An Iterative Combinatorial Auction Design for Fractional Ownership of Autonomous Vehicles

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Abstract

This study designs a new market for fractional ownership of autonomous vehicles (AVs), in which an AV is co-leased by a group of individuals. We present a practical iterative auction based on the *combinatorial clock auction* to match the interested customers together and determine their payments. In designing such an auction, we consider continuous-time items (time slots) which are defined by bidders and naturally exploit driverless mobility of AVs to form co-leasing groups. To relieve the computational burdens of both bidders and the auctioneer, we devise user agents who generate packages and bid on behalf of bidders. Through numerical experiments using the California 2010–2012 travel survey, we test the performance of the auction design. We also compare various bidding strategies and study the effect of *activity rules* on the bidders' payoffs. We find that the designed activity rules successfully remove the strategic behavior of bidders. We also find that *core-selecting payment rule* brings the largest revenue to the auctioneer in most cases.

Keywords: Auction Design; Autonomous Vehicles; Combinatorial Clock Auction

1 Introduction

Since the invention of the assembly line, vehicle ownership has been a distinctive part of the American culture, placing the country on the leading positions by vehicles per capita. The current model of vehicle ownership is, however, neither cheap nor efficient. In fact, the total auto-loan amount in the country exceeded \$1.24 trillion in 2018, while in 95% of the time cars are parked (Center Microeconomic Data , 2018; Shoup, 2005). Nevertheless, in recent years collaborative consumption has been accounted for dramatic changes in transportation. The wide spectrum of new services ranging from ride-sharing (Uber, Lyft and etc.) to peer to peer car renting (Buzzcar,

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Drivy and etc.) has influenced the traditional view of vehicle ownership. This can be observed in the increase of 7 years on the average age of new vehicle buyers (Kurz et al., 2016). Consequently, to satisfy the potential shift in consumer demand, car-manufacturers have launched various car-sharing services (Maven by GM, Audi on Demand, etc.), thus seeking new forms in vehicle ownership.

Motivated by the recent advancements in autonomous transportation technologies (Akbar et al.; Gu et al.), this study investigates a novel form of vehicle ownership called *fractional ownership* of autonomous vehicles (AVs), where an AV is co-leased by a group of individuals. Thus, 'fractional ownership' means that customers co-lease a vehicle and 'co-owners' mean co-lessees. The benefits of such ownership include reduced costs and increased utilization of vehicles coming from collaborative consumption. A fractional ownership model had been tested with regular vehicles (Ford Credit Link, Nissan Micra Go & Get) and it had encountered significant challenges. The most critical challenge came from the need to relocate a vehicle from the location of one customer to the location of another customer. Therefore, only customers living in the closest neighborhoods were allowed to group together, thus offering limited options. In addition, in the fractional ownership of the regular vehicles market, customers were asked to find co-leasing groups themselves, highlighting the absence of market design mechanisms. Because of such limitations, the above-mentioned programs have been discontinued.

In this study, we propose using AVs for the fractional ownership market. AVs naturally solve the above-mentioned relocation issue since they have driverless mobility. Therefore, using AVs in fractional ownership model allows to attract a large pool of customers from various neighborhoods with diverse travel needs. Also, when using AVs, *co-leasing* may be a more viable service due to the maintenance and parking costs. As compared to ride-sharing (Smet, 2021) or car rental services, the fractional ownership of AVs do not require customers to look for rides or rental cars every time, but instead provides a long-term predictable service on fixed rates. The premise of the fractional ownership model is in its convenience for customers to use the same AV for each ride while co-owning it with the fixed set of customers. For instance, a customer may store a child safety seat in the AV while taking low health risks associated with a shared vehicle. Lastly, the long term commitment nature of the fractional ownership allows both for customers and service companies to have a long planning horizon.

For a practical fractional AV ownership model to be successful, there must be no time conflicts among co-owners. In fact, the origins and destinations of trips also need to be considered to incorporate traveling times of empty AVs. Therefore, a suitable mechanism is needed to match customers with non-overlapping time-schedules together and avoid conflicts. There are two popular mechanisms to match customers. The first is matching-theoretic approaches, such as stable or maximal matching, commonly used by ride-sharing services like Uber and Lyft; see Wang et al. (2018) and Zhang et al. (2020), for example. In this case, a leasing company needs to solve two problems: to determine matching groups of customer and to set prices for each customer. Since fractional ownership of autonomous vehicles (AV) is a novel service, there is no benchmark for pricing, which poses substantial challenges for leasing companies. The second mechanism is based on auction theory. The outcomes of auctions not only determine matching groups of customers, but also determine the prices. Indeed, auction mechanism has a potential to generate more revenue compared to matching mechanism and it allocates time slots efficiently offering them to customers who value them the most. Also, we need to note that customers are most likely interested in using AVs for a combination of time slots (e.g. from home to work and from work to home). With these in mind, in this paper, we design a *combinatorial* auction market for fractional ownership of AVs as an alternative to the traditional full ownership model. In particular, the proposed auction exploits the unique feature of AVs, thus their driverless mobility in forming co-leasing groups.

Combinatorial auctions are suitable mechanisms to sell items or allocate resources in packages, instead of single items separately. They have been used widely across various industry sectors (De Vries and Vohra, 2003; Pekeč and Rothkopf, 2003; Milgrom, 2019) including allocation of the spectrum right licenses to telecommunication companies and Internet pricing (Hershberger and Suri, 2001). In transportation and logistics, combinatorial auctions have gained attention for selling airport departure and arrival slots (Rassenti et al., 1982), truckload transportation (Zhang et al., 2015), city bus route market (Cantillon and Pesendorfer, 2006), and railway industry (Kuo and Miller-Hooks, 2015). Recently, researchers suggest combinatorial auctions in the ride-sharing market for designing a more efficient shared mobility system (Hara and Hato, 2018), collaborative vehicle routing (Gansterer and Hartl, 2017) and public transportation systems (James et al., 2018).

In the proposed auction market, the auctioneer is a car manufacturer or leasing company that sells AVs, and the bidders are customers who are interested in co-leasing a car. The auctioneer sells time-slot packages to bidders through an auction that gives the winners the right to use the same vehicle in these time-slots within a week for a certain period. Each time-slot package includes several time-slots covering the travel needs of customers.

Combinatorial auctions involve complex package valuation problems for bidders and allocation problems for the auctioneer. Customers have to value time slot packages. The value of these packages may be different from the summation of the values of the individual time slots. Iterative combinatorial auctions have been introduced to address this *preference elicitation problem*. In particular, in iterative auctions bidders can bid iteratively, receive feedback based on their rivals' valuation and adjust their valuations (Pekeč and Rothkopf, 2003). This feedback information is valuable for the new products, where there is no benchmark for the pricing. Indeed, fractional ownership of AVs can be considered as a new product with limited valuation insight for a customer. Furthermore, the dynamic nature of iterative combinatorial auctions, where valuation information of time slots is exchanged between customers, may potentially lead to higher revenue compared to a single round combinatorial auction (Parkes, 2006). Because of the above-mentioned reasons, the majority of combinatorial auctions with applications in various industries (spectrum auction, real estate and etc.) are iterative in nature.

The existing combinatorial auction designs, however, do not fully capture the nature of the bids in the market for fractional ownership of AVs. For instance, in determining winning bids, the problem under study involves additional constraints to avoid time-conflicts between bidders. To address such issues, Takalloo et al. (2020) proposed a single-round, combinatorial auction market with user defined continuous-time items for the fractional ownership of AVs. The auction design of Takalloo et al. (2020) is based on the well-known Vickrey-Clarke-Groves (VCG) mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973), which possesses many desirable properties, but can suffer from low revenue for the auctioneer. In addition, due to the complex nature of combinatorial auctions, a single-round auction may limit opportunities for bidders to fully learn about the market and express their preferences. In this paper, we extend the settings of Takalloo et al. (2020) and design an *iterative* auction, as opposed to a single-round auction, to overcome these limitations of the VCG mechanism.

The proposed Combinatorial Clock Auction (CCA) includes two stages: the clock stage and the supplementary stage, which are designed to help customers to bid efficiently. The *clock stage* consists of multiple rounds and gives insight to the bidders about the market value of items. In each round, bidders submit bids for a package of time slots and observe the ask prices from an auctioneer. Even though it is possible to design a single round auction and avoid iterative bidding, in that case, customers have only a single shot to bid. Thus, in a single round auction customers bid without any insights about the competitiveness of their bids. Instead, in the proposed auction the clock stage serves as the *price discovery* for bidders (Ausubel et al., 2006). In the *supplementary stage* consisting of a single round, bidders submit their final bids considering the ask prices from the clock stage. Consequently, the auctioneer solves the *winner determination problem* (WDP) considering the bids from both clock and supplementary stages and calculates the payments. To suppress the strategic behavior of bidders such as when bidders avoid bidding until the last round or bid intentionally on undesired time slots, we use *activity rules* in both the clock stage and the supplementary stage. In the auction, to elevate the burden of iterative bidding, customers are given a choice to use user agents, a software tool designed to bid on behalf of bidders. The proposed user agents offer several bidding strategies to customers and propose bidding packages based on customers' travel needs. We also consider different payment rules and compare them together to study the revenue of auctioneer. To the best of our knowledge, this is the first study investigating the CCA as a market design for applications in transportation.

1.1 Unique Challenges and Contributions

The setting of the problem under study is unique in several aspects. In most existing combinatorial auctions, products are pre-defined discrete items. In the proposed auction in this study, items are neither pre-defined nor discrete; instead, we consider *bidder-defined continuous-time items*. Every possible time interval becomes an item when a bidder finds it valuable. While bidder-defined continuous-time slots serve the interests of customers best and result in greater social welfare compared to discrete-time slot (Takalloo et al., 2020), the infinite number of possible items cause new challenges as well. In particular, customers face large scale valuation problems for all possible items (time slots). Selecting a suitable set of time-slots requires substantial computational resources. In addition, due to its iterative nature the proposed auction, compared to a single round auction by Takalloo et al. (2020), results in the substantial increase in the number of bids, thus requiring new solution techniques both for customers and the auctioneer.

The contributions of this study can be summarized as follows. First, we design a unique iterative combinatorial auction for the market design of fractional ownership of AVs, namely *combinatorial clock auction with bidder-defined items*. Second, we devise a fast algorithm for determining the ask prices in the proposed auction setting. Third, we develop user agents with different bidding strategies for generating packages as a support tool for customers. Fourth, we test the performance of the proposed auction design under different *payment rules* with the simulation studies.

Our numerical experiments show that core-selecting payment rule results in a relatively high revenue compare to the other payment rules. We also find that activity rules, which are based on the eligibility points, are effective under the *bidder-defined continuous-time items* setting as they support consistent bidding while suppressing the strategic behavior of customers.

1.2 Outline

The remainder of the paper will proceed as follows: In Section 2, we describe the proposed auction design in the clock and the supplementary stages. In Section 3, we investigate several payment rules and their effect on the auction outcome. In Section 4, we introduce user agents who assist bidders in bidding in the clock stage and generating packages in the supplementary stage. In Section 5, we present numerical experiments based on the California 2010–2012 travel survey dataset and derive insights into the auction market for fractional AV ownership. Lastly, in Section 6, we conclude the paper.

2 The Auction Design for Fractional Ownership of AVs

In this section, we design CCA for the fractional ownership of autonomous vehicles. First, we define the continuous-time items and packages in the fractional ownership CCA in Section 2.1. Next, we describe the first stage in the CCA which is the clock stage in Section 2.2. The clock stage can be viewed as a multi-round auction. In each round, the auctioneer announces the ask prices, and the bidders bid on the desired items. The auctioneer calculates the ask price based on the supply-demand balance. We present Algorithm 1 for calculating the ask prices in Section 2.2.1. To ensure that bidders bid actively throughout the clock stage and to remove the strategic behavior of bidders, we design some activity rules for the auction which are based on the eligibility points of the bidders. We describe the activity rules in the clock stage in Section 2.2.2. Next, we describe the supplementary stage which is the final round of the auction in Section 2.3. In the supplementary stage, bidders submit new bids based on the information they gain through the clock stage. Then, the auctioneer considers all the bids submitted in the clock stage and the supplementary stage to solve the WDP considering the spatial information of the bidders and the continuous-time items.

2.1 Auction Setting

To better understand the fundamental elements of the proposed auction design, consider a customer who is interested in using an AV in the following time slots: 7:30–8:00 AM, 4:40–5:10 PM. In this case, the items are two mentioned time slots. The customer may submit the following set of packages from the items: {7:30–8:00 AM}, {4:40–5:10 PM} and {7:30–8:00 AM, 4:40–5:10 PM}. However, each bidder can receive at most one package. More formally, we introduce the definitions of an *item*, a *package* and a *bid* in the combinatorial auction as follows:

Definition 1. **An item** in the CCA for fractional ownership of vehicles is a continuous time-slot uniquely defined by its start and end time that a customer selects based on her schedule. Unlike the traditional CCA, there are no predefined items in the proposed auction and a customer is given a choice to define an item.

Definition 2. **A package** is a combination of continuous time slots defined by a customer.

Definition 3. **A bid** is a package submitted by a customer indicating start and end time of her desired time slots along with spatial information of her trips.

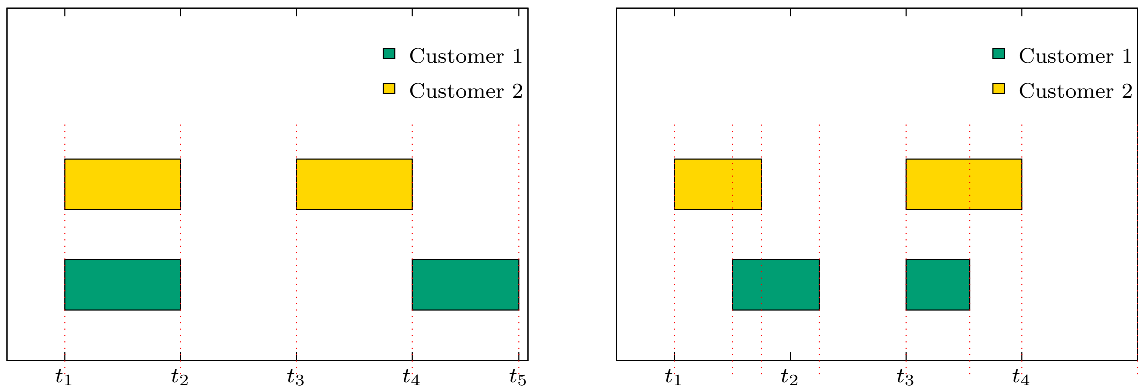
In the proposed combinatorial auction, the auctioneer is a car leasing company who wants to sell AVs, and the bidders are customers who are interested in co-leasing a car. The auctioneer offers a homogeneous fleet of vehicles for co-leasing, while interested customers enter into the iterative bidding. Customers submit their bids as combinations of time slots and provide spatial information for their trips. The *bidding language* in the auction is XOR; that is, the auctioneer will accept at most one bid from each bidder. The auction itself consists of two stages: *clock* and *supplementary* stages with a set of rules specified below.

2.2 The Clock Stage

The clock stage in the proposed auction serves as *price discovery* for each possible item and subsequently for each package, where package's ask price equals to the summation of ask prices of its items. In particular, at the beginning of each round of the clock stage, the auctioneer announces ask prices for items and bidders bid on the desired items. Note that the collection of items submitted by a bidder in round r form a package, which we call a clock stage package. The bid price for a clock stage package is computed as the summation of ask prices for items in the package. The customers can bid on the items that they bid on previously or on the new items. Ask prices increase for the items with excess demand. The clock stage continues until we have demand-supply balance or bids do not change in two consecutive rounds. At the end of the clock stage, bidders learn the minimum bid price required to win a particular item.

2.2.1 Ask Price Calculation

In the traditional CCA setting, finding supply-demand balance and determining the ask prices is relatively easy. Since items are predefined, the auctioneer can easily count the total demand for



(a) Discrete auctioneer-defined items

(b) Continuous bidder-defined items

Figure 1: Comparing the items in the conventional CCA and the proposed auction

each item. Figure 1a shows the conventional CCA, in which items are defined as hourly time slots. The auctioneer counts demand for each hour and indicates demand-supply balance. Figure 1b represents the proposed CCA with continuous bidder-defined items (time slots). Each time slot can be identified uniquely by a start time and an end time. Placing the start time and the end time of all submitted time slots in the time horizon results in a set of *time slices*. Each item (time slot) includes a set of consecutive time slices. To determine the ask prices for each item, the auctioneer specifies its corresponding set of time slices. For a particular time slice, if the demand (the number of items which include that time slice) is greater than the supply (the fleet size), then there will be a price increment ϵ_p for that particular time slice. The ask prices for an item in each round will be updated by adding the summation of price increments for the corresponding time slices. Note that we need to calculate the ask prices in each round of the auction. Algorithm 1 shows the pseudo-code of the algorithm for determining the ask price efficiently, which is explained below.

An intuitive explanation of the algorithm is illustrated in Figure 2a which represents a CCA with three customers and a single vehicle. To find the ask price increments, we first collect all the bids and place all the time slots into the time horizon and construct the time slices (from t_1 to t_2, from t_2 to t_3, and so on). Next, we set a price increment ϵ_p, where p stands for price, for each time slice with excess demand. Note that the value of ϵ_p is the same in all rounds of clock stage. Then, the ask price for each item is updated by adding the summation of price increments for its corresponding time slices. For example, as Figure 2a represents, the second customer's bid consists of a single item which overlaps in three time-slices with other items. In this case, the ask price for the second customer will be increased by $3\epsilon_p$.

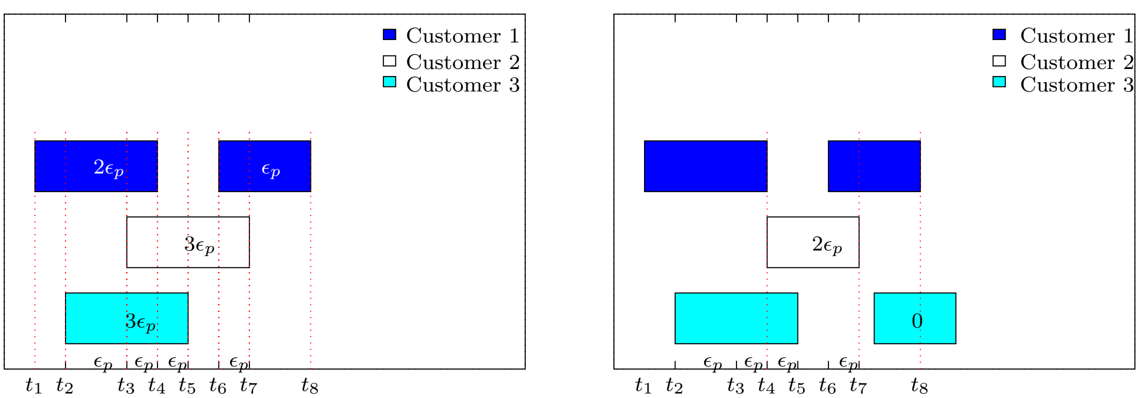
While determining the ask prices for the items that have been present in the previous rounds is a relatively easy task, it is not the case for new items. For example, as shown in Figure 2b, the second customer submits a single item bid again in the next round, but with a new item. Since this item has not been present in the previous rounds, the auctioneer does not know the initial

Algorithm 1: Ask price calculation in round r

Input: All submitted items in round r denoted by $\mathcal{I}$, fleet size $|\mathcal{H}|$, and price increment ϵ_p

Output: Vector of ask price increment in r-th round $\mathbf{p}$

```
Initialization: times ← ∅, soe ← ∅ ; // soe for 'start or end'
for j = 1 to length(I) do
    push(times, s_j); push(soe, 1); // 1 for start time
    push(times, e_j); push(soe, −1); // -1 for end time
sorted_times, sorted_idx = sort(times);
sorted_soe = soe[sorted_idx]; // soe is sorted in the same order as sorted_times
j ← 1; count ← 0; conflicts ← ∅; p ← 0;
while j ≤ length(sorted_times) do
    same_time_idx ← findall(sorted_times, sorted_times[j]) // Finding a vector of indices
        with the same time value as sorted_times[j]
    foreach m ∈ same_time_idx do
        if sorted_soe[m] = 1 then
            count ← count + 1; // for start time
            push(conflicts, same_time_idx[m]);
        else
            count ← count − 1; // for end time
            remove(conflicts, same_time_idx[m]);
    if count > |H| then
        foreach n ∈ conflicts do
            p_n ← p_n + ε_p;
    j ← j + length(same_time_idx);
```



(a) Regular bids (b) New bids

Figure 2: Price determination through conflict detection in an auction with a single vehicle ($|\mathcal{H}| = 1$)

ask price. The auctioneer may set the initial ask price to 0 for all new items, but it may lead the customers to submit new items all the time by slightly changing their previous bids. Therefore, the auctioneer requires a more effective way to determine the initial ask prices for the new items. We propose an alternative approach for setting the ask prices for new items. As can be seen in Figure 2b, for the new items submitted by the second customer, we consider the corresponding time slices in the previous rounds. The ask price for the new item will be set to the sum of price increments for the corresponding time slices in the previous rounds. For example, if we consider Figure 2a as the first round, and Figure 2b as the second round, the initial ask price for Customer's 2 new items in the second round will set to $2\epsilon_p$. Similarly, the initial ask price for Customer 3's new item will be 0, since there is no conflict in any corresponding time slice in the first round. In the clock stage, as the auction proceeds, the ask prices for the items increase and as a result, the demand decreases until we achieve the supply-demand balance.

2.2.2 Activity Rules for the Clock Stage

Iterative combinatorial auctions, in general, are vulnerable to strategic behaviors of bidders. For instance, a bidder may not bid on her preferred items until the last round to keep their prices from rising. To suppress such behavior, the CCA has another design element called activity rules which are present in both the clock stage and the supplementary stage. These rules restrict bidders to enforce them to bid actively and constantly through the clock stage in order to be able to submit competitive bids in the supplementary stage.

Ofcom (2011) proposes some activity rules that have gained popularity among practitioners because of their simplicity. These activity rules are based on *eligibility points* that bidders purchase with their initial deposit before entering an auction. The auctioneer assigns eligibility points to each item based on historical demand for that item. High historical demand for a time-slot results in a high value of the eligibility point for that time slot. In particular, the auctioneer determines the eligibility point for each hour (12:00 AM–1:00 AM, 1:00 AM–2:00 AM, ...) based on the historical data for the demand. Then the eligibility point for each item i (time slot) will be determined as:

$$e_i = \sum_{t \in \mathcal{T}} \delta_i^t e_t$$

where e_t is the eligibility for each hour time-slot, and e_i is the eligibility for item i, and δ_i^t is the proportion of item i that falls in an hour time slot t. Formally, we can define the eligibility point of items and packages as follows:

Definition 4 (Eligibility Points of Items and Packages). Let $\mathcal{I}$ denote the set of items in an auction and let e_i denote a preassigned eligibility point of item i. Then the eligibility point of package k denoted by $E(k)$ is defined as the sum of Eligibility Points of items in the package, i.e. $E(k) = \sum_{i \in k} e_i$.

Definition 5 (Eligibility Points of Bidders). If bidder j has submitted a bid on package k in round $r-1$, we say that bidder j possesses eligibility point $E_r^j = E(k)$ at the beginning of round r. This

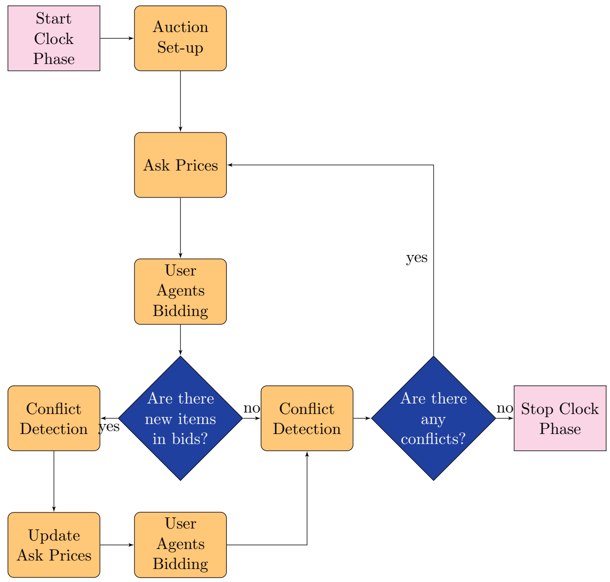


Figure 3: Clock stage flow chart

quantity is known as *eligibility* of bidder j in the literature (Ausubel et al., 2011).

The bidders start with an initial eligibility point based on the bidders' initial deposit. We can state the following activity rule for the clock stage: for each bidder, the eligibility points of her package at any round should not exceed her eligibility points at the beginning of the round. Thus, bidders are required to bid large quantities to maintain their eligibility in future rounds.

According to the activity rule mentioned above, the bidders need to bid constantly throughout the clock stage. If they do not bid actively in each round of the clock stage, their eligibility level decreases and they will lose the chance to submit packages in the supplementary stage. Figure 3 summarizes the clock stage of the proposed auction.

2.3 The Supplementary Stage

The supplementary stage is the final round for the submission of bids. Unlike the clock stage packages, whose bid prices are calculated as the summation of ask prices for the corresponding items, the bid prices of supplementary packages are directly determined by the bidders. In fact, bidders submit new packages based on the price discovery in the clock stage (where they learn the ask price for each item) and their budget.

Next, considering the clock stage packages and supplementary stage packages, the auctioneer solves the winner determination problem (WDP) to determine the set of winners in the auction. With the aim to relate the clock stage bids with the supplementary stage bids, the following activity rules for the supplementary stage were introduced in Ofcom (2011):

- If a bidder submits a package exactly the same as in the final clock round (FCR), the bidding price for that package should be greater or equal to the FCR package bidding price

- If the FCR package is a subset of the submitted package in the supplementary stage, in other words, new bids are introduced in the supplementary stage, then the following condition should hold:

$$c_j(b) \leq C_j(b^r) + p^r(b) - p^r(b^r) \tag{1}$$

where $c_j(b)$ is the bidding price of bidder j for package b in the supplementary stage; r is the last round when bidder j was eligible for package b; b^r is the package submitted by bidder j in the r-th round of clock stage; $C_j(b^r)$ is the maximum bid price by bidder j for package b^r in any round of the clock stage or in the supplementary stage; $p^r(b^r)$ is the ask price in round r for the clock stage package b^r; and $p^r(b)$ is the ask price for package b in round r (note that if this package has not been bid in round r, we determine the bidding price for that package in the same way as we determine the ask price for newly introduced items in round r).

As mentioned above, the activity rules in the supplementary stage, put some restriction on bidders' bidding based on their bidding history in the clock stage. Thus, from Equation (1) we can see that the difference between the bids prices for the newly introduced bid b and a bid b^r cannot exceed their respective ask prices' difference in round r. As for the posted bids, their bid price in the supplementary stage should be no less than their maximum bid price in the clock stage. This way the supplementary stage activity rules impose a relative cap on a bid amount, which can strongly limit the competitiveness of packages not submitted in the clock stage.

We also note that the proposed auction design may include reservation prices for an auctioneer. Thus, if the trips' costs for the winning bids exceed the bids' prices, an auctioneer may not accept the auction outcome. In order to avoid such situation, all bidders before entering the auction will be provided with the cost calculation formula. Then in the supplementary stage of the auction, the bidders submit bid prices no less than the summation of the costs of trips in their packages. Next, we formulate the WDP for the fractional ownership CCA.

2.3.1 Winner Determination Problem for the Fractional Ownership CCA

Note that in the supplementary stage, customers submit start and end times of their trips along with their bidding prices and spatial information. This information can be used to obtain the approximated commuting time between the bidder's locations. Also, only a single bid of each bidder can be determined as a winner as mentioned before. Then each winning bidder is assigned to a single AV, which will serve all her winning trips. Next, a simple algorithm can be used to determine the conflicting bids, which are defined as bids with overlapping trips. In particular, each pair of bids are examined to check if they overlap by considering the start location, start time and end location, end time of each trip in the bids. Based on the locations of the bidders, we estimate the travel time of an empty AV between locations of two bidders. If the end time of the first bidder plus the travel time of an empty AV is greater than the start time of the second bidder, we consider two bids as conflicting. Once the conflicting bids determined, the auctioneer can solve the following winner determination problem (WDP) with the objective to maximize social welfare, as formulated in Takalloo et al. (2020), considering all the bids from the clock and supplementary stages:

$$\text{(WDP)} \quad \max_{x_{bh}} \quad \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} \sum_{h \in \mathcal{H}} c_j(b) x_{bh} \tag{2}$$

$$\text{s.t.} \quad \sum_{b \in \mathcal{B}_j} \sum_{h \in \mathcal{H}} x_{bh} \leq 1 \quad \forall j \in \mathcal{J} \tag{3}$$

$$x_{bh} + x_{lh} \leq 1 \quad \forall h \in \mathcal{H}, j, q \in \mathcal{J}, b \in \mathcal{B}_j, l \in \mathcal{B}_q : b, l \text{ are conflicting} \tag{4}$$

$$x_{bh} \in \{0, 1\} \quad \forall j \in \mathcal{J}, b \in \mathcal{B}_j, h \in \mathcal{H} \tag{5}$$

In the above formulation, $\mathcal{J}$ denote the set of bidders and $\mathcal{B}_j$ denote the set of bids submitted by bidder $j \in \mathcal{J}$. Similarly, $\mathcal{B}_q$ is the set of bids of bidder q, who has a conflicting bid $l \in \mathcal{B}_q$ with bid $b \in \mathcal{B}_j$ of bidder j. A binary decision variable x_{bh} indicates whether bid b is assigned to vehicle $h \in \mathcal{H}$ or not where $\mathcal{H}$ is the set of vehicles. Constraint (3) ensures that at most one bid of each customer will be determined as a winner. Constraint (4) is a conflict constraint, which ensures that two conflicting bids do not share a vehicle. It usually takes a considerable amount of time to find a high-quality feasible solution for problem (WDP) for large instances.

However, we can use a sequential heuristic to solve the problem by decomposition. Since we assume a homogeneous fleet of vehicles, we can decompose the combinatorial auction problem into a $|\mathcal{H}|$-round single vehicle combinatorial auction, following the approach of Takalloo et al. (2020). At each round, considering the set of remaining bidders, we solve the winner determination problem for a single vehicle and find the winners. Then, we update the set of bidders by excluding the winners from the list of bidders and go to the next round. This procedure continues until we assign all the vehicles to the bidders.

At the end of the supplementary stage, after determining the winners, the auctioneer calculates payments based on payment rules that will be discussed in Section 3.

3 Payment Rules

Choosing a suitable payment rule is an important part of the auction design, as it influences the bidding strategy of bidders and the revenue for the auctioneer. However, there is no universal pricing scheme that guarantees both incentive compatibility and high revenue. For instance, even though setting payments to submitted bid amounts is an intuitive choice, Ausubel et al. (2014) have shown that such payment rule results in demand reduction. In this study, we consider three pricing rules; namely, proxy payments, VCG payments (Vickrey-Clarke-Groves payments) and core-selecting payments. We introduce these payment rules in this section.

3.1 Proxy Payments

The proxy phase (Ausubel et al., 2006) has been proposed as an alternative to the supplementary stage bidding. In the proxy phase, which follows the clock stage, first, each bidder submits her packages and their valuations to her own *proxy agent*. Then, the proxy agents enter an iterative proxy auction on behalf of bidders. After each round r, the auctioneer determines the provisional winners and increases the ask price p_b^r for package b by ϵ_p. Then the agents select the set of packages with the nonnegative utility values to bid in the next round by solving the following problem.

$$\max_{x_b} \sum_{j \in \mathcal{J}} \sum_{b \in \mathcal{B}_j} (v_b - p_b^r - \epsilon_p) x_b \tag{6}$$

$$\text{s.t. } x_b \in \{0, 1\} \qquad \forall j \in \mathcal{J}, b \in \mathcal{B}_j \tag{7}$$

where x_b indicates whether bid b is selected by proxy agent or not. The auction finishes when there are no more bids offered by any agents and the final outcome determines winners and their payments. Thus, we can look at *proxy payments* as iterative first-price payments that are in the core; there is no other set of bidders willing to pay more than the selected set of winners.

3.2 VCG Payments

The *Vickrey-Clarke-Groves (VCG)* mechanism is known to satisfy incentive compatibility. Under the VCG payment rule, truthful bidding is the dominant strategy for the bidders. VCG payment π_j for bidder j can be determined according to the following rule:

$$\pi_j^{\text{VCG}} = Z^*_{\text{WDP}_{-j}} - \left(Z^*_{\text{WDP}} - \sum_{b \in \mathcal{B}_j} c_j(b) x_b^* \right) \tag{8}$$

where Z^*_{WDP} and $\mathbf{x}^*$ are the optimal objective function value and optimal allocation of winner determination problem under the set of bidders $\mathcal{J}$ and $Z^*_{\text{WDP}_{-j}}$ is the optimal objective function value of the winner determination problem under the set of bidders $\mathcal{J} \setminus \{j\}$.

Although theoretically interesting, the VCG mechanism has shown serious practical problems (Rothkopf, 2007). It usually takes a considerable amount of time to solve the winner determination problem optimally. However, it is possible to solve relatively large problems with a small optimality gap. When the solution is suboptimal, incentive compatibility and rationality do not necessarily hold under the VCG mechanism.

Next, we consider core-selecting payment (Cramton, 2013), which is more practical compared to VCG and entails the desirable auction properties such as rationality, efficiency, and core property (Day and Cramton, 2012).

3.3 Core-Selecting Payments

Payments are in the core, if there is no other set of bidders willing to pay more than the set of selected winners (Day and Cramton, 2012). Suppose b^* denote the winning bid of bidder j, and $c_j(b^*)$ denote the bidding price for package b^*. We wish to determine the payment amount for b^* which satisfies the core property. Day and Cramton (2012) propose the following quadratic program to compute the core-selecting payments by minimizing the Euclidean distance between VCG payments and core-selecting payments:

$$\min_{\pi_j} \sum_{j \in \mathcal{W}} \left(\pi_j - \pi_j^{\text{VCG}} \right)^2 \tag{9}$$

$$\text{s.t.} \sum_{j \in \mathcal{W} \setminus \mathcal{S}} \pi_j \geq \beta_{\mathcal{S}} \qquad \forall \mathcal{S} \in \mathcal{S}' \tag{10}$$

$$\pi_j \leq c_j(b^*) \qquad \forall j \in \mathcal{W} \tag{11}$$

where $\mathcal{S}$ denotes any set of bidders who are willing to offer more than the total payment of winners in $\mathcal{W}$ and $\mathcal{S}'$ denotes the set of all such possible sets. Variable π_j is the core-selecting payment for winner $j \in \mathcal{W}$, π_j^{VCG} is the VCG payment for bidder j and $c_j(b^*)$ is the bidding price for the winner j. Parameter $\beta_{\mathcal{S}}$ denote the aggregate payment offered by set of bidders $\mathcal{S}$. Constraint (10) ensures that the total payment of the winners is greater than or equal to the payments offered by any other set of bidders. Constraint (11) makes sure to satisfy rationality by enforcing the individual payments of the winners to be not greater than their offered bid amounts.

Day and Cramton (2012) propose an iterative approach to solve (9)–(11) which finds a set of bidders $\mathcal{S}$ in each iteration by changing the bids' prices of the bidders in set $\mathcal{W}$. In particular, the WDP is resolved at each iteration by setting the bid prices of the bids of each bidder in set $\mathcal{W}$ to $c_j(b) + (c_j(b^*) - \pi_j^k)$. The resulting new set of winners, $\mathcal{S}$ is then used to solve (9) - (11). We can summarize this procedure as follows:

- Step 0. Set $k = 0$, $S' = \emptyset$ and $\pi_j^0 = \pi_j^{\text{VCG}}$.
- Step 1. Set $c_j^{\text{new}}(b) = c_j(b) + (c_j(b^*) - \pi_j^k) \quad \forall j \in \mathcal{W}, b \in \mathcal{B}_j$.
- Step 2. Solve the WDP in (2)–(5) and define $\mathcal{S}$ as the corresponding set of winners. Add $\mathcal{S}$ to set $\mathcal{S}'$.
- Step 3. Set $\beta_{\mathcal{S}}^{k+1} = \sum_{j \in \mathcal{S}} c_j(b^*) - \sum_{j \in \mathcal{S} \cap \mathcal{W}} (c_j(b^*) - \pi_j^k)$ in (10)–(11).
- Step 4. Solve (9)–(11) to generate a new payment π_j^{k+1}, go to Step 1.
- Stopping rule: The process repeats until the WDP with modified bidding prices does not generate any new set of bidders $\mathcal{S}$.

It is worth mentioning that in core-selecting payment method, we do not change the original set of winners $\mathcal{W}$, but we update their payments each time we solve (9) - (11).

4 User Agents in the CCA for Fractional Ownership of AVs

In general, vehicle ownership requires a substantial investment from household income. As a result, customers may need to spend a considerable amount of time (e.g. several weeks) in bidding in CCA for AVs. Moreover, it is not uncommon to see hundreds to thousands of bids from each bidder in combinatorial auctions (Olivares et al., 2012). Considering the time-consuming and the complex bidding process of the CCA, customers may be interested in choosing user agents. User agents communicate with bidders to assist them in the bidding process. In particular, as a supporting tool, user agents assist bidders by bidding through the clock stage on behalf of them and by generating competitive packages in the supplementary stage.

4.1 Bidding Strategies in the Clock Stage

The user agents relieve the computational burden of bidders. In particular, user agents select items to bid in the clock stage according to some strategies which are determined by the bidders. We propose the following bidding strategies for the user agents.

- **Strategy 1**: Under the first bidding strategy, it is assumed that bidders know the customers' exact trip schedule. Under this strategy, customers submit the set of must-have and optional trips to the user agents. Considering the budget restriction and the eligibility points, user agents first consider only the must-have items and solve the following binary optimization problem to select the must-have items to bid in the r-th round of the clock stage for bidder j:

$$\max_{\mathbf{w}} \sum_{i \in \mathcal{I}_j} e_i w_i \tag{12}$$

$$\text{s.t.} \sum_{i \in \mathcal{I}_j} e_i w_i \leq E_j^r \tag{13}$$

$$\sum_{i \in \mathcal{I}_j} p_i^r w_i \leq B_j \tag{14}$$

$$w_i \in \{0, 1\} \qquad \forall i \in \mathcal{I}_j \tag{15}$$

  where $\mathcal{I}_j$ is the set of bidder j's must-have items, w_i is a binary decision variable which indicates whether item $i \in \mathcal{I}_j$ is selected for a bid or not, p_i^r is the ask price of item i in round r, e_i is the eligibility point required for item i, and E_j^r is the eligibility of customer j at the beginning of round r. Constraint (13) satisfies activity rules in the clock stage and Constraint (14) considers the budget limitations of the customer. The objective function maximizes the eligibility of the bidder, which helps her maintain her eligibility in the future rounds and bid on more items. After selecting must-have items, if any eligibility points remained, the user agent solves Problem (12)–(15) for optional trips, by letting $\mathcal{I}_j$ be the set of optional trip items of bidder j. In such a case, we must first update the values of both E_j^r and B_j to

Table 1: Bidding strategies item selection in clock stage

Trips	EP	Ask Price	Trip type	Valuation	Time range
M 7:15–8:00 AM	15	16	must-have	11.03	6:45–8:30 AM
W 5:00–6:00 PM	20	14	must-have	15.72	4:30–6:30 PM
Th 12:30–1:00 PM	10	8	optional	10.90	12:00–1:30 PM

account for used resources to select must-have items.

- **Strategy 2**: In the second bidding strategy, customers are required to submit their valuation for desired time slots along with the budget constraints. We believe this is not a demanding task for customers given the ride-sharing services benchmark prices. Then user agents solve the following optimization problem for customer j in each round r:

$$\max_{\mathbf{w}} \sum_{i \in \mathcal{I}_j} (v_{ij} - p_i^r) w_i \tag{16}$$

$$\text{s.t. } (13), (14), (15) \tag{17}$$

where v_{ij} is the value of item i for bidder j. In the second strategy, the user agent maximizes the utility of customer j by taking into account the ask prices for items in the current round.

- **Strategy 3**: Under the third strategy, at the beginning of auction, instead of submitting the exact time schedule, customers submit an acceptable time range for each trip. Then, in each round of the clock stage, the user agents select time slots in the submitted ranges with the lowest ask prices. In order to find the lowest priced time slots, the user agents consider the demand in each hour provided by the auctioneer. Algorithm 2 presents a suitable approach to find a set of items with the lowest price for the bidders. (Note that since the number of continuous time-slot in each range is infinite, the user agents consider only a finite subset of them by discretizing the possible start time within the range). After obtaining the set of items with the lowest prices, user agents solve the same optimization problem as in Strategy 2 to determine the items.

Table 1 gives an example of the bidding strategies in fractional ownership CCA. A customer provides as an input three trips with start time and end time information, out of which a user agent needs to select some items for bidding in the next round. The customer also indicates her budget as \$50 and her current eligibility as 40 points.

After receiving the given input, a user agent calculates eligibility points and ask prices for each trip (columns EP and Ask Prices in Table 1). If the customer selects the first strategy, the user agent bid on the first and the second trip, since both of them are must-have trips and have high eligibility. Under the second bidding strategy, the customer provides her valuation for each trip (see Valuation column). In this case, the user agent bid on the second and the third item, because they generate nonnegative payoffs. Lastly, under the third strategy, the customer provides a time

Algorithm 2: Package generation algorithm in the r-th round of Clock Stage under Strategy 3

Input: The set of desired items for the customer $\mathcal{I}^r$, aggregate demand until r-th round each item i, D_i^r, demand in r-th round for item i, d_i^r, set $\mathcal{R}^r$ which includes the desired acceptable time range (l_i, u_i) for each item i, number of vehicles $|\mathcal{H}|$, price for item i submitted in r-th round p_i^r, time increment by user agents ϵ_t, and trip length $(e_i - s_i)$ for each item i.

Output: Set of items with the lowest prices $\mathcal{I}^{\min}$

Initialization: $\mathcal{I}^{\min} \leftarrow \emptyset$;
foreach $i \in \mathcal{I}^r$ **do**
    $D_i^r = D_i^{r-1} + d_i^r$;
    if $p_i^r = p_i^{r-1}$ **then**
        `push`($\mathcal{I}^{\min}, i$);
    else
        if $D_i^r > |\mathcal{H}|$ **then**
            `remove`($\mathcal{I}^r, i$);
            $i \leftarrow$ `find_item`($i, \mathcal{R}^r, \epsilon_t$); `// finding set of items with the lowest prices`
            `push`($\mathcal{I}^r, i$);
        else
            `push`($\mathcal{I}^{\min}, i$);

function `find_item`($i, \mathcal{R}^r, \epsilon_t$)**:**
    $k \leftarrow [l_i, l_i + e_i - s_i]$;
    $k_{\min} \leftarrow k$;
    $p_{\min} \leftarrow p_k^r$;
    $t \leftarrow l_i$;
    while $t + e_i - s_i \leq u_i$ **do**
        $t \leftarrow t + \epsilon_t$;
        $k \leftarrow [t, t + e_i - s_i]$;
        if $p_k^r < p_{\min}$ **then**
            $k_{\min} \leftarrow k$;
            $p_{\min} \leftarrow p_k^r$;
    return $k_{\min}$;

Table 2: Time slots submitted by a customer to the user agent

Trips	Monday	Tuesday	Wednesday
1	8:00–9:00 AM	8:00–9:00 AM	8:00–9:00 AM
2		5:00–6:00 PM	12:00–1:00 PM
3			5:00–6:00 PM

range for each trip, which suits customer travel needs (see Time range column). For instance, for the first trip, a customer may consider 6:45–8:30 AM as her acceptable time range. Then, user agent may consider 6:45–7:30 AM, 7:15–8:00 AM and 7:45–8:30 AM as possible items to bid. To choose the best item, the user agent determines the time slot with the lowest price.

Note that as discussed before, the activity rules aim to remove the strategic behavior of bidders. Hence, under an effective set of activity rules, we expect that the bidders' payoff does not differ from each other significantly, where payoff is defined as the difference between a customer's valuation of the bid and bid price. However, without any set of activity rules, we expect that the bidder's payoff under the third strategy becomes higher. We will study the impact of activity rules on bidders' payoff under different strategies by presenting numerical experiments in Section 5.

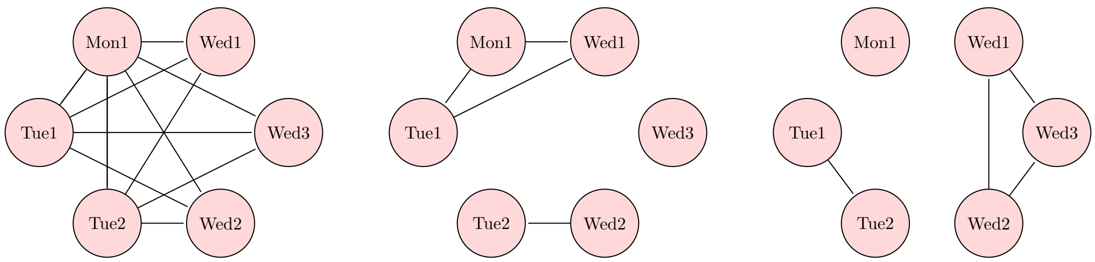
4.2 Automatic Package Generation in the Supplementary Stage

Generating competitive packages is challenging for bidders. The problem intensifies under the proposed customer-defined continuous time slots, where an infinite number of items exist. While iterative bidding in the clock stage reveals the minimum bid amount to win a certain package of time slots, customers may find it useful to bid on other packages which are generated by the user agents in the supplementary stage. In this section, we propose two sets of automatically generated packages by user agents.

The first set of automatically generated packages rely on the bidding strategy selected by a customer in the clock stage. For example, for customers who select the first bidding strategy, the user agent may include all must-have trip time slots while generating other packages which also include several optional trips. For the second strategy, we propose to use a package generation technique similar to the internal-based strategy presented in An et al. (2005), by adding items with the highest utility first while considering the budget constraints. For the third bidding strategy, we propose selecting items that were chosen most frequently in the clock stage by the user agent, since these items tend to have lower prices.

The second group of packages are generated based on the day and the time of the trips submitted by bidders. These packages are common among all bidding strategies. We propose generating *diverse, consistent,* and *single-day* packages. Diverse packages include time-slots from different days. Consistent packages include the trips in different days which take place in the exact same time. Single-day packages include only the trips within the same day.

To understand the package generation procedure for the second group, let us look at the trips



(a) A diverse package (b) A consistent package (c) A single-day package

Figure 4: Diverse, consistent and single-day packages

Table 3: Summary of all types of packages.

Packages	Strategy 1	Strategy 2	Strategy 3
Must-have trips only	✓		
Must-have trips plus optional	✓		
Highest utility trips		✓	
Frequently chosen trips			✓
Diverse package	✓	✓	✓
Consistent package	✓	✓	✓
Single-day package	✓	✓	✓

submitted by a customer given in Table 2. Figure 4 shows graphs used for generating packages, where nodes represent time slots for submitted trips. For instance, Figure 4a visualizes a diverse package, where time slots from different days have been connected with edges while time slots from the same days do not. We use maximum cliques to generate the second group of packages. Note that in graph theory a clique is a complete subgraph of a given graph. A maximum clique is a clique with the maximum number of vertices. By looking for a maximum clique in the graph, user agents generate a package, which consists of all time slots within the clique. Another example is given in Figure 4b, which displays a network for a consistent package. In this case, we connect nodes (time slots) with similar start and end time of trips such as the first trips on Monday, Tuesday and Wednesday. Then user agents again look for a maximum clique in the graph and select time slots within the clique for a consistent package. Lastly, in the single-day package, we connect time slots from the same day as shown in Figure 4c. The summary of all automatically generated package types can be found in Table 3.

5 Numerical Experiments and Case Studies

The goal of the numerical experiments is two-fold. First, we investigate the efficiency of the proposed auction elements such as activity rules, the ask price algorithm and pricing rules. Second, we discuss the auction outcomes to bidders and an auctioneer in terms of winning bids and generated revenue.

For the numerical studies, we build a simulation module based on 2010–2012 California Household Travel Survey, which indicates start and end time of trips as well as miles for 2908 vehicles for one week. Based on this data, we generate bids for customers. In practical, we remove trips with a length less than 15 minutes from the data. Then each vehicle represents a customer with the respective trips information out of which we generated bids as discussed in Section 4. The location distances of each pair of customers are randomly selected between 0 and 60 minutes. Further, we use Caltrans Traffic Counts data, namely Annual Average Daily Traffic 2016, for assigning required eligibility points for each hour of a week. All instances are run 3 times using seeds 1, 3 and 5. For all our experiments we set the ask price increment $\epsilon_p = 2$.

We use the Julia Language (Bezanson et al., 2012) to implement the simulation model and CPLEX 12.5 to solve integer programs via the JuMP.jl package (Dunning et al., 2017). We also use the heuristic algorithm by Takalloo et al. (2020) to solve WDPs not only to determine the winners, but also solve sub-problems arising in various payment rules.

The simulation model includes three modules that have different functionalities; namely the general module, the auctioneer module, and the user agent module.

The general module in the simulation aims to generate trips, bidders, bids and etc. The general module takes as input the number of bidders and the percentage of bidders with a particular bidding strategy. Based on the given input, the module generates a set of bidders and assigns them their initial eligibility points, budget constraints, selected trips, including their start time, end time and miles.

The auctioneer module performs all the auctioneer's tasks such as calculating ask prices in the clock stage, enforcing activity rules, solving WDP and calculating payments. This module takes as an input the fleet size to be auctioned and price increment for ask prices in the clock stage in the absence of supply-demand balance.

The user agent module performs all the user agent's tasks. As discussed in Section 4, user agents bid on behalf of bidders both in the clock and the supplementary stages. User agents also generate packages for bidders. This module takes as an input the bidding strategies of the bidders.

5.1 Managerial Insights

To compare the different bidding strategies offered by user agents and check the efficiency of the proposed activity rules, we conduct experiments under 4 different scenarios discussed below. In each scenario, we simulate an auction with 21 bidders and a single AV. We let a bidder select strategy 1, 2, or 3 for the clock stage and consider the following scenarios for the rest of bidders:

Scenario 1: The rest of bidders select Strategy 1.

Scenario 2: The rest of bidders select Strategy 2.

Scenario 3: The rest of bidders select Strategy 3.

Scenario 4: The rest of bidders are split to select Strategy 1, 2, or 3.

Table 4: The strategies comparison with Activity Rules

	The total wins			The percentage of full wins		
Scenario	Str1	Str2	Str3	Str1	Str2	Str3
1	55	44	47	57.69%	19.23%	23.08%
2	62	46	51	48.78%	21.95%	29.27%
3	59	51	52	42.86%	28.57%	28.57%
4	58	48	50	44.44%	25.00%	30.56%

Table 5: The strategies comparison without Activity Rules

	The total wins			The percentage of full wins		
Scenario	Str1	Str2	Str3	Str1	Str2	Str3
1	32	36	42	23.08%	38.46%	38.46%
2	34	48	47	14.29%	47.62%	38.10%
3	27	38	50	5.88%	23.53%	70.59%
4	43	42	51	28.57%	32.14%	39.29%

We run the simulation for 3 instances for each class (strategy-scenario combination) and count the number of times (in 3 instances) a bidder has been selected as a winner under different scenarios and strategies.

Table 4 reports the total number of times bidders win (in 3 instances) using different strategies and under 4 scenarios and the percentage of full wins (when a bidder wins in all 3 runs) when the proposed eligibility based activity rules are used. We observe that the greatest number of wins is achieved under Strategy 1 regardless of any configuration of scenarios. Similarly, bidders who select Strategy 1 are more likely be selected as a full winners under any scenario followed by Strategies 3 and 2.

From computational studies, we may conclude that the activity rules effectively induce the first bidding strategy. When the bidders with the third bidding strategy (who seeks to bid for items with the lowest ask prices) enter the supplementary stage, they face a relative cap on their bidding price induced by the supplementary stage activity rules.

Thus, submitting competitive bids under Strategy 3 is relatively harder compared to bidding under the first strategy which is not restricted by activity rules. Indeed, under the first strategy bidders consistently bid for must-have trips and gradually increase bid prices, which is a favored behavior according to activity rules. This explains why a customer with Strategy 1 tends to be a winner. Similarly, since under Strategies 2 and 3, item selection is based on the value of ask prices, when entered into the competition for conflicting time slots, customers with such bidding strategies most likely to choose less competitive items. Then a bidder with the first bidding strategy encounters less competition with the bidders with the second or the third bidding strategy, which increases her chance of winning.

Table 6: Payments rules comparison using exact solutions

$\|\mathcal{J}\|$	Bidders' Strategy	Revenue, \$			Time, sec		
		VCG	Core	Proxy	VCG	Core	Proxy
30	Strategy 1	1,017	4,850	2,960	9.89	10.96	112.55
	Strategy 2	889	1,193	1,055	85.83	91.39	387.10
	Strategy 3	1,090	2,063	1,728	102.58	108.73	5,404.54
	Mixed	779	2,930	1,469	63.87	67.31	286.86
60	Strategy 1	3,370	7,189	4,858	57.95	60.24	193.32
	Strategy 2	1,478	2,267	2,012	457.23	471.25	656.90
	Strategy 3	2,429	4,121	3,666	682.35	703.44	11,695.41
	Mixed	1,970	4,582	3,052	247.25	254.72	662.30
90	Strategy 1	4,308	8,940	6,405	150.66	155.08	827.98
	Strategy 2	2,191	2,954	2,711	17,573.46	17,898.58	15,056.72
	Strategy 3	3,776	5,767	5,304	7,468.69	7,641.55	14,730.09
	Mixed	3,003	6,172	4,313	751.17	769.80	5,875.49

We also run similar experiments in the absence of the proposed activity rules to investigate the auction outcomes under different strategies and scenarios. In particular, we removed eligibility constraints in selecting bids presented in (13) for Strategies 1 and 2 and eliminated the objective function of Strategy 1 shown in (12). Also, we removed the relative caps imposed by (1). Table 5 presents the results, which clearly indicate the prevalence of Strategy 3 in the number of wins and in the percentage of full wins, while Strategy 1 results in the lowest number of wins in almost all scenarios. These experiment results clearly indicates the importance and efficiency of the proposed activity rules to prevent strategic bidding.

As discussed before, in order to select payment rules for the proposed auction, we used the simulation model. In particular, we measure the generated revenue and the computation time to compare VCG, core-selecting and proxy payments. We run the simulation model multiple times by generating instances based on the different number of bidders, vehicle numbers and bidding strategies in the clock stage while using the exact and heuristic approaches in solving the WDP. We implement proxy-auction using safe start with VCG prices (Hoffman et al., 2006). We also implement quadratic core-selecting payments suggested by Day and Cramton (2012).

Table 6 reports the results when the WDP is solved exactly and $|\mathcal{H}| = 1$. In this case, core-selecting payments generate the highest revenue under all bidding strategies, while proxy payments also demonstrate competitive revenues. We have to note that for proxy payments calculation, we place a time limit of 4 hours and report the revenues generated within the time limit. Nevertheless, the computation time of the proxy payment method is significantly larger than those of other payment methods, while core-selecting payments dominate both in terms of revenue and calculation time.

Table 7 reports the revenue and the computation time when the WDP is solved using the

Table 7: Payments rules comparison using heuristic solutions under mixed strategy

		Revenue, $			Time, sec		
$\|\mathcal{H}\|$	$\|\mathcal{J}\|$	VCG	Core	Proxy	VCG	Core	Proxy
2	90	2,756	8,771	6,564	1,315	1,335	3,064
	120	4,904	11,018	8,805	3,038	3,077	10,986
3	90	170	10,069	6,694	1,649	1,671	10,561
	120	17,496	12,948	19,986	4,486	4,533	6,168

heuristic method discussed in Section 2.3.1 for $|\mathcal{H}| = 2$ and $|\mathcal{H}| = 3$. We observe that both under the exact and heuristic solutions the VCG payments dominate in terms of computation time. When the WDP is solved using the heuristic approach, core-selecting payments generate higher revenues compared to other payments except when $|\mathcal{H}| = 3$ and $|\mathcal{J}| = 120$ with the proxy payments being the highest. Since the WDP is solved using the heuristic solution, the VCG payments may violate individual rationality, thus resulting in higher payments compared to core-selecting payments which enforce individual rationality. Consequently, the warm start of proxy payments with the VCG payments results in larger proxy payments compared to core-selecting payments. We place a time limit of 4 hours for the proxy payments calculation and took the average run time for 3 replicates. Even though the time limit may cause lower revenues under the proxy payments, in general, we conclude that core-selecting payments generate the highest payments using the heuristic solutions. Therefore, based on the computational study we recommend using core-selecting payments as a pricing scheme for fractional ownership of AVs.

We also study the auction outcomes to bidders measuring a number of co-leasers defined as customers sharing the same AV and some statistics of the winning bids. For instance, average values of payments, miles, time slots' lengths and the number of trips are calculated by summing their respective values for the winning bids and dividing by the total number of winners. The number of bids indicates the average number of bids submitted by all winners. We report the results of 3 replicates for each instance taken as their average values. As shown in Table 8, the average payments increase with the increase in the number of participating bidders due to the rise in the competition. From the used dataset containing trips of customers, the average number of trips in the winning bids is above 2. We also note that the average number of trips in the winning bids change slightly across all instances. Such outcomes are related to the structure of the bids. For instance, bids with a large number of trips encounter a significant number of conflicts with other bids, thus, requiring large bid prices to win. In contrast, bids with a small number of trips have fewer chances to overlap with other bids. Then when solving the WDP selecting a large pool of bids with a small number of trips may contribute more to the social welfare compared to accepting a small pool of bids with a large number of trips.We also note that the trip costs per mile and per minute increase with the increased number of bidders. We may also enforce the minimum trip length requirement in the WDP. For instance, the auctioneer may require customers to use AVs at

Table 8: The auction outcomes

		Average Value						
$\|\mathcal{H}\|$	$\|\mathcal{J}\|$	# co-leasers	Payments	Miles	Time, min	# trips	# bids	WDP sol.
1	30	23.30	38.76	39.65	74.46	2.88	19.38	exact
1	45	31.33	47.91	30.33	60.66	2.49	17.01	exact
2	60	26.33	22.56	33.91	69.36	2.82	15.85	heuristic
2	90	34.67	69.80	29.04	58.93	2.45	16.15	heuristic

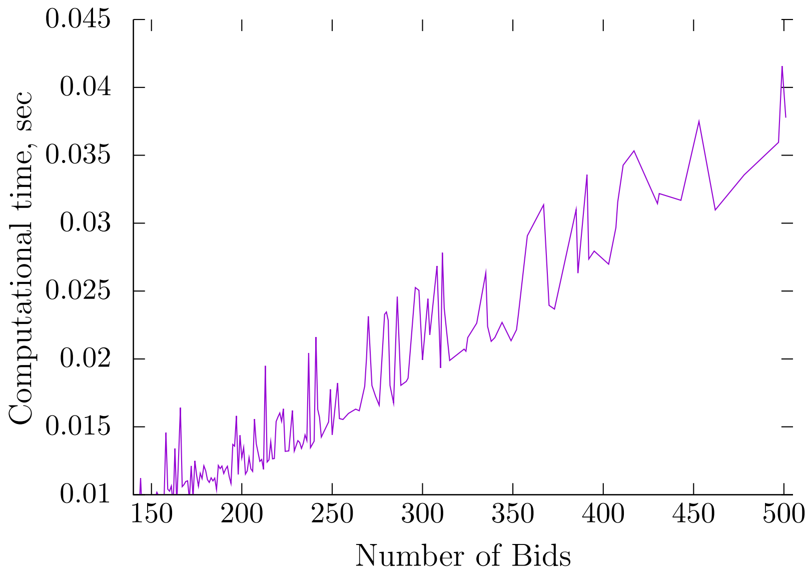


Figure 5: The performance of the ask price algorithm

least for 30 minutes per use. This may reduce the revenue of the auctioneer as it limits options for serving small trips and increases the idle time of AVs. However, it also offers more flexibility to winners in their schedule offering a wide time window between serving two customers and making such an option more attractive. In return, such a setting actually may boost the revenue.

Finally, to demonstrate the performance of the proposed ask price algorithm, we run the algorithm under various bid numbers, while measuring the computational time. As has shown in Figure 5, the algorithm solves all instances in a fraction of a second. Also, the computational time increases linearly with the number of bids, suggesting $O(n)$ complexity.

6 Conclusion

Motivated by recent advancements in the fractional ownership of regular vehicles and its potential for the increased resource utilization, this study investigates a market for fractional ownership of AVs. In this novel form of vehicle ownership, a group of individuals agrees to lease a single AV together. We propose a combinatorial clock auction to match interested customers to use an AV without any time conflicts. In the proposed auction design we use driverless mobility of AVs to form co-leasing groups and allow customers themselves to select items to bid as continuous time slots. The resulting auction becomes a combinatorial clock auction with bidder-defined times. Further, we deliver practical tools and techniques both for an auctioneer and customers to effectively deploy the proposed auction. In particular, we devise a novel ask price calculation algorithm to support the decision making of an auctioneer. Similarly, to help customers with complex valuation problems, we introduce user agents, a software tool designed to bid on behalf of customers throughout the auction.

The simulation study of the auction tests the effects of bidding strategies offered by user agents on the payoff of customers. The computational studies indicate that eligibility points based activity rules favor consistent bidding among all other proposed bidding strategies. In addition, the total revenue of an auctioneer has been evaluated under exact and heuristic solutions of the WDP. In both cases, we found that core-selecting payments dominate the VCG and proxy payments in terms of solutions time and generated revenue.

One of the future research directions can be developing exact solution methods for the WDP with a large number of AVs and bidders. Further, the introduced user agents can be enhanced with learning capabilities based on the bidding history of a customer. In the future research, we may consider including the spatial information of bidders in calculating ask prices and include uncertainty in the travel time of AVs between the locations of two customers.

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