated optimal integer solutions—without subtours—for instances with 5 to 25 number of customers, while there were optimality gaps for the larger instances. Table 3 demonstrates the results for instances with 5 to 25 medical centers shown in Table 2. Due to the presence of a randomness factor in the penalty function, we solved the heuristic approach 30 times for every instance of Table 3. In Table 4, the Optimality Gap is defined by comparing the best integer and best bound found by CPLEX within 24 hours. The optimality gap denoted by Gap\* in Table 4 shows the difference between the best bound found by CPLEX within 24 hours and the objective value found by the heuristic algorithm.

To better present the efficiency of our heuristic approach, we report the minimum, mean, and maximum optimality gaps obtained for each test instance. The minimum optimality gap varies from 0% to 3.15% and shows the impact of applying randomness in selecting customers in our heuristic algorithm. The mean gap varies from 0% to 6.06%, and shows the effectiveness of the heuristic. CPLEX run time exponentially grows with the number of healthcare centers and the number of time

Table 3: Computational Performance of the Proposed Heuristic Algorithm

Test	Exact		Heuristic		Exact vs. Heuristic		
	Risk	Run Time	Risk	Run Time	Min Gap	Mean Gap	Max Gap
1	502.9	0 sec	502.9	0 sec	0%	0%	0%
2	2461.2	11 min 33 sec	2508.0	23 sec	1.87%	5.98%	7.41%
3	9401.7	19 hr 17 min	9826.9	18 min 21 sec	4.25%	5.24%	5.87%
4	6270.4	24 hr 10 min	6474.1	9 min 12 sec	3.15%	6.06%	9.57%

Table 4: Computational Performance of the Proposed Heuristic Algorithm for Large-sized Instances

Instance	CPLEX				Heuristic						
	$ \mathcal{A} $	$ \mathcal{V} $	$C$	$ \mathcal{K} $	Best Integer	Best Bound	Optimality Gap	Run Time	Objective Value	Gap*	Run Time
5	290	40	180	10	221.6	200.5	9.50%	24 hr	210.3	4.66%	4 min, 21 sec
6	290	40	150	8	245.8	209.5	14.75%	24 hr	224.7	6.76%	2 min, 15 sec
7	290	40	200	6	254.7	206.4	18.94%	24 hr	234.9	12.13%	4 hr, 5 min
8	344	50	180	7	326.7	269.2	17.60%	24 hr	315.2	14.59%	3 hr, 8 min
9	344	50	150	8	-	274.2	-	24 hr	319.5	14.17%	6 hr, 5 min
10	344	50	160	10	-	272.6	-	24 hr	283.8	3.94%	4 hr, 15 min

periods. Although the problem becomes more complicated by increasing the number of healthcare centers, Table 3 shows that the run time of our heuristic for all test problems is significantly less than those of the exact method. For example, when there are 25 healthcare centers to be served, the solution can be obtained by the heuristic algorithm within 9 minutes and 12 seconds, where as the exact method (CPLEX) requires 24 hours and 10 minutes to obtain the solution. Thus, the decomposition-based heuristic approach is capable of finding high quality solutions in notably shorter run time. To be able to improve the optimality gap obtained by CPLEX for large size instances, we applied the solution found by our heuristic algorithm as an MIP-Start in CPLEX. Although providing CPLEX with a good feasible initial solution helped to improve its starting best integer, the lower bound improvement became worse.

In Table 4, we present the results for 6 large-sized instances with 40 and 50 number of medical centers to better judge the efficiency of each solution method. Evidently, the exact method (CPLEX) is intractable for large size problem instances. The branch and bound algorithm is inefficient in reducing the optimality gap after a certain amount of time because, the size of the tree and number of nodes grow exponentially in PLCVRP. The bottleneck in the results obtained by CPLEX is due to the lack of tangible improvement in the lower bound. Table 4 shows the results found by CPLEX after 24 hours running the program for every instance. Comparison of the final solution obtained by heuristic algorithm with its corresponding best bound and best integer found by CPLEX, demonstrates the efficiency of our proposed heuristic approach. Comparison of two columns, Optimality Gap and Gap\* in Table 4, indicates that the solutions obtained by heuristic algorithm strikingly reduce the optimality gap. Moreover, the decomposition based algorithm is

Table 5: Results Obtained by the Decomposition Based Heuristic for Different Parameter Setting

Instance	$ \mathcal{K} $	Vehicle Capacity	Max Route Length	Total Risk	Run-time
11-a	2	$C$	$L$	113.5	5 min 30 sec
11-b	2	$C$	$\frac{1}{2}L$	114.8	2 min 25 sec
11-c	4	$C$	$\frac{1}{2}L$	93.3	4 min 46 sec
11-d	4	$\frac{1}{2}C$	$\frac{1}{2}L$	95.2	1 min 15 sec

able to find the near-optimal solutions or probably optimal solutions in an acceptable time interval. As one can see from Table 4, computational time of the heuristic approach is highly dependent on the size of the graph and the value of parameters.

## 5.2 Analysis of Experiments with Different Resource Limitations

We now show how the number of available vehicles, vehicle capacity, and maximum allowable route length affect the routing decision in the context of the PLCVRP. We choose an instance with 15 healthcare centers, 77 links, and a 3-day planning horizon from the instance pool. We see from Table 5 that four different combinations of resource availability are considered. The solutions demonstrate the minimum total risk obtained after implementing 10 iterations of the heuristic algorithm for each of the test instances. The comparison of instance 11-a and 11-b reveals that if we set a tighter limit on the maximum allowable distance a vehicle can travel, the total risk on the network will be increased. This likely happens due to the decrease in flexibility in visiting farther healthcare centers which have less amount of hazardous medical waste.

Moreover, the comparison of instance 11-b with instances 11-c and 11-d indicates that increase in the number of available identical vehicles can result in a decrease of total risk. The solution obtained for instance 3 provides valuable managerial information. If we suppose that each vehicle’s travel distance during a time-period is at most equal to  $L$ , then, changing  $L$  to  $L/2$  is equivalent to doubling the total number of available vehicles. Alternatively, we can assume that the drivers work 2 shifts with maximum allowable travel distance of  $L/2$ . This strategy leads to a remarkable reduction in total risk of medical waste collection for instance 3.

## 6 An Illustrative Case Study: Dolj, Romania

We now illustrate our method on a case study that determines weekly routing schedules for medical waste collection in the county of Dolj, Romania. The locations of 10 hospitals (healthcare centers) and the treatment center are as specified on the map in Figure 2. The locations are shown by colored pins and their corresponding unit numbers as presented in Table 6. In our case study, we use the real data set previously presented by Bulucea et al. (2008) for the assessment of the biomedical waste situation in the hospitals of Dolj. The case study focuses on applying our model and its solution methodology to medical waste (hazardous and non-hazardous) pickup for the 10

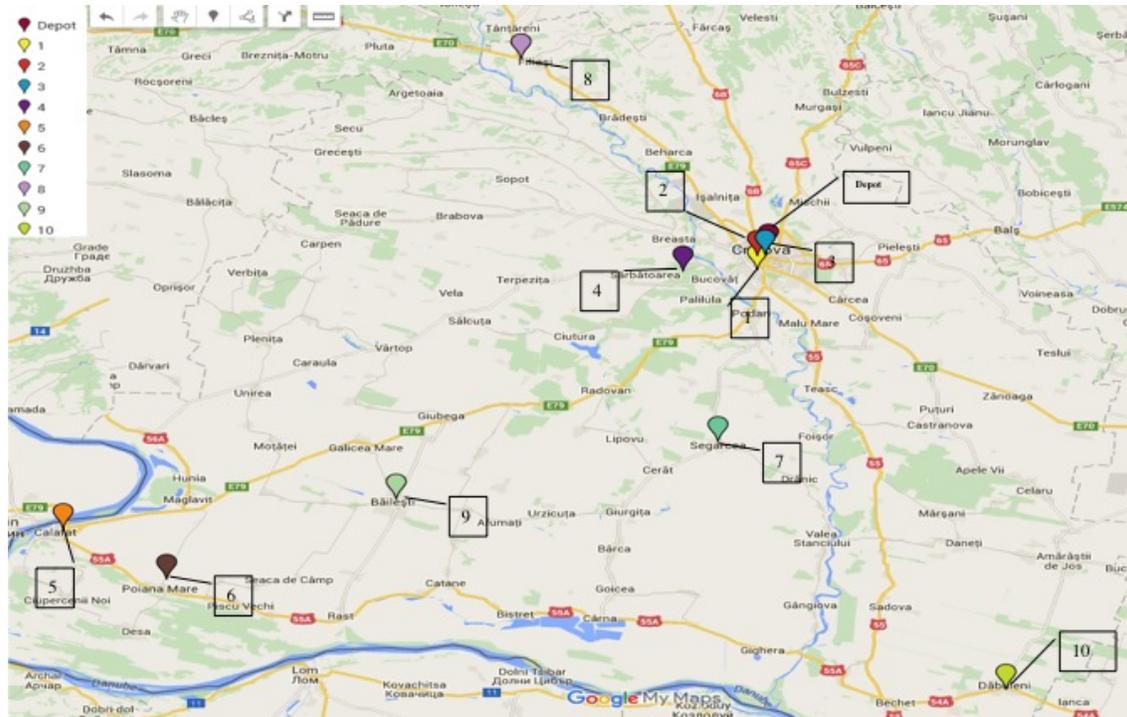


Figure 2: Hospitals' locations in Dolj, Romania

hospitals of Dolj County. Our focus is on the portion of the waste stream termed hazardous such as, pathological waste, chemical waste, genotoxic waste, and radioactive waste. Our findings indicate that PLCVRP provides a different set of route alternatives which result in a significant reduction in risk for medical waste transportation.

## 6.1 The Data Set

Table 6 shows the survey results of the average daily medical waste generated and waste handling corresponding to a month of observation. Since we were able to find the exact location only for 10 out of 11 hospitals using Google Maps, we only use 10 hospitals in our case study. We chose a medical waste treatment center in Dolj as the depot for our PLCVRP (Basel Convention, 2011). The shortest path between any pair of nodes represent the corresponding link on the complete graph, which consisted of 11 nodes (10 hospitals and depot). Using street address as of nodes, shortest path lengths were obtained using google map (See Appendix A). We defined a weekly schedule with 5 working days (or periods) from Monday to Friday. The daily medical waste accumulated at each hospital is randomly generated from a uniform distribution with the mean equal to the average daily amount of medical waste presented in Table 6. The portion of generated medical waste at each hospital that is hazardous waste is found by dividing the hazardous waste by the total non-hazardous and hazardous generated medical waste. In order to make the one-time pick-up possible for all the vehicles, we assume that the maximum storage capacity at hospitals is equal to the vehicle's capacity. Moreover, the maximum allowable travel distance on a route for a vehicle is

Table 6: Average Daily Hazardous Medical Waste Generated in Hospitals of Dolj District

No.	Hazardous Waste (kg/24 h)
1	443.00
2	71.50
3	336.00
4	32.95
5	53.00
6	26.50
7	4.50
8	10.00
9	8.90
10	7.77

considered to be 300 miles. The case study involves routing two identical medical waste collection vehicles in 5 days. For our testing, we assume that the consequence of a hazardous material accident is proportional to the average population density in the county of Dolj (230/square mile), and generate reasonable estimates of the parameters, including  $\rho_{ij}$ , and  $\theta_i$ .

## 6.2 Results

The decomposition based heuristic is used to solve the PLCVRP. As in Section 5, we apply Java and CPLEX 12.6.1 on a 2.4 GHz computer to implement our algorithm. We investigate the characteristics of the PLCVRP’s solutions with respect to four different criteria: (i) transport and occupational risk, (ii) transport risk, (iii) occupational risk, and (iv) transportation cost. Tables 7 to 10 summarize the implications of these four alternative objectives. In Table 7, the medical waste collection routes corresponding to vehicles in each day are represented. Each of the four objectives has a different routing schedule during the planning horizon. Our goal of considering the last objective, transportation cost, is to indicate how the route schedules would change in terms of defining different objective functions for PLCVRP. Transportation cost in this study is described as follows:

$$\sum_{i \in \mathcal{V}'} \sum_{j \in \mathcal{V}'} f w_{ij} l_{ij} \tag{17}$$

where  $f$  is the fuel cost per kilogram per mile and  $w_{ij}$  denotes the vehicle’s load in kg on link  $(i, j)$ . Similar to the model of Kara et al. (2008), the transportation cost is a function of vehicle’s load, which is assumed to be linear for simplicity. As we can see from Table 9, when the objective of PLCVRP is occupational risk, all the medical centers should be served in every single day of a week. This implies that medical centers have no tendency to store the medical wastes even for a single period. Another interesting observation is increase in the daily number of unvisited medical centers when the objective is the transport risk. Since our planning horizon is finite, we force the PLCVRP

Table 7: Implications of different routing schemes

Objective of PLCVRP	Vehicle No.	Monday	Tuesday	Wednesday	Thursday	Friday
Transport Risk & Occupational Risk	1	(0,10,7,2,1,0)	(0,9,4,1,0)	(0,9,4,1,0)	(0,10,7,4,2,1,0)	(0,10,6,9,4,1,0)
	2	(0,6,9,3,0)	(0,10,7,2,0)	(0,10,3,2,0)	(0,9,6,5,3,0)	(0,5,7,8,2,3,0)
Transport Risk	1	(0,7,9,2,1,0)	(0,4,1,0)	(0,10,2,3,0)	(0,6,9,7,4,2,3,0)	(0,9,5,6,4,2,3,0)
	2	(0,10,4,0)	(0,9,7,3,2,0)	(0,6,4,1,0)	(0,10,5,1,0)	(0,10,7,8,1,0)
Occupational Risk	1	(0,1,4,8,0)	(0,1,4,8,0)	(0,4,3,8,0)	(0,1,3,8,0)	(0,1,3,8,0)
	2	(0,2,3,7,6,5,9,10,0)	(0,2,3,7,5,6,9,10,0)	(0,2,1,7,6,5,9,10,0)	(0,2,4,7,5,6,9,10,0)	(0,2,4,7,5,6,9,10,0)
Transportation Cost	1	(0,9,7,1,0)	(0,9,4,0)	(0,8,4,0)	(0,7,10,1,0)	(0,8,10,7,4,1,3,0)
	2	(0,10,2,3,0)	(0,8,3,2,1,0)	(0,7,1,2,3,0)	(0,8,5,4,2,3,0)	(0,5,6,9,2,0)

Table 8: Vehicles' load (travel distance) for two routing schemes

Objective of PLCVRP	Vehicle No.	Monday	Tuesday	Wednesday	Thursday	Friday
Transport Risk & Occupational Risk	1	776 (178)	1744 (141)	1326 (141)	1470 (195)	1798 (287)
	2	616 (181)	882 (175)	1092 (164)	1334 (198)	904 (276)
Transportation Cost	1	652 (129)	692 (133)	610 (90)	1474 (173)	1732 (270)
	2	581 (164)	1652 (84)	1764 (65)	1323 (263)	1462 (199)

to find the route schedules such that all the medical centers in the last day of time horizon are visited. This assumption results in having more work load on the last day of the week. To address this issue we solve the PLCVRP for a longer planning horizon and extract our desired solution for a shorter time horizon. Since PLCVRP imposes a storage limit on the amount of accumulated medical waste at hospitals, the solution obtained for a longer time horizon necessitates serving all the customers after some time periods.

From Table 8, one can conclude that the solutions impose a good work balance on each vehicle, such that, any of two drivers who has to pick up more amount of medical waste from hospitals travels a shorter distance compared to the other vehicle. In this situation, the drivers of two vehicles might be able to finish their work in an equal time-interval during a time period. Figures 3, 6, and 7 in Appendix B show the weekly route schedules obtained by PLCVRP for Monday, Wednesday, and Friday.

A valuable observation regarding Table 10 is that by solving the version of PLCVRP that aims to minimize both transport and occupational risk on the network we can aid decision makers to develop a better routing schedule in terms of the imposed risk of hazardous medical waste. In order to obtain the risk values presented in Table 10, a summation of transport risk and occupational risk corresponding to each PLCVRP's solution is calculated. A comparison of the solution found by minimizing the transportation cost with the solution obtained by minimizing the total risk (transport risk and occupational risk) demonstrates a 26.25% reduction in risk value. Moreover, the run time in Table 10 shows that the PLCVRP is computationally more difficult to solve compared with the single-objective PLCVRP. Although the routing schedule with the minimum total risk is not necessarily a schedule with minimum transportation cost, the remarkable reduction in total risk

Table 9: Unvisited medical centers for three routing schemes

Objective of PLCVRP	Monday	Tuesday	Wednesday	Thursday	Friday
Transport Risk & Occupational Risk	4,5,8	3,5,6,8	5,6,7,8	8	-
Transport Risk	3,5,6,8	5,6,7,8	5,6,7,8,9	8	-
Occupational Risk	-	-	-	-	-
Transportation Cost	4,5,6,8	5,6,7,10	5,6,9,10	6,9	-

Table 10: Risk values for all routing schemes

Objective of PLCVRP	Risk Value	Run Time
Transport Risk	101.71	2 min 20 sec
Occupational Risk	4.42	2 min 25 sec
Transport Risk & Occupational Risk	110.92	5 min 12 sec
Transportation Cost	150.12	5 min 17 sec

can convince decision makers to implement the obtained schedule. Although, solving the PLCVRP expends more efforts and entails complicated analysis, the route schedule and the risk value look to be plausible according to our observations.

## 7 Conclusion and Future Research

We introduced a periodic load-dependent capacitated vehicle routing problem (PLCVRP) to find the least risk routing schedule for medical waste collection. We proposed a decomposition based heuristic approach to solve the PLCVRP, where each decomposed sub-problem itself is solved by a column generation approach. Computational results using the decomposition based heuristic confirmed its efficiency and tractability. We applied our PLCVRP and the heuristic approach to a case study to verify the importance of this study in real applications. We consider our proposed model and algorithm a step towards solving other types of multi-period inventory routing problems.

To improve the solution quality and computation time, we can consider an exact algorithm for solving each decomposed sub-problem. Each sub-problem is a load-dependent capacitated vehicle routing problem (LCVRP), for which (Fukasawa et al., 2015) developed a branch-cut-and-price algorithm. Considering such an exact solution approach within the proposed time decomposition framework with penalty functions will be a valuable future research direction.

A suggested refinement of our model is to consider stochasticity in medical waste generation, because changes in demand may result in a different optimal solution. The load dependency assumption in PLCVRP can probably spread out the application of our model in other areas of transportation such as, hazardous materials transportation and green transportation. Future

research suggestions include collection of real data from healthcare centers and medical waste shipping companies. Future work can also consider multi-criteria decision making techniques and Pareto optimal solutions to find ideal routing schedules with respect to minimization of both risk and cost.

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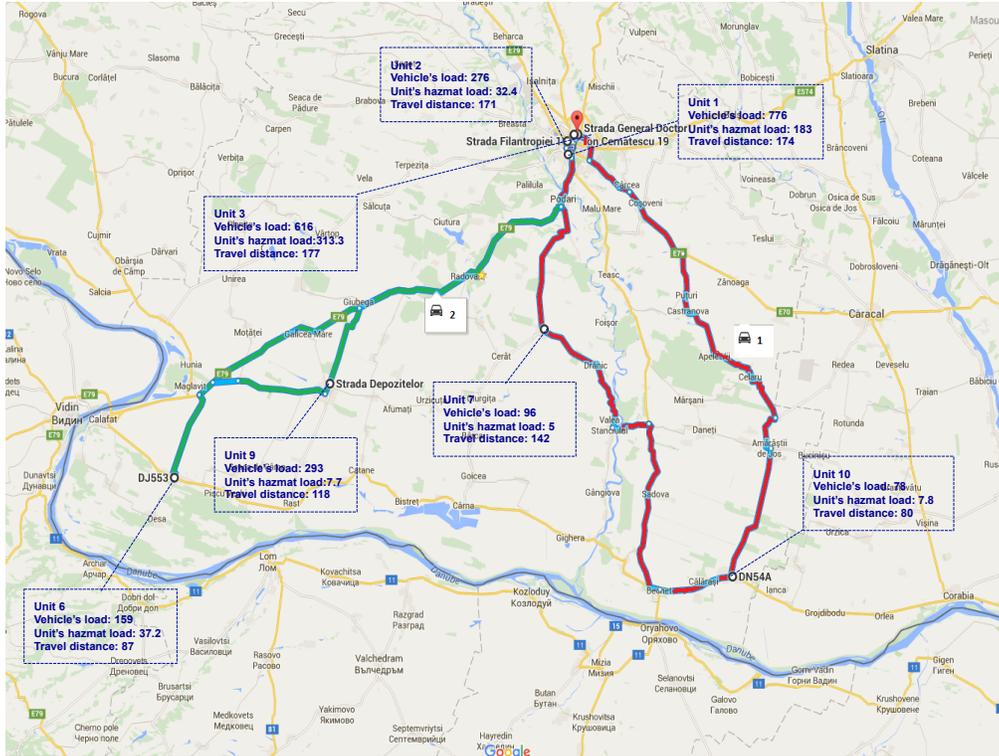
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# Appendices

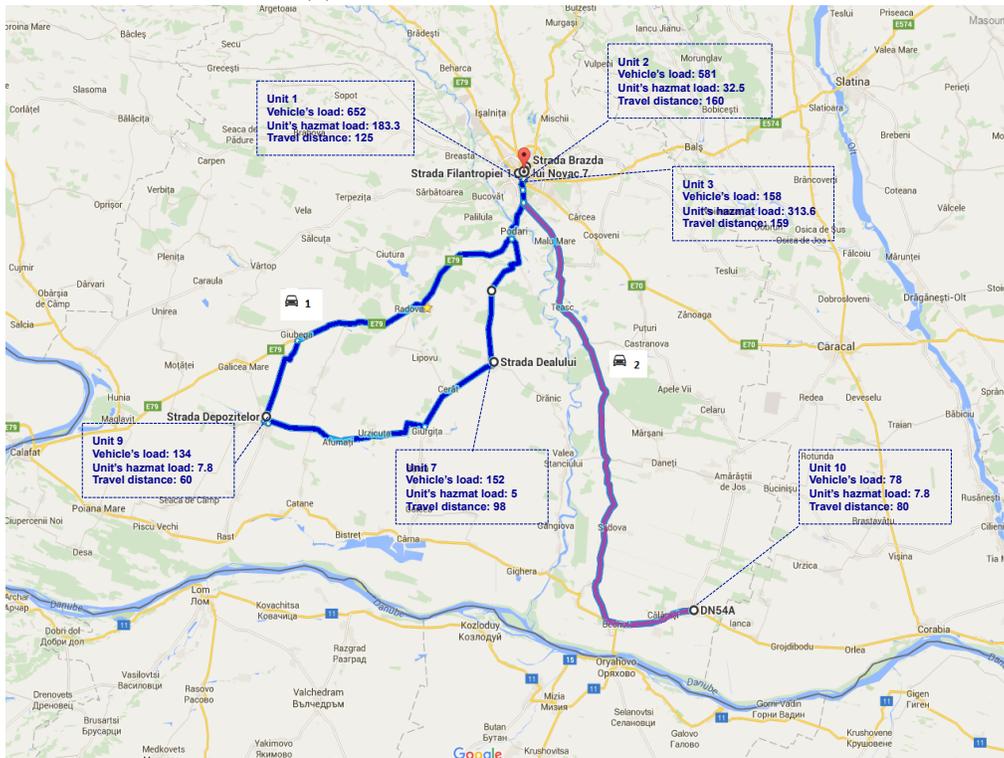
## A Locations of 10 hospitals and a treatment center in the city of Dolj, Romania

Unit No.	Unit Name	Location
0	Treatment Center	S.C. SIGMAFLEX S.R.L. DJ, Craiova, str.Brazda Novac, BL. 7
1	Emergency Clinical Hospital of Craiova	Spitalul Clinic Judetean de Urgenta Strada Tabaci 1 Craiova 200642 Romania
2	Municipal Clinical Hospital of Craiova	Spitalul Clinic Municipal Filantropia Strada Filantropiei 1 Craiova 200143 Romania
3	Infectious Diseases Clinical Hospital of Craiova	Hitmed Strada Stefan cel Mare 23 Craiova 200129 Romania
4	Lungphysiology Hospital of Leamna	Spitalul de Pneumofiziologie Leamna de Sus 207129 Romania
5	Municipal Hospital of Calafat	Spitalul Municipal Calafat Strada Traian 5 Calafat 205200 Romania
6	Psychiatry Hospital of Poiana Mare	Spitalul de psihiatrie DJ553 Poiana Mare 207470 Romania
7	Urban Hospital of Segarcea	Spitalul orasenesc Strada Dealului Segarcea Romania
8	Urban Hospital of Filiasi	Filiasi City Hospital Bulevardul Racoteanu 216 Filiasi 205300 Romania
9	Urban Hospital of Bailesti	Spital Strada Depozitelor Bailesti Romania
10	Hospital of Dabuleni	Spitalul Orasenesc Asezamintele Brancovenesti Dabuleni DN54A Dabuleni Romania

## B Detailed route-schedules in the map in Figures 3–7

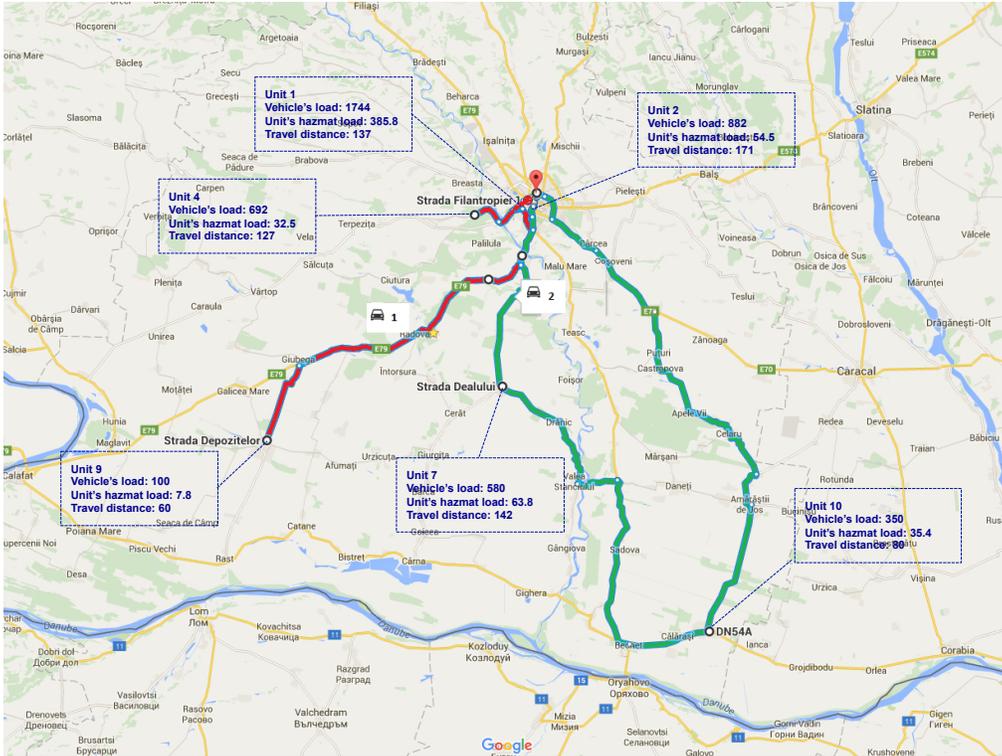


(a) Routes with minimum total risk

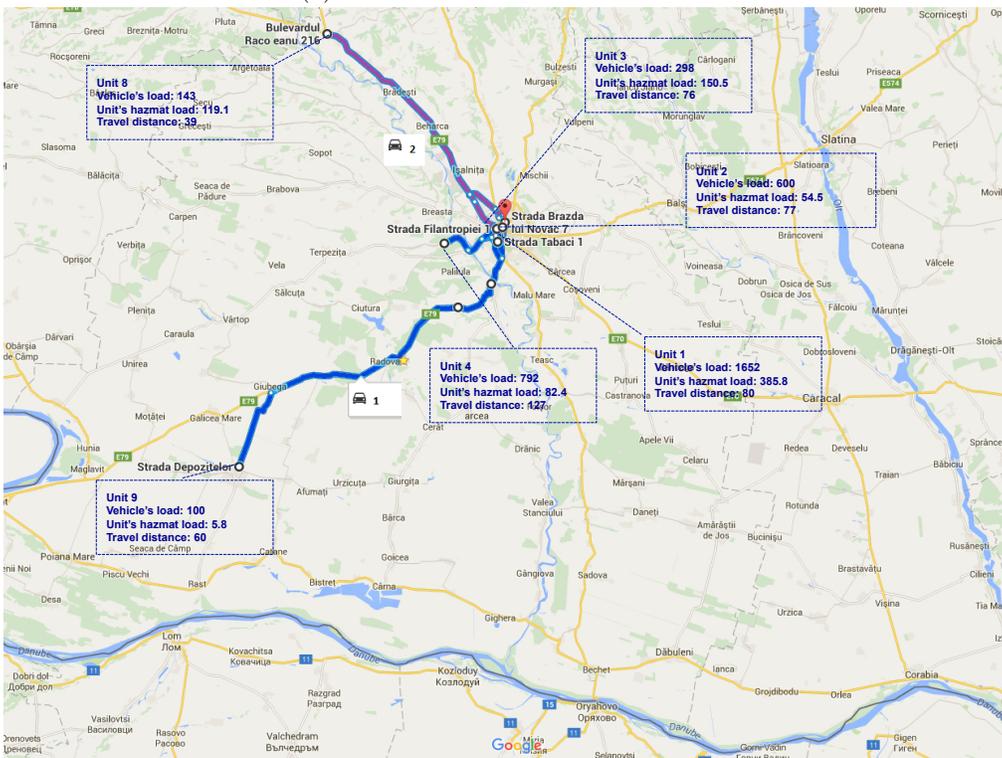


(b) Routes with minimum cost

Figure 3: Route schedule for medical waste collection on Monday in Dolj, Romania.

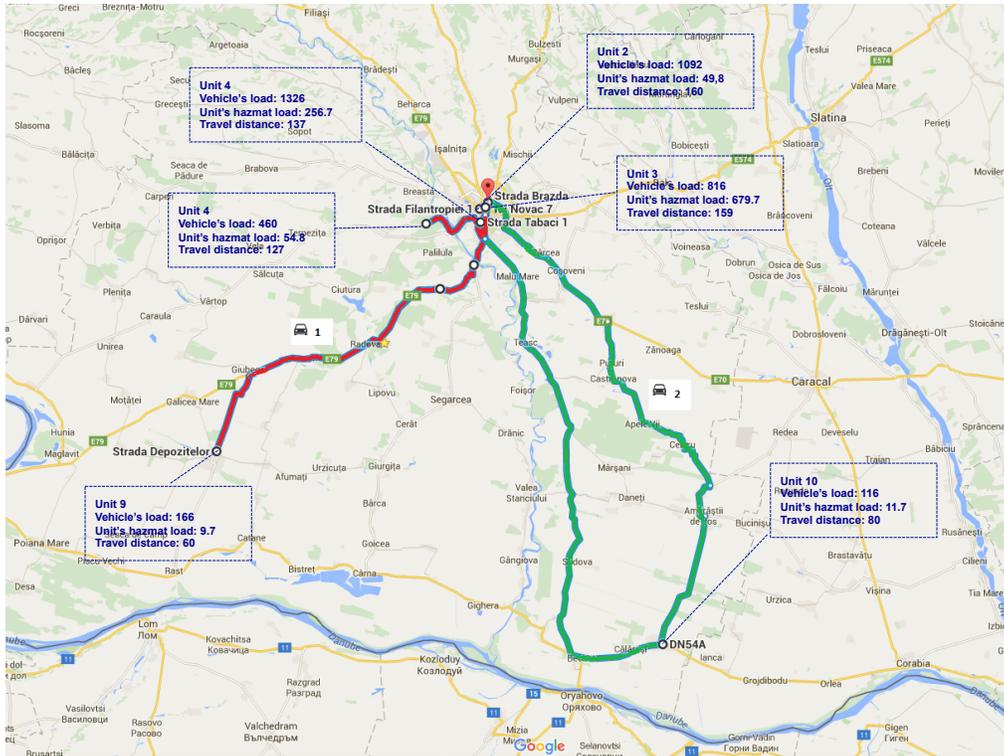


(a) Routes with minimum total risk

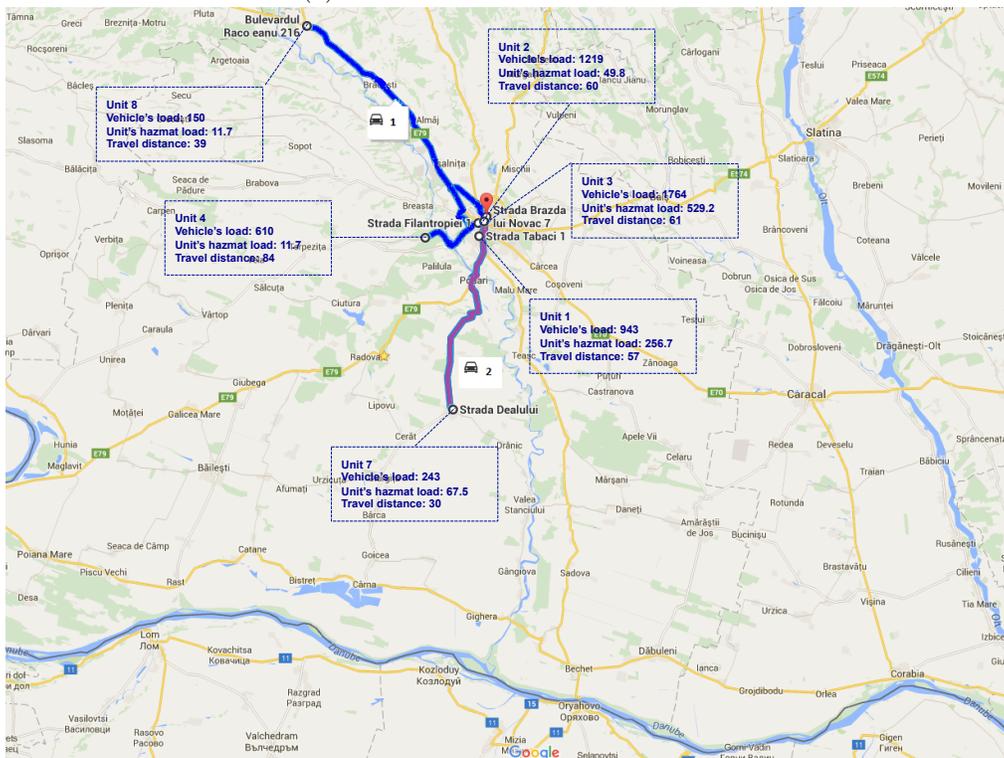


(b) Routes with minimum cost

Figure 4: Route schedule for medical waste collection on Tuesday in Dolj, Romania.

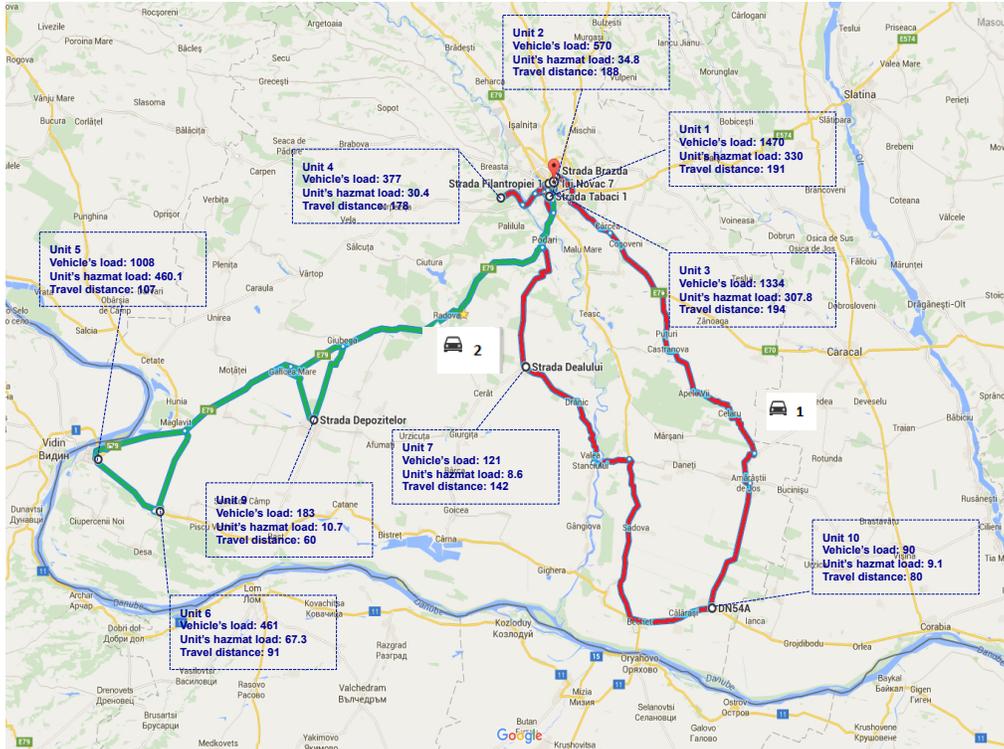


(a) Routes with minimum total risk

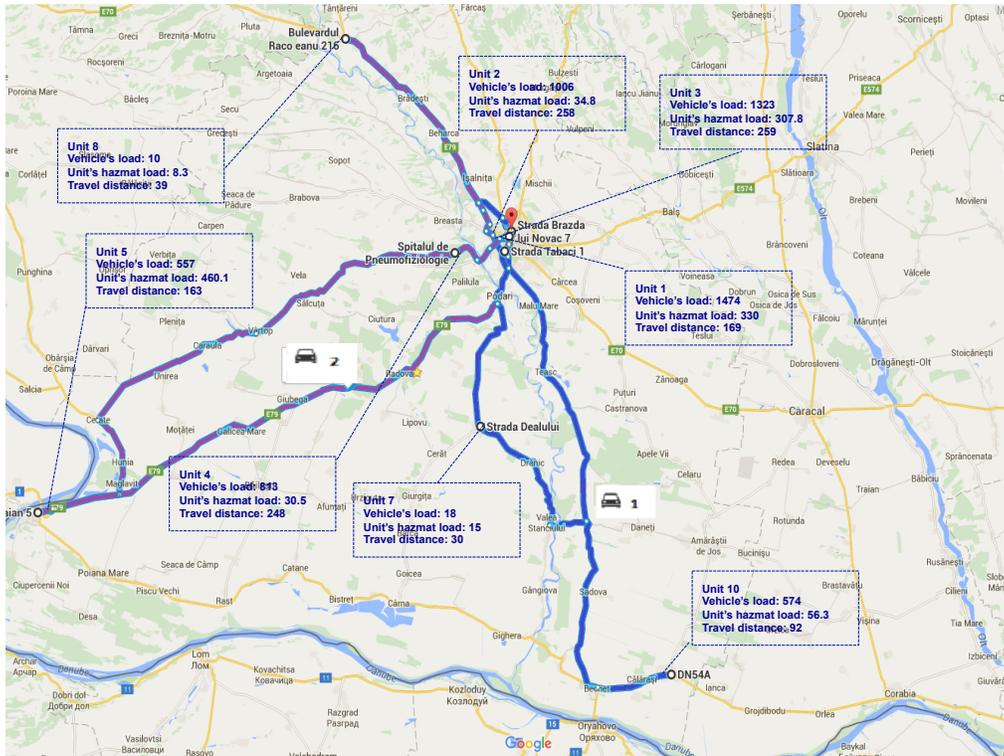


(b) Routes with minimum cost

Figure 5: Route schedule for medical waste collection on Wednesday in Dolj, Romania.

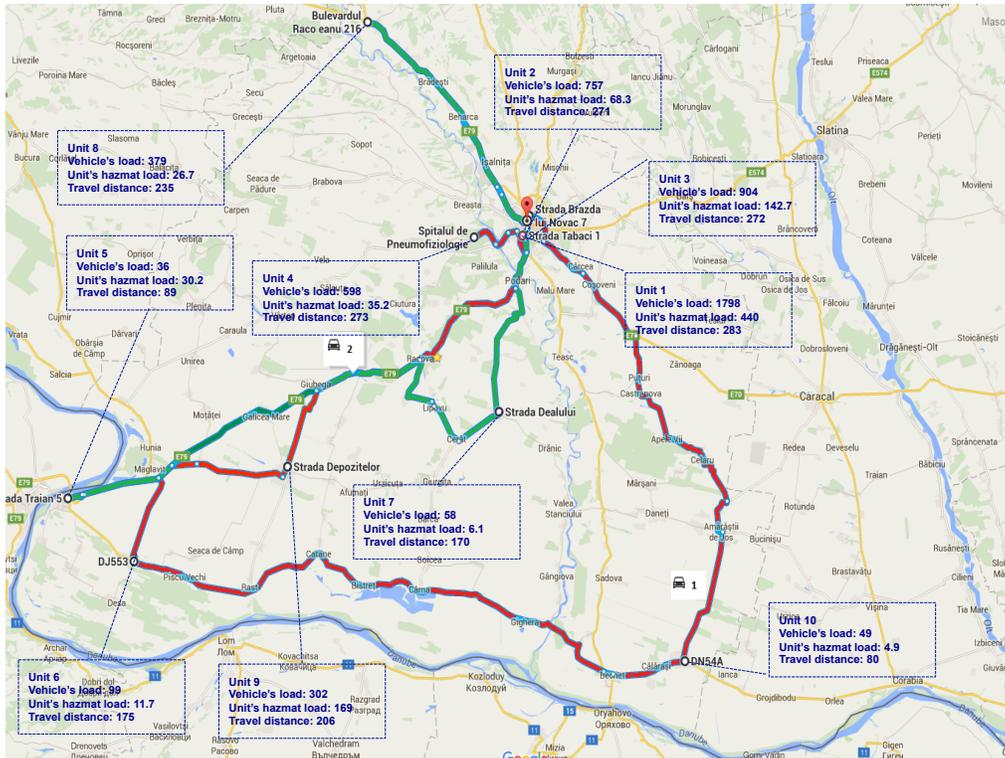


(a) Routes with minimum total risk

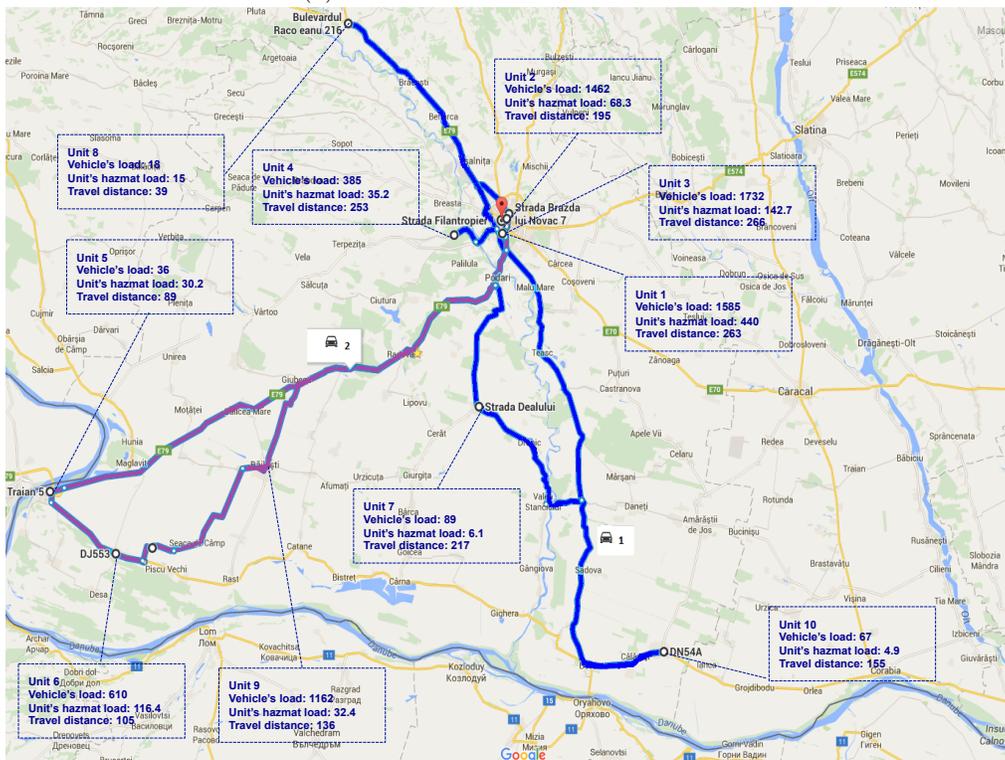


(b) Routes with minimum cost

Figure 6: Route schedule for medical waste collection on Thursday in Dolj, Romania.



(a) Routes with minimum total risk



(b) Routes with minimum cost

Figure 7: Route schedule for medical waste collection on Friday in Dolj, Romania.