

# Drone-Truck Routing Problems

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## 1 Introduction / Background

Recent advances in aerial vehicle technologies made drones useful in many practical applications. While most vehicle routing problems are defined for ground vehicles such as delivery trucks, the drone-truck routing problems refer to vehicle routing problems that involve two types of vehicles: unmanned aerial vehicles, called drones, that can perform tasks with limited flying range or capacity, and ground vehicles, called trucks, that can perform tasks with much longer range and larger capacity. While drones usually refer to aerial vehicles, they can be any mobile robots that work with trucks in coordination. Drones may be launched from trucks to perform tasks and then picked up by trucks for payload replenishment or battery replacement. Depending on the applications, a drone can be primary with the support of a truck, or vice versa, or they may be independent working units. A common objective is to minimize the makespan (job completion time).

## 2 Problems

When customers or tasks are located at nodes, the corresponding vehicle routing problems are particularly called node routing problems. On the other hand, when

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they are at arcs, the problems are called arc routing problems. While the current literature on drone-truck routing problems mostly focuses on node routing problems, the literature for drone-truck arc routing problems is gaining traction.

## 2.1 Node Routing

Applications of drone-truck node routing problems include last-mile delivery, logistics, disaster management, infrastructure monitoring, and many others.

Let  $\mathcal{V}$  be the set of customer vertices, also called nodes, and  $\mathcal{A}$  be the set of arcs, also called edges. Let  $\mathcal{V}_0$  denote the set of all nodes, including depots and customer vertices. Drones and trucks depart from a depot and must return to the depot. It is possible that some customer nodes can only be served either by trucks or drones, depending on the nature of the customer requirements. The different traveling times for drone and truck are applied for each arc  $(i, j) \in \mathcal{A}$ :  $c_{ij}^{\text{dr}}$  and  $c_{ij}^{\text{tr}}$ , respectively. In most drone-truck routing problems, the objective is to minimize the makespan, i.e., the time that the last vehicle returns to the depot.

Many drone-truck routing problems are based on the traveling salesman problem (TSP) or the vehicle routing problem (VRP). A few variants of the drone-truck node routing problems are introduced.

**FS-TSP:** *Flying side-kick traveling salesman problem.* This problem is the first drone-truck routing problem reported in the literature [12]. There is a single truck and a single drone, and the drone can depart from the truck, serve a customer, and return to the truck to replace the battery or cargo. Both the drone and the truck are required to be present at some particular node at the same time for rendezvous, which is called the synchronization condition. The concept of sortie, a drone excursion supported by a truck, is used for the basis of the formulation. When there are multiple drones, the problem is also called mFS-TSP.

**PDS-TSP:** *Parallel drone scheduling traveling salesman problem.* This is when a single drone and a single truck can perform the tasks independently from each other. The drone is used to serve customers located near the depot, while the truck may serve other customers. Drone service scheduling and truck routing are to be tackled in this problem. This problem can be extended to consider multiple drones, called PDS-TSP-DP.

**TSP-D:** *Traveling salesman problem with drones.* This problem, very similar to FSTSP, is for when there is a single truck and one or more drones. The concept of operation, where some constraints are relaxed from the sortie used in FS-TSP, is used to simplify the formulation [1].

**VRP-D:** *Vehicle routing problem with drones.* This general group of routing problems is for the multiple-truck and multiple-drone cases. Several variants are created with assumptions such as drones can serve multiple customers in a single fly-out, only drones can serve customers, and trucks support drones by supplying batteries, parcels, or any other items necessary to fulfill the demand.

Optimal solutions to the drone-truck combined routing problems can exhibit quite distinct behavior compared to the solutions to the truck-only problems. In TSP-D, if a node is visited by both the drone and the truck, it is called a combined node. An optimal solution may require the truck and the drone to visit a combined node multiple times, while it is not necessary to visit a truck-only or drone-only node twice [1]. In addition, it is commonly assumed that the drone launch and retrieval from and by a truck can be done only at customer nodes, which gives rise to synchronization issues. Some consider the use of ground hubs where drones can be stored temporarily until a truck picks them up. These properties make the drone-truck combined routing problems significantly more challenging to solve than the truck-only vehicle routing problems.

The computational methods to solve the drone-truck routing problems are mostly heuristics or meta-heuristics such as adaptive large neighborhood search [6, 11, 14]. More recently, exact methods are being actively developed, such as Benders decomposition [8, 15], Branch-and-Price [13, 16], and Branch-and-Cut [2, 5], although they are currently limited to problems with a small number of vehicles and a small number of customer nodes.

## 2.2 Arc Routing

Arc routing problems [7] involving drones have applications in energy transmission line inspection and monitoring, police patrolling, rural mail delivery, roadway pavement inspection, and so on. In the classical arc routing problems, a vehicle must cover each demand arc fully in a single pass. On the other hand, drones in arc routing problems can cover the arc partially in the first pass and the remaining part in the next pass, and they can enter and exit an arc at any point [3]. With the drone's limited flying range and the coordination between the truck and the drone, the drone-truck arc routing problems have additional complexity.

We are given a graph  $G(\mathcal{V}, \mathcal{A})$ , where  $\mathcal{V}$  is the set of vertices, and  $\mathcal{A}$  is the set of arcs. The set of required (demand) edges is denoted by  $\mathcal{A}_R \subseteq \mathcal{A}$ . It is possible that arcs are traveled by a vehicle more than once to complete the task. Some required edges may only be covered by the drone, and some others only by the truck. While the node routing problems are defined over a complete graph typically, the arc routing problems are defined over a sparse graph usually.

**D-ARP** *Drone arc routing problem.* When there are only drones performing the arc routing tasks, the problem is called D-ARP. When there is a single drone, it is an extension of the rural postman problem. Drones can cover each required arc in a split, while the optimal split point is unknown a priori. Therefore, an approximate solution procedure would require solving multiple rural postman problems with added split points. The limited flying range of drones also introduces additional complexity.

**DT-ARP** *Drone-truck arc routing problem.* If both drones and trucks are involved either to cover the required arc or to support the other vehicle, the problem is

called DT-ARP. When there is a single truck in the problem, it may be called the Chinese postman problem with drones (CPP-D) when  $\mathcal{A}_R = \mathcal{A}$  and the rural postman problem with drones (RPP-D) when  $\mathcal{A}_R \subsetneq \mathcal{A}$ .

Since partial arc covering is possible in D-ARP and DT-ARP, the shape of arcs is a critical piece of information, and one requires to obtain GIS data for each problem instance as well as the network connectivity data, especially when the arcs are curvy [3]. A special case of D-ARP is when there are  $K$  drones with limited flying range, called the Length-Constrained  $K$ -Drone Rural Postman Problem (LC  $K$ -DRPP) and solved by a Branch-and-Cut algorithm and a heuristic algorithm [4]. Applications of DT-ARP are presented for powerline inspection [9] and traffic patrolling [10] without considering partial arc covering, both of which are solved by heuristic algorithms.

### 3 Side Constraints and Extensions

While the distance constraint to consider the limited flying range of drones is an obvious constraint in drone-truck routing problems, some drone-specific constraints may be added, such as restricted air space, safety and privacy concerns, and environmental issues. Furthermore, many practical constraints may be added as in the vehicle routing problems. Examples include capacity constraints, time-window constraints, and precedence constraints. A generic form of drone-truck routing problems can consider a heterogeneous fleet of trucks and a heterogeneous fleet of drones to serve the diverse needs of the customers.

#### Cross References (to other articles)

- Traveling Salesman Problem
- Vehicle Routing
- General Routing Problem

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