STRATEGIC FREIGHT NETWORK PLANNING MODELS AND DYNAMIC OLIGOPOLISTIC URBAN FREIGHT NETWORKS

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1. Introduction

In this chapter we focus on the important topic of mathematical models for strategic freight network planning. Such models are not meant for use in managing the moment-to-moment or even day-to-day operations of freight companies or freight infrastructure. Rather such models are employed primarily to forecast, months or years into the future, freight traffic over specific network links and routes and through specific network nodes and terminals. The fundamental decision variables of these models are expressed as flows (volumes per unit time) and are entirely continuous in nature. The time frame is that of months or years. The perspective is generally that of a multimodal partial equilibrium of the transport market, with alternatives being evaluated according to the comparative statics paradigm. Our discussion is restricted primarily to those models that have been commercially available and are well documented in the open literature.

In strategic freight network modeling, traffic forecasts are not made by statistical inference or econometric methods; neither is discrete event simulation typically used. Instead network models expressed in a closed mathematical form as optimization and game theoretic problems are the usual formalism. Furthermore, these models -- because of their large size and complexity -- are solved numerically using adaptations of powerful algorithms developed for nonlinear mathematical programming and noncooperative mathematical games.

The fact that freight network models are not based on statistical inference means that they have the important capability of examining the implications of structural changes in underlying markets, something which is very difficult if not impossible to do with econometric methods. To be sure, the specific parameters needed to articulate the constituent submodels of any freight network model are obtained by statistical and time series methods; yet the behaviors of individual agents active on the freight network of interest are not based on trends or historical conduct. Rather these behaviors are modeled mathematically using results from mathematical programming and game theory. The resulting mathematical models are the basis for numerical calculations with modern high-speed digital computers, which determine the end result of various forms of cooperation and competition among those agents.

Because strategic freight network models are primarily concerned with the freight transport market, they have historically been viewed as distinct from computable general (CGE) models, which determine prices and consumption and production activities for the entire economy. Yet this distinction is somewhat artificial, as the demand for freight transportation services is derived from the spatially separated production and consumption activities associated with individual commodities. It is therefore not a surprise that some of the most recent work on strategic freight network planning attempts to bridge this gap between freight models and general equilibrium models. The models emerging from this synthesis have come to be called spatial computable general equilibrium models and are one of the main categories of models we review below.

2. Some Background

In the last four decades, very significant progress has occurred in the understanding and modeling of passenger trip making behavior over networks. Corresponding advances in understanding and modeling of freight transportation decision making over inter-regional, inter-modal networks have been much slower in coming. This fact is illustrated by noting that the accuracy with which urban passenger travel demand and route/mode choice decisions on a network can be forecast appears to be very substantially greater than that possible for the inter-regional freight case. The most accurate large scale U.S. freight network model is able to predict equilibrium network link volumes agreeing with Federal Railway Administration (FRA) density codes (reported data describing annual tonnages on every physical link of the rail system) with a frequency of only about $60% (Friesz et al, 1981, 1983a, 1983b, 1985). This performance leaves much to be desired since density codes simply denote upper and lower bounds for link volumes; the difference between those upper and lower bounds is frequently of the same order of magnitude as the predicted volumes themselves. Poor as this accuracy is, it is nonetheless significantly greater (about three times greater) than that reported for earlier models (Bronzini, 1980). Because this accuracy increase was achieved by relatively straight-forward extensions of the urban passenger network modeling paradigm, there is reason to believe that still greater accuracy may be obtained from a model designed specifically for freight applications from the outset. The main goal of this article is to outline the various efforts made to date to realize this promise of strategic freight network models.

To help understand the various strategic freight network modeling efforts which have been reported in the literature, it is useful to proffer some hypotheses regarding the reasons for the accuracy disparity between predictive urban passenger network models and predictive inter-regional, inter-modal freight network models noted in the previous paragraph. In particular, the accuracy disparity may be attributed to the following factors:

1. freight-related databases needed for calibrating and validating predictive network models are not as extensive and probably not as accurate as those maintained for passenger travel;
2. freight transportation decisions are decidedly more complex and correspondingly more difficult to model than passenger travel decisions;
3. the predictive freight network models developed and applied to date continue to be heavily influenced by the passenger network paradigm, whose assumptions are simply erroneous for many freight applications;
4. efficient and inexpensive algorithms for solving mathematically rigorous freight network models have not been widely available nor well understood by practitioners; and
5. large scale predictive freight network models are poorly integrated with computable general equilibrium models, causing inconsistencies among forecasts of national/regional economic activities and prices on the one hand and detailed freight flows on the other.

Other reviews of freight models which contain substantial infromation on strategic freight network models are: Crainic and Laporte (1997) and Friesz and Harker (1985).

3. The Key Commercial Models

The history of freight network modeling is a rich one. It is generally agreed that the first significant strategic freight network planning model was developed by Kresge and Roberts (1971); that model is referred to in Table 1 as the Harvard-Brookings model. All subsequent freight network models have been heavily influenced by the essential observation of Kresge and Roberts (1971): the multitudinous interactions of freight infrastructure and the decision making agents active on a freight network can be analyzed using powerful results from mathematical programming for the study of problems with network structure. The Harvard-Brookings model is now obsolete and no longer in use.

Another historically important freight network model is that developed by Bronzini (1990) for CACI. The CACI model was notable for its use of a nonlinear programming formulation based on nonlinear cost and delay functions obtained by simulation of different railway and waterway operating environments. This model was used to perform most of the freight-related calculations of the U.S. National Energy Transportation Study; it is also obsolete.

The Princeton Rail Network Model [Kornhauser et al (1979)], developed by ALK Associates, is one of the important current freight network planning models. It originally relied on a very simple linear carrier model, although options for certain types of equilibrium congestion calculations have been recently added. Although this model does not explicitly treat the interaction of shippers and carriers, it does contain the best available U.S. multi-modal freight network database.

The current version of the Freight Network Equilibrium Model (FNEM) was developed by George Mason University under funding from the U.S. Department of Energy and the U.S. Central Intelligence Agency. It employs a rather sophisticated game theoretic model of shipper and carrier interactions and has data bases for the U.S., China, Africa, the Middle East and the countries of the former Soviet Union. It is used routinely by the U.S. Government for defense and intelligence related freight forecasts. FNEM was re-designed in the early 1990s to employ satellite imagery. The most advanced versions of FNEM and its most current databases are classified. The foundations of FNEM are explained in Friesz (1981, 1985).

STAN [Crainic *et al* (1990a, 1990b)] is a freight network planning model developed by the University of Montreal in association with a private consulting firm. It is qualitatively very similar to FNEM, as Table 1 reveals. It differs from FNEM primarily in treating only carriers (but not shippers) explicitly and in having an explicit mechanism for backhauling. The use of STAN has been limited to a few developing countries and to Canada. It, like The Princeton model and FNEM, is still in active use.

4. Typology of Models

Friesz et. al (1983a,1998) describe an idealized freight network planning model which is a useful pedagogical device for developing an appreciation of the many compromises involved in constructing and applying an actual model of this sort. In particular, Table 1 presents 17 criteria, which when addressed favorably lead to an ideal freight planning model.

Table 1. Typology of Predictive Freight Network Models

MODEL	CRITERIA																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Harvard-Brookings	Y	Y	Y	Y	N	Y	Y	N	*	*	Y	N	N	N	N	N	N
CACI	Y	Y	Y	N	N	N	N	N	*	*	Y	N	N	N	N	N	N
Princeton-ALK	N	Y	Y	N	N	N	N	Y	*	*	Y	N	N	Y	N	Y	N
NETLAB (FNEM)	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	Y	N	Y	N	N	N	N
CRT Montreal (STAN)	N	Y	N	Y	Y	N	N	Y	*	*	Y	N	N	Y	N	Y	N

Symbols: Y = yes; N = no; * = not applicable

Criteria:

1. multiple modes

2. multiple commodities
3. sequential loading of commodities
4. simultaneous loading of commodities
5. explicit congestion
6. elastic transportation demand
7. explicit shippers
8. explicit carriers
9. sequential shipper and carrier submodels
10. simultaneous shipper and carrier submodels
11. sequential computable general equilibrium (CGE) and network models
12. simultaneous computable general equilibrium (CGE) and network models
13. nonmonotonic functions
14. explicit backhauling
15. blocking strategies
16. fleet constraints
17. imperfect competition

Some of these criteria depend on the dichotomy of freight decision-making agents: shippers and carriers. Shippers are those decision-making entities desiring a particular commodity at a particular destination; carriers are those decision making entities that actually effect the transport of commodities, satisfying the transportation demands of the shippers. Note that Table 1 describes how each of five models fairs relative to these criteria. Friesz et. al (1983a, 1998) offer the following summaries of each criterion:

Criterion 1 recognizes that multiple modes compete for and are used to carry freight shipments. The data in Table 1 indicate that four of the five models address multimodal interactions whereas the remaining model is a unimodal (rail) model.

Criterion 2 incorporates the fact that freight transportation involves multiple commodities with distinct transportation cost characteristics and different shipping time requirements that prevent meaningful treatment as a single commodity.

Criterion 3 refers to the fact that it is sometimes possible to prioritize commodities and assign them individually to the network in order from highest to lowest shipment priority. Some commodity disaggregation schemes will lead, however, to commodities of identical shipment priority but with distinct unit cost characteristics; for these commodities, a simultaneous loading procedure is required Criterion 4).

Criterion 5 recognizes the general variation of relevant costs and delays with flow volumes due to congestion economies and diseconomies.

Criterion 6 refers to the fact that demand for transportation will generally vary with transportation costs and delays. Two of the models incorporate elastic demand functions in the form of trip distribution models to determine origin-destination (O-D) flow levels. The remainder of the models require fixed trip matrices as input.

Criteria 7 and 8 address the fact that routing and modal choices in freight systems are the result of decisions of both shippers and carriers and that these groups obey distinct behavioral principles and may, at times, have conflicting goals. Only one of the five models explicitly treat shippers and multiple carriers.

Criteria 9 and 10 refer to whether one ascertains the decisions of the shippers first and then the decisions of the carriers or determines both simultaneously. Only a simultaneous determination gives a true equilibrium; otherwise there exists the possibility of further adjustments by shippers whose perceptions of freight transportation levels of service differ from those actually provided by carriers.

Criteria 11 and 12 recognize that virtually all reported freight network models use as input fixed supplies and demands of individual commodities obtained from a separate general equilibrium model. Generally, such general equilibrium models employ assumptions about freight transportation costs, and the question naturally arises of whether the network model outputs are consistent with those costs. Iteration between the general equilibrium model and the network model in an attempt to produce consistency is, of course, an heuristic device with no rigorous convergence properties; only simultaneous solution of the general equilibrium model and the network model will always result in the desired consistency.

Criterion 13 refers to the ability of a given model to treat nonmonotonic functions, particularly nonmonotonic cost and delay functions that are expected to occur as a result of average rail operating costs which initially decline as volume increases and then begin to increase as capacity is approached. When nonmonotonic functions are used in a user-optimized situation, the associated mathematical formulation may possess multiple equilibria. It is a commonly held myth that equilibrium problems with nonmonotonic functions cannot be solved efficiently. In fact such problems can be solved nearly as efficiently as those with strictly monotonic functions so long as one is content to compute only a single, nonunique equilibrium point.

Criterion 14 recognizes that a large portion of traffic is made up of empty rolling stock, empty barges, and empty trucks that contribute to costs and congestion. Freight transportation is dependent on the availability of empties, and this necessitates considerable attention to backhauling operations if carriers are to be able to satisfy shippers' transportation demands.

Criterion 15 recognizes that rail freight flows are composed of trains of varying length, made up of different types of rail cars that are frequently ``blocked'' into groups bound for common or similar destinations. This blocking has a significant effect on yard delays encountered by a shipment.

Criterion 16 refers to the fact that there are generally restrictions on the supply of rolling stock and vehicles that cannot be violated in the short run; as such, this criterion is intimately related to Criterion 14 dealing with backhauling. Note that only Princeton and STAN models explicitly treat fleet constraints.

Criterion 17 recognizes the tendency of carriers to collude with one another and to bargain with shippers in setting rates.

Table 1 helps us to define key research issues in predictive freight network modeling, namely those associated with Criterion 10 and Criteria 12-17. Although some models have addressed the issues raised by these criteria, improvements – as will be argued below – are still needed. To these criteria we add the need for model validation and for dynamic extensions to obtain the following list:

1. simultaneous shipper and carrier submodels;
2. simultaneous computable general equilibrium (CGE) and network models;
3. nonmonotonic functions;
4. backhauling;
5. fleet constraints; and
6. imperfect competition
7. validation
8. dynamic extensions

Consequently, the balance of our discussion is devoted to the above eight considerations

5. Shipper-Carrier Simultaneity

It is a common misconception that no simultaneous shipper-carrier freight network models have been developed. In fact there have been three significant efforts to develop simulataneous shipper-carrier network models. Of these, the model by Friesz and Viton (1985) is purely theoretical in nature, demonstrating that a marginal cost pricing scheme for carriers can be treated simultaneously with a shippers' equilibrium submodel for carrier selection.

By contrast, the simultaneous shipper-carrier model developed by Harker (1982,1983) and Harker and Friesz (1986a, 1986b) has been applied to study the coal industry of the United States. This model employs a spatial price equilibrium submodel for shippers in conjunction with a profit-maximizing submodel for each carrier. The resulting framework, known as the generalized spatial price equilibrium model (GSPEM), is perhaps the most advanced freight network model developed to date in terms of the mathematical formalism employed to model the behavior of the decision making agents active on freight networks. GSPEM has been validated in a partial equilibrium context, although its goodness of fit statistics are substantially weaker than those developed for FNEM. GSPEM, as mentioned previously, has been used to assess the overall efficiency of coal transport in the United States. GSPEM has, however, not been applied by any governmental agency and remains essentially a prototype; for this reason it is not included among the models listed in Table 1.

A third simultaneous shipper-carrier model is presently under development for the Chilean Ministry of Railways [Fernández et al (1998a, 1998b)]. It is the result of a deliberate effort to review all antecedent models and synthesize the best features of each.

6. Integrating Static CGE and Network Models

A major impediment to the wide spread use of strategic freight network models is that they frequently are incompatible with CGE models at the both the regional and national levels. CGE models are frequently the result of much labor. Regional and national authorities often do not have the resources to maintain both a CGE model and a large scale freight network model; as a consequence, it is usually the large scale freight model which languishes or is abandoned altogether.

CGE models typically represent the transport sector in a very aggregate fashion and cannot provide any information at the link, node and fleet level. By contrast, freight network planning models use a very detailed representation of the transportation sector and its infrastructure. Freight network planning models also tend to employ exogenous consumption and production data; those that generate production and consumption data endogenously use only a few commodity groupings and a partial equilibrium perspective. It is therefore almost inevitable that the predictions of the two categories of models will be inconsistent. Specifically, CGE models employ transport cost data which will typically not agree with the transport costs computed from a freight model using the commodity production and consumption numbers output by the CGE model.

It is possible to overcome the aforementioned inconsistency by carefully crafting an equilibrium model which uses the full supply and demand sectoral detail of the CGE model and the full network detail of the transport model. This must be done with great care to avoid double counting of activities and costs in the transport sector. Such combined models are known as spatial computable general equilibrium models, a name which can be traced to the *International Workshop on Transportation and Spatial CGE Models* held in Venice in 1993 [Roson (1994)]. Probably the first strategic freight SCGE model is that proposed by Friesz et al (1994,1998). A related formulation is that of Goldsman and Harker (1990). However, much work remains to be done on SCGE models, especially as regards existence, uniqueness and convergence of algorithms.

7. Non-monotonic Models

It is well known that economies as well as diseconomies of scale and scope exist in freight systems for specific flow regimes, leading to nonmontotonic unit cost and delay functions which in turn lead to nonconvex mathematical programming models of carrier behavior. The presence of such nonconvexities is simply unavoidable and has significant computational implications. In particular, we must adandon aspirations of finding globally optimal carrier strategies and we are unable to establish uniqueness of shipper-carrier network equilibria. Nonetheless, we are able to find locally optimal carrier strategies and nonunique shipper-carrier equilibria by modifying the methods of setting step sixes in feasible direction methods devised for convex mathematical programs and monotonic variational inequalities. Efforts need to be made to apply newly emerging global optimization methods based on artificial intelligence, neural networks, tabu search and nontraditional paradigms to strategic freight network planning models. Although these global methods tend to be slow to converge, there application in this context is entirely practical since real time calculation is not required.

8. Backhauling and Fleet Constraints

A major aspect of a freight carrier's strategy is the choice of a scheme for backhauling: that is, for the relocation of empty and near empty vehicles and rolling stock to meet subsequent transportation demand. The treatment of backhauling and of fleet constraints go hand-in-hand, as the size of the pool of vehicles and rolling stock dramatically influences the choice of a backhauling strategy by a carrier. The Princeton and STAN models are notable for explicitly dealing with these important issues. FNEM, by contrast, treats these considerations indirectly by including backhauling and fleet size considerations in the cost and delay functions it employs, Specifically, FNEM employs cost and delay functions for each of several categories of freight movements; the functions for these categories are the result of fits to data obtained from simulation model outputs. The categories are defined for various ranges of relevant attributes which include fleet size and backhauling [Friesz et al (1981) and Bronzini (1980)]. We need comparative numerical studies to ascertain which formulations of backhauling and fleet management are the most accurate and computationally efficient.

9. Imperfect Competition

In reality, few freight markets can be described as perfectly competitive. Most are oligopolistic and regulated in significant ways. As a consequence, the assumptions of perfect competition employed by some freight models are highly questionable. Yet, the theory of mathematical games presently only allows us build numerically tractable large scale models of network equilibria which correspond to pure non-cooperation or full collusion This circumstance severely limits the realism of strategic freight network models and is an important research frontier. Although there is a rich economics literature on different forms of freight competition and organization of freight firms [see for example Friesz and Bernstein (1991)], virtually none of this theoretical work has been made operational. One exception is an effort by Argonne National Laboratory (1985) to introduce endogenous freight rate setting in FNEM; the models and software associated with this effort have not been applied in any real world setting and remain essentially prototypes.

10. Validation

It is important to be clear about the fact that strategic freight network models are fundamentally predictive in nature. As such they need to be validated; that is, we need to see how well these models replicate observed freight flows before they are used for strategy setting and policy evaluation. To date only FNEM has been vetted by a thorough validation effort that includes goodness of fit statistics [Friesz et al (1985)]. It is notable that FNEM predicts flows very well for certain classes of commodities and rather poorly for other classes, suggesting that specification errors may exist and underscoring the poor quality of available calibration data. Much greater effort and resources must be expended to calibrate and validate each of the extant freight network planning models described above. Only when more validation efforts have been completed and reported will we know the value of adding (or deleting) various model features.

11. Revenue Management

Revenue management (RM), sometimes referred to as revenue optimization, has been in existence as long as the concept of money. While RM was done on an intuitive level for centuries, it has become a science within the latest century. The idea of RM, in general, is to improve revenues of the firms by efficiently managing the pricing and allocation of service capacity. The growth of revenue management was boosted by the deregulation of U.S. domestic and international airlines in the late 1970's. Airlines, car rentals and hotels typically exercise quantity based RM techniques by controlling the number of resources to be sold during the booking period at a fixed, pre-specified price. On the other hand, retailers use price based RM techniques by using price as an instrument to control demands over the selling period. The first comprehensive book on this subject by Talluri and van Ryzin (2004) provides good detailed information on price and quantity based RM techniques. Today, RM is widely used in certain classes of business and its applications are ever expanding. RM is experiencing both a breadth and depth growth as more and more industries such as car rental, hotels, and retail are employing it to a greater extent. McGill and van Ryzin (1999) provide a detailed survey of the research advancements in this field since 1970.

Network RM arises in airline, railway, hotel and cruise-line revenue management where customers buy service or products, which are bundles of resources, under various terms and conditions. Each product will use a subset of resources which gives rise to a network topology. Overbooking is one of the oldest and most important RM tactics where firms accept more reservations than their physical capacities to serve to hedge against cancellations and no-shows. Most of the past works on overbooking models have considered a single product/service type, where as Karaesmen and van Ryzin (2004) consider an overbooking model with multiple substitutable inventory and production classes where they determine the overbooking limits for the reservation classes taking into account substitution options.

12. Dynamic Extensions

All of the models reported above are essentially static or quasi-static in nature. Very clearly, an important next step for the models we have reviewed here is to make them dynamic. This will require that consideration be given to both dynamic disequilibrium models and to dynamic equilibrium models, leading us into the world of optimal control models for freight system. This step will likely involve integrating freight models with the theory of economic growth and with so-called non-tatonnement models from microeconomic theory. Preliminary steps in this direction have been taken by Friesz and Holguin-Veras (2005). We proceed by defining three classes of spatially separated firms: sellers, transporters and receivers. The sellers are those firms who produce goods that are sold to receivers. The transporters are the firms that are contracted to deliver the goods from the sellers to the receivers. These interactions take place on a network formed by the relationships among the different classes of firms. We assume that both the sellers and transporters are Cournot-Nash agents in a network economy and they are profit optimizers with pricing power. Each seller of commodities competes with other sellers and each transporter competes with other transporters. However, the sellers and transporters do not compete with each other.

The receivers' input factor demands are fixed for the time scale of one abstract " day" (which might be several real days), so the sellers have to compete for that demand which depends on delivered factor prices which in turn depend on transportation prices (tariffs) which are also competitively set. Likewise, each transporter's demand function depends on its own price as well as its competitors' prices. The demand for the transporters is derived from the spatial separation of supply and consumption activities. Similar to the sellers, the transporters must compete with each other to procure this demand for services. Receivers are those entities who desire delivery of goods. In particular, receivers dictate the volume of the delivery and the desired time of the delivery of the goods. Demand for the goods and desired time of delivery are taken exogenous to this model as they are considered fixed for the time scale of the model. Our model considers homogeneous goods only; however, this model may be extended to a more general model with nonhomogeneous goods.

The extremal problem for each seller and transporter is formulated as a continuous time optimal control problem that depends on the strategies of the other firms. This leads to a set of coupled optimal control problems that describe the game. This set of continuous optimal control problems is then discretized to obtain a set of coupled mathematical programs. Using the Karush-Kuhn-Tucker (KKT) conditions for each mathematical program, the problem can be recast as a nonlinear complementarity problem (NCP).

Using the notation from the Appendix and discretizing time, the Cournot-Nash noncooperative game among the agents takes the form of a nonlinear complementarity problem. The complete non-linear complementarity problem (NCP) describing the Cournot-Cournot game is created by concatenating complementarity conditions that were obtained through the analysis of the seller and transporter models.

$$G(z) = \begin{pmatrix} G_s(z^s) \\ G_c(z^c) \end{pmatrix} \perp z \begin{pmatrix} z^s \\ z^c \end{pmatrix}$$

where

$$
0 \le \begin{pmatrix} \Theta_s\left(\overline{\psi}^s; \beta_{i,t}^{r,+}, \beta_{i,t}^{r,-}, \gamma_{i,t}^{r,s,+}, \gamma_{i,t}^{r,s,-}, \zeta_t^{s,+}, \eta_t^{+}\right) \\ I_{j,t}^{s} \\ -\sum_{s\in\Sigma} d_{i,t}^{r,s}\left(p_t\right) + D_{i,t}^{r} \\ \sum_{s\in\Sigma} d_{i,t}^{r,s}\left(p_t\right) - D_{i,t}^{r} \\ -d_{i,t}^{r,s}\left(p_t\right) + \sum_{j\in \mathrm{N}_s} v_{i,j,t}^{r,s} \\ d_{i,t}^{r,s}\left(p_t\right) - \sum_{j\in \mathrm{N}_s} v_{i,j,t}^{r,s} \\ v_t^{r,s} \\ p_{\max}^{r,s} - \overline{p}_t^{r,s} - p_{\min}^{r,s} \\ \overline{p}_t^{r,s} \\ q_{\max}^{s} - q_t^{s} \\ q_t^{s} \end{pmatrix} = G_s\left(z^s\right) \perp z^s = \begin{pmatrix} \overline{\psi}^s \\ \xi_{j,t}^{s} \\ \beta_{i,t}^{r,+} \\ \beta_{i,t}^{r,-} \\ \gamma_{i,t}^{r,s,+} \\ \gamma_{i,t}^{r,s,-} \\ \delta_t^{r,s} \\ \zeta_t^{r,s,+} \\ \zeta_t^{r,s,-} \\ \eta_t^{s,+} \\ \eta_t^{s,-} \end{pmatrix} \ge 0
$$

and

$$
0 \le \begin{pmatrix} \Theta_c\left(\overline{\psi}^c; \phi_t^{c,s}, \vartheta^{c,s,+}, \vartheta^{c,s,-}, \lambda_{i,j,t}^{s,+}, \rho_{i,j,m,t}^{c,r,s}, \nu_t^{c,s,-}\right) \\ x_t^{c,s} \\ -x_N^{c,s} \\ x_N^{c,s} \\ -\sum_{c\in \mathrm{X}} u_{i,j,t}^{c,r,s}\left(\pi_t\right) + v_{i,j,t}^{r,s} \\ \sum_{c\in \mathrm{X}} u_{i,j,t}^{c,r,s}\left(\pi_t\right) - v_{i,j,t}^{r,s} \\ \overline{\pi}_t^{c,r,s} \\ -\overline{\pi}_t^{c,r,s} + \pi_{\min}^{c,r,s} + \pi_{\max}^{c,r,s} \\ \left(\rho_{m,t}^{c,r,s}\right) \end{pmatrix} = G_c\left(z^c\right) \perp z^c = \begin{pmatrix} \overline{\psi}^c \\ \phi_t^{c,s} \\ \vartheta^{c,s,+} \\ \vartheta^{c,s,-} \\ \lambda_{i,j,t}^{r,s,+} \\ \lambda_{i,j,t}^{r,s,-} \\ \nu_t^{c,r,s,+} \\ \nu_t^{c,r,s,-} \\ \mu_{m,t}^{c,r,s} \end{pmatrix} \ge 0
$$

Such a complementarity problem can be solved using a commercial solver such as PATH (Ferris and Munsun, 1998) via a modeling language such as GAMS. Because both the seller and transporter models are linear in the constraints, we may use the sequential linearization option in PATH to solve this complementarity problem and be guaranteed convergence.

13. Illustrative Numerical Example

Friesz and Holguin-Veras (2005) report a small example problem with the following parameters:

Parameter	Range	Parameter	Range
$a_1^{r,s}$	$47-57$	$b_1^{c,s}$	$0.05-0.15$
$a_2^{r,s}$	$0.45-0.525$	$b_2^{c,s}$	$0.02-0.12$
$a_3^{r,s,g}$	$0.025-0.075$	$\omega_1^{c,r,s}$	$9-10$
e_j^s	$0.45-0.55$	$\omega_2^{c,r,s}$	$0.45-0.525$
$f_{1,j}^s$	$0.25-0.35$	$\omega_3^{c,g,r,s}$	$0.1-0.15$
$f_{2,j}^s$	$0.05-0.15$	$I_{j,0}^s$	$35-75$
$f_{3,j}^s$	0	D_t^r	$50-70$
$l_{1,m}^{c,r,s}$	$15-15.5$	Δ	0.5
$l_{2,m}^{c,r,s}$	$0.3-0.4$	N	21
$p_{\min}$	0	$\pi_{\min}$	0
$p_{\max}$	100	$\pi_{\max}$	75
$q_{\max}$	100		

The following graphics summarize same of their numerical findings:

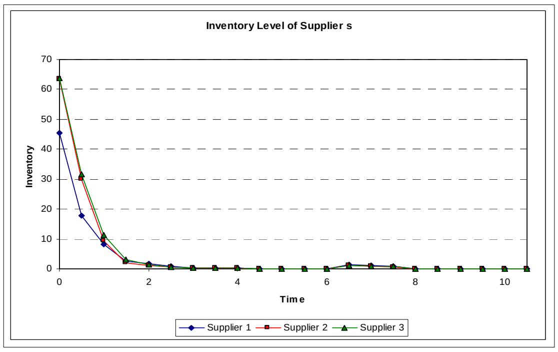

Figure 1: Supplier Inventory

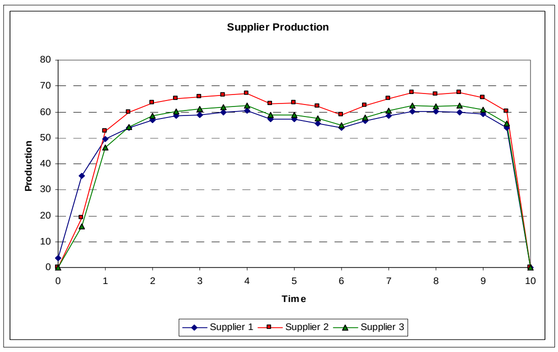

Figure 2: Supplier Production

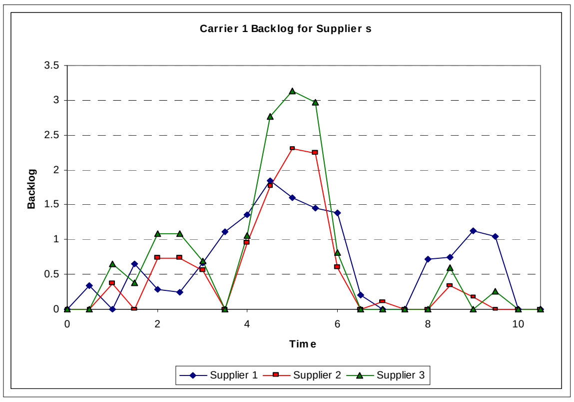

Figure 3: Carrier 1 Backlog

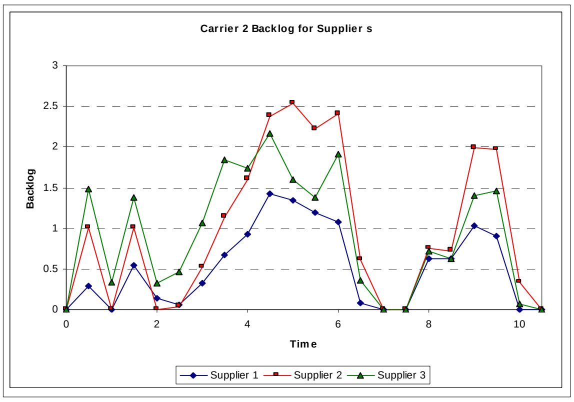

Figure 4: Carrier 2 Backlog

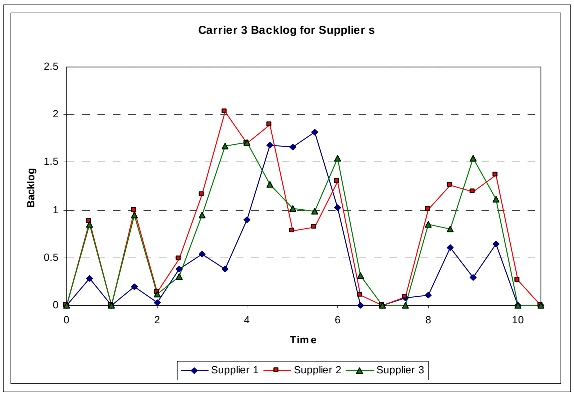

Figure 5: Carrier 3 Backlog

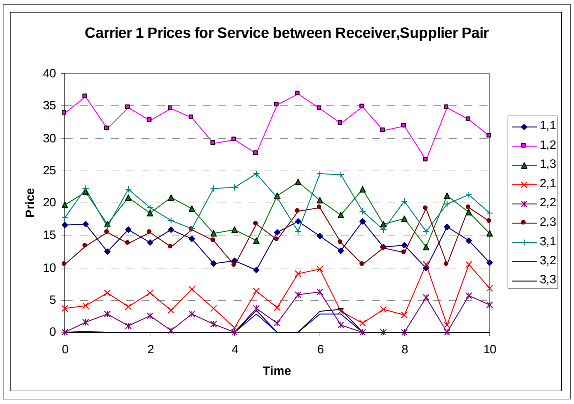

Figure 6: Carrier 1 Prices

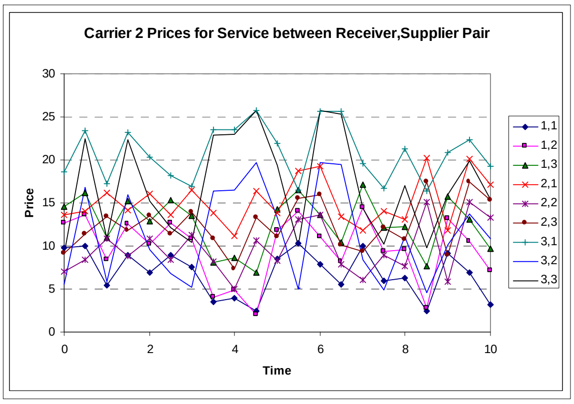

Figure 7: Carrier 2 Prices

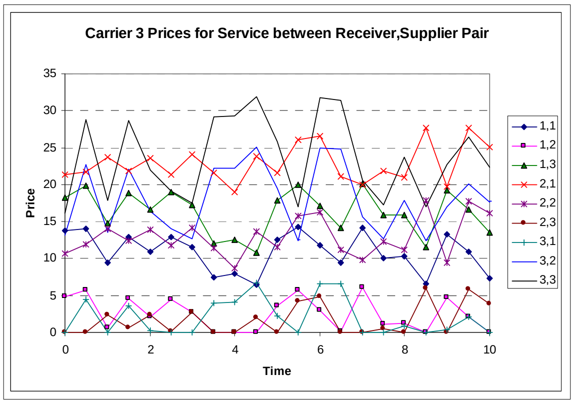

Figure 8: Carrier 3 Prices

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Appendix: Notation

1. Parameters

- Σ : set of sellers
- X: set of transporters
- P : set of receivers
- N_s: set of nodes where seller s is located
- N_r: set of nodes where receiver r is located
- M : set of transportation modes available to each transporter
- t_0: start of the planning horizon
- t_1: end of the planning horizon
- $t \in [t_0, t_1]$: clock time
- $D_i^r(t)$: amount of goods desired by receiver r at its facility $i \in \mathrm{N}_r$ at time t
- $I_{j,0}^s$: starting inventory held by seller s at its location $j \in \mathrm{N}_s$
- $p_{\min}^s$: lower limit of price for firm s
- $p_{\max}^s$: upper limit of price for firm s
- $q_{j,\max}^s$: upper limit of production at node $j \in \mathrm{N}_s$ of seller s
- $\pi_{\min}^c$: lower limit of price for transporter c
- $\pi_{\max}^c$: upper limit of price for transporter c

2. Variables

- $p_i^{r,s}(t)$: delivered price charged by the seller s charged to the receiver r located at node $i \in \mathrm{N}_r$
- $q_j^s(t)$: production rate of seller s at location $j \in \mathrm{N}_s$
- $v_{i,j}^{r,s}(t)$: flow of goods sent by seller s from its location $j \in \mathrm{N}_s$ for delivery at receiver r at its location $i \in \mathrm{N}_r$
- $I_j^s(t)$: inventory level of seller s at location $j \in \mathrm{N}_s$ at time t
- $d_i^{r,s}(p,t)$: demand of goods by receiver r located at location $i \in \mathrm{N}_r$ fulfilled by seller s
- $\Psi_j^s\left(I_j^s(t)\right)$: inventory holding cost of seller s at location $j \in \mathrm{N}_s$ when inventory level is $I_j^s(t)$
- $\theta_j^s(q_j^s(t))$: unit production cost of seller s located at node $j \in \mathrm{N}_s$ when production level is $q_j^s(t)$
- $\pi_{i,j}^{c,r,s}(t)$: price charged by transporter c for delivering goods from location $i \in \mathrm{N}_s$ to the location $j \in \mathrm{N}_r$ at time t
- $\rho_{i,j,m}^{c,r,s}(t)$: flow of goods delivered by transporter c at time t to the receiver r at

its location $i \in \mathrm{N}_r$ using the transportation mode m shipped by the seller s from location $j \in \mathrm{N}_s$

- $x^{c,s}(t)$: total backlogged service of transporter c for seller s at time t
- $u_{i,j}^{c,r,s}(\pi(t), x(t))$: amount of demand of service produced by transporter c to deliver goods from location $i \in \mathrm{N}_s$ to the location $j \in \mathrm{N}_r$ at time t
- $w^{c,s}\left(x^{c,s}(t)\right)$: cost of lost goodwill from seller s for transporter c due to the level of backlogged shipments at time t.
- $k_m^c(\rho^c(t), t)$: unit transportation cost of transporter c while using mode m transferring ρ units of goods at time t