A Nash-bargaining Model for Trading of Electricity Between Aggregations of Peers

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***Abstract*—In the last several years, the growth in household solar generation and the lack of success of the feed-in-tariff programs have led to the rise of peer-to-peer (P2P) energy trading schemes among prosumers. However, a change that has started more recently is the growth of smart homes and businesses, of which loads are IoT controlled and are supported by advanced metering infrastructure (AMI). This has created a new opportunity for smart homes and businesses to form aggregations (coalitions) and participate in cooperative load management and energy trading. Unlike energy trading among individual prosumers in most P2P networks, a new trading opportunity that is emerging is between aggregations of peers of smart homes and businesses and electric vehicles (EVs). In this paper, we consider one such trading scenario between two aggregations, of which one has smart homes and businesses with load consuming entities (not prosumers), and the other has EVs only. The aggregation with smart homes and businesses derive cost reduction through optimal load scheduling based on load preferences, market-based pricing of electricity, and opportunity to trade (buy) energy from the aggregation with EVs. Whereas the aggregation of EVs optimally schedules charging to meet EV needs and uses stored energy to trade (sell). A generalized Nash bargaining model is developed for obtaining optimal trading strategies in the form of plain or swing option contracts. A sample numerical problem scenario is used to show that suitable contracts can be derived that allow aggregations of peers to mutually benefit from energy trading. It is shown that there exist numerous alternative optimal solutions to the Nash bargaining problem. The solutions comprise different combinations of strike price and option value, for all of which savings to the parties remain constant. For plain option, with a contract quantity of 1 MWh, the total savings generated is equivalent to the average price of 1.62 MWh of electricity. Interactions among contract parameters (such as strike price, option value, and option quantity) and the relative market power of the aggregations are also examined.**

Keywords: Aggregation of peers, peer-to-peer energy trading, option contract, Nash bargaining solution

I. Introduction

Increasing adoption of internet-enabled load control and the move towards real-time pricing of electricity are creating opportunities for end-use consumers (peers) to form aggregations [17]. These aggregations are intended to reduce cost of the participating peers by optimally scheduling their loads and by trading energy. As these aggregations pay time varying market prices set by the system operator, strategies for load scheduling and energy trading depend on the hourly market price variations, consumption preferences of the participants, and any other prevailing network constraints. An aggregation can comprise a variety of participating peers including smart households and businesses, collection of electric vehicles, battery banks, wind mills, and solar farms. Depending on their composition, aggregations may have different characteristics, such as load consuming only, load consuming with storage, storage only, load consuming with storage and generation, among others. These characteristics in turn guide their cost minimization and trading approaches. For example, an aggregation of only load consuming entities (ALCEs) will optimally schedule its deferrable loads when the market prices are lower. Also, to reduce the risk from spikes in market prices, ALCEs may enter into contracts for electricity trading with aggregations having storage capacities. Such trading contracts can be drafted as option contracts. In this paper, we consider energy trading between two aggregations, one comprising smart load consuming entities only (ALCE) and the other is an aggregation of EVs (AEV).

For the energy trading contract to be fair to both, we develop a Nash bargaining model using the operational strategies of both ALCE and AEV. We limit our model to two aggregations in order to get better insight into their interactions. However, our model can be used to obtain optimal contracts for the general case in which an ALCE may be interested to bargain with more than one AEVs or vice versa. This can be achieved by considering a finite sequence of bilateral bargaining sessions in which two players bargain for a partial agreement. The effectiveness of this sequential approach was studied in [32] and it was shown that for any given bargaining strategy the solution obtained is a subgame perfect equilibrium. The Nash bargaining model presented here can be used as a bargaining strategy in the sequential approach.

We first develop cost minimizing operational models for ALCE and AEV. We consider that both aggregations buy needed electricity from the grid (paying the market price) and trade electricity using option contracts. The operational models are then used to develop a generalized Nash bargaining solution (GNBS) approach for designing option contracts. The approach also incorporates relative market powers of the aggregations. The ALCE is considered to have two types of loads, fixed and deferrable, of which only the deferrable load schedules are optimized. Deferrable loads in turn are considered to be of two types: shiftable and adjustable. Shiftable loads are scheduled at any time within their respective predefined time windows. Whereas for adjustable loads, both the time of operation and the level of power consumption can be altered while satisfying their total power requirement within a predefined time window. The EVs in the AEV are considered heterogeneous with different battery sizes, arrival and departure times, maximum and minimum allowable state of charge (SOC), and maximum rates of charge/discharge. The price of electricity is consider to be available to ALCE and AEV at the start of each time interval of a day (say, an hour). Price is assumed to vary with time based on demand, supply, and network conditions and is considered as an exogenous input to our model.

It is assumed that ALCE and AEV predict the time-varying electricity prices and optimize their daily operations. The ALCE's operational model is formulated as a mixed integer program that schedules all loads of the day to minimize cost. Electricity consumed by the ALCE loads is drawn from both the network (based on the price) as well as from the AEV (based on the contract). The AEV consumes electricity from the network to meet the SOC requirements of the EV owners and also to store excess energy in the batteries to trade with ALCE. The operational model of AEV is formulated as a mixed integer program. It considers both cost and revenue. The cost is the amount paid to the grid for electricity, and the revenue comprises the payments it receives from the EV owners and the ALCE. The AEV is also considered to pay overcharge and undercharge penalties to the EV owners. These penalties are incurred if at the time of departure from the parking-lot, the SOC of an EV battery is either above or below the charge level desired by the EV owner. A schematic of the interactions between the ALCE, AEV, and the network is presented in Figure 1. We consider both plain and swing option contracts between ALCE and AEV. An option contract is defined by its time window, strike price, option quantity, and option value. The use of financial instruments like option contracts is common in trading electricity between market constituents (see for example [29, 34]). However, the design of such instruments in the context of various forms of energy trading among peers is still a growing research area.

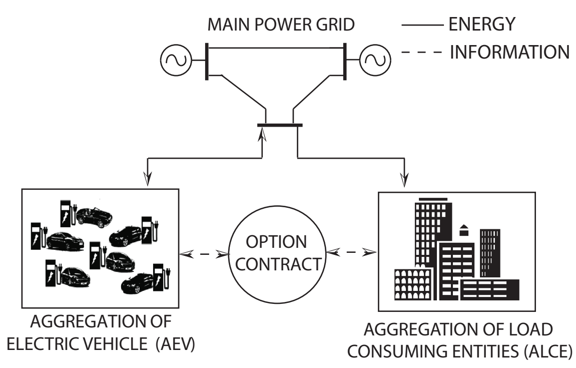

Fig. 1: Energy and information flow between ALCE and AEV

A. Contributions of this paper

Peer-to-peer trading in a power network is a well examined topic, where peers are commonly assumed to be prosumers (households with generation and storage capabilities). However, growth of IoT and advanced metering infrastructure has lead to the rise of a set of newly empowered peers, namely smart homes and businesses and EVs. In our previous work [17], we have shown how these peers are now able to form coalitions and reduce their operational cost by engaging in demand response, in markets with hourly price variations and price spikes. In this paper, we demonstrate that, in markets with price variations and spikes, aggregations formed by these peers can also effectively engage in energy trading and derive financial benefits that are fair to all. This is demonstrated by developing a generalized Nash bargaining solution (GNBS) model for obtaining trading contract between two aggregations and implementing it on a sample problem. The operational optimization models for the aggregations that we have formulated as input for the GNBS model are also novel in how they schedule load consumption and plan for energy sharing. Our contributions also include a computational strategy where instead of solving the GNBS model as one single nonlinear mixed integer program, we separately solve the operational models and use their results to solve GNBS model and obtain the contract parameters.

B. Related literature

Peer-to-peer trading in power markets is an ongoing area of research. Excellent reviews of the various aspects of prosumer-based peer-to-peer energy trading can be found in [34, 35]. However, effective mechanisms to schedule load and trade electricity in P2P networks are still in their early stages of development [16, 35]. The concept of nonprosumer peers forming coalitions to trade energy among themselves, as considered in this paper, is relatively new. One such mechanism has recently been modeled in [17], which demonstrates how an aggregation of peers comprising smart homes and businesses and EVs can generate cost savings and share it in a fair and equitable manner among the participating peers. The aforementioned paper considers a number of practical features of the power market, including modeling of price and demand data, network constraints, day ahead commitment, and real time price spikes. In what follows, we review some of the relevant literature in three categories: P2P trading mechanisms, use of options contract in power markets, and EV based P2P trading.

Bilateral contract networks as a new scalable market design for P2P energy trading is presented in [20]. It considers both real-time and forward markets for energy trading contracts between generators and consumers with fixed and deferrable loads and/or renewable sources. They show that utility-maximizing preferences for these contracts satisfy conditions essential for establishing the existence of a stable outcome from which agents do not wish to mutually deviate. Forward and real-time energy contracts were also considered in [40]. A coalition formation game framework is proposed in [33], to help prosumers decide whether or not it is profitable to bid its battery capacity in the P2P market for energy trading. The proposed mechanism allows prosumers to compare the benefit from participating in the P2P trading with and without using its battery, allowing the prosumer to form suitable social coalition groups. A discrete-time double-sided auction model to enable energy trading between prosumers in near real-time and forward markets was proposed in [8]. This device-oriented bidding strategy considers the physical characteristics and the technical limitations of each device type, such as EVs or heatpumps, and use them maximize the system reliability. It was shown that prosumers can reduce their cost on average by 23% using the proposed approach. The main challenges towards real world implementations of P2P mechanisms are to scalability and asynchronicity of the negotiation process. The work in [19] analyzed these challenges, by comparing distributed community-based market approaches to decentralized and distributed versions of P2P electricity markets. The computational properties of distributed and decentralized algorithms (ADMM and RCI) for market clearing were also assessed. It was shown that community-based distributed approaches are faster and more robust. Centralized and distributed P2P markets are also contrasted in [14]. The paper characterizes the solution of the P2P market as a variational equilibrium problem (without price arbitrage) and shows that the solution corresponds to that of social welfare optimization.

A recent paper [11] proposes a Nash-type non-cooperative game model between residential and commercial prosumers to guarantee fairness in P2P trading. The proposed model considers commonly used energy supply technologies, three categories of storage, and various demand-side management measures. Trading of both electricity and heating are considered. It was shown that trade-off can be achieved between a totally fair trade (sacrificing the total cost saving) and a minimal-cost trade with an unfair benefit allocation. In [15] a P2P trading mechanism is proposed to encourage prosumers to trade electricity in a community microgrid. The decision making process for the participating prosumers is modeled using a game theoretic approach and the Shapley value.

A bargaining-based energy trading model among interconnected microgrids is presented in [36]. Microgrids with surplus power generations trade with other microgrids in need of power for mutual benefit. The amount of energy to trade as well as the payments are decided cooperatively through a decentralized algorithm, which solve the bargaining problem. The authors showed that the total cost reduction is 22% when compared with the case of no trading. The work in [37] also studied the energy trading problem among interconnected microgrids, but using Nash bargaining theory. In the proposed framework, microgrids with excessive renewable generations can trade with other microgrids for profit. A distributed solution method is also proposed. The main goal of the paper is to design an incentive mechanism using Nash bargaining theory to encourage proactive energy trading and fair distribution of the economical benefits. The bargaining problem is solved by decomposing it into two sequential problems.

Choice of option contract for trading of electricity in the day-ahead market is studied in [30]. It is shown that the power producers participating in both day-ahead and option markets can get a higher share of the profit than those who only participate in the day-ahead market. A multi-stage stochastic model is proposed in [27] to determine optimal option and forward contracts for risk-averse producers. It is shown that option contracts can reduce price and availability risks in power markets. Real options approach is used in [9] to evaluate the economic value of demand response programs (DRPs). A probabilistic model for a real option contract between an aggregator and its customers can be found in [29]. The model determines the option value i.e., the incentive paid to the customers by the aggregator, for the right to engage in demand response by shifting loads. Other examples of option contract for electricity trading can be found in the literature, for example, between generators and SO [3], demand-side customers and SO [25], and among microgrids [24].

Although energy storage using stand-alone batteries is still quite expensive, impending growth of EVs may soon offer an alternative. It is estimated that the number of EVs in the U.S. will grow to 145 million by 2030 [38]. Considering an average battery capacity of 70 kWh, these vehicles have the potential to optimally store and share up to 1015 GWh per day. There is a significant body of literature on optimal operation of EVs i.e., optimal charging and discharging for trading (arbitrage). An auction-based game theoretic approach for optimal charging of a group of EVs over a finite horizon is examined in [42]. In [1], a stochastic programming methodology is developed to maximize aggregator's profit by optimally scheduling charging of EVs under varying market prices. Auction games for P2P energy trading using EVs in smartgrids is explored in [12]. A double-auction based noncooperative game approach in [28] examines how groups of PHEVs can benefit by selling a part of their stored energy back to the power market. The work in [2] examines P2P trading of electricity between two sets of EVs resulting in a significant reduction of the impact of charging process on the power network during business hours. It is shown that the trading also greatly reduces the energy cost paid by the EV owners.

The remainder of this paper is organized as follows. Section II presents mixed integer linear programs to obtain optimal operational strategies for ALCE and AEV for both plain and swing option contracts. In Section III, the GNBS model is presented and its solution approach is discussed. A case study in Section IV developed using price and demand data from PJM market (in the U.S.) demonstrates the efficacy of the GNBS model. Section V presents the concluding remarks.

II. Operational Models for ALCE and AEV with option contract

In this section, we develop separate operational models for ALCE and AEV considering two different types of call option contracts (plain and swing) for bilateral trading of electricity.

A. Notation

Parameters:

- α: Relative market power of the ALCE
- δ_{bT_b}: Minimum required state of charge at time of departure of EV b
- ρ_{bT_b}: Desired state of charge at time of departure
- f_{ijt}: Magnitude of the fixed load (i,j) during time interval t
- g: Price paid by the EV owners for charging
- K: Strike price
- P^+, P^-: Charge and discharge rate of the EVs, respectively

- ϕ_b: Maximum battery capacity of EV b
- Π_t: Market price of electricity
- Q: option quantity for plain option contract
- $\underline{Q_t}, \overline{Q_t}$: Lower and upper bounds for energy purchase at time t for swing option contract
- $\underline{Q}, \overline{Q}$: Lower and upper bounds for total energy purchase for swing option contract
-
- $\overline{R_{ij}}$, $\underline{R_{ij}}$: Maximum and minimum level of consumption of adjustable load (i, j), respectively
- $\underline{R_j}, \overline{R_j}$: Start and finish time intervals within which an adjustable load j can be scheduled, respectively
- $\underline{S}_{bt}, \overline{S}_{bt}$: Minimum and maximum alowabe state of charge, respectively, of EV b at time t
- s_{ij}: Consumption level per unit time of shiftable load (i, j)
- T_b: Departure time of EV b
- t_s, t_e: Start and final time intervals within which the option contract can be exercised, respectively
- σ_{ij}: Total required consumption of the adjustable load (i, j)
- τ_{ij}: Length of operation of shiftable load (i, j)
- $\underline{T_j}, \overline{T_j}$: Start and finish time intervals within which a shiftable load j can be scheduled, respectively
- μ_1: Overcharge penalty
- μ_2: Undercharge penalty
- ω_{bt}: Binary parameter equal to one if EV b is in the parking lot at time t
- V: Option value

Decision variables:

- d_{it}: Total energy consumed by LCE i at time t
- d_t: Energy bought from the grid by ALCE at time t
- p^+_{bt}, p^-_{bt}: Energy charged and discharged, respectively, from EV b at time t
- r_{ijt}: Energy consumption level of the adjustable load (i, j) during time interval t
- q_t: Energy bought by the ALCE from the AEV at time t
- $\tilde{q}_t$: AEV estimate of energy requested by ALCE
- s_{bt}: State of charge of EV b at time t
- s^1_b, s^2_b: Dummy variables to compute the overcharge and undercharge penalties, respectively
- w^+_{bt}: Binary variable equal to one if EV b is charging at time t, and zero otherwise
- w^-_{bt}: Binary variable equal to one if EV b is discharging at time t, and zero otherwise
- x_{ijt}: Binary variable indicating on/off status of the shiftable load (i, j) during time interval t
- y_{ijt}: Binary variable indicating on/off status of the adjustable load (i, j) during time interval t
- z: Binary variable equal to 1 if the contract is exercised at least once, and zero otherwise
- z_t: Binary variable equal to 1 if the option contract is exercised at time t, and zero otherwise

Sets:

- $\mathcal{A}_i$: Set of adjustable loads of LCE i
- $\mathcal{B}$: Set of EV batteries in the AEV
- $\mathcal{C}$: Set of load consuming entities
- $\mathcal{F}_i$: Set of fixed loads of LCE i Then, the total energy that LCE i consumes at a given time interval t is:
- $\mathcal{S}_i$: Set of shiftable loads of LCE i
- $\mathcal{T}$: Set of all time intervals of a day

Objective functions:

- u^{ALCE}: ALCE objective function
- u^{AEV}: AEV objective function

B. Operational Models with plain option

By participating in a plain call option, the ALCE holds the right, not the obligation, to acquire a fixed amount of electricity from AEV at a prespecified strike price during a time interval within a given time window. The ALCE pays a fee (option value) to the AEV for the right.

1) ALCE's model for plain option: We consider that the ALCE loads are of two types: fixed and deferrable loads. Schedules of fixed loads are not controlled. Deferrable loads are considered to have two subcategories, shiftable and adjustable loads. Operation of shiftable loads can be scheduled at any time within their respective time windows. Whereas, for adjustable loads, both time as well as level of power consumption can be altered, while satisfying their total power requirement during operational time windows.

Each load consuming entity (LCE) i within an ALCE (denoted by $\mathcal{C}$) has a set of shiftable loads denoted by $\mathcal{S}_i$. For an individual load $j \in \mathcal{S}_i$, the consumption level per unit time is s_{ij} and its length of operation is τ_{ij}. The start and finish time intervals within which shiftable load j can be scheduled are denoted by $\underline{T_j}, \overline{T_j} \in \mathcal{T}$, where $\mathcal{T}$ denotes the set of all time intervals of a day over which the loads are scheduled. Let x_{ijt} denote a binary variable indicating on/off status of the shiftable load j of i^{th} LCE during time interval $t \in \mathcal{T}$. Then, we can write:

$$\sum_{t=\underline{T_j}}^{\overline{T_j}} x_{ijt} = \tau_{ij}, \qquad \forall i \in \mathcal{C},\ \forall j \in \mathcal{S}_i. \tag{1}$$

We denote the set of adjustable loads within a LCE i as $\mathcal{A}_i$. The maximum (minimum) level of consumption per unit time of individual loads $j \in \mathcal{A}_i$ is denoted by $\overline{R_{ij}}$ ($\underline{R_{ij}}$) within the allowable time window $[\underline{R_j}, \overline{R_j}]$. Let y_{ijt} be a binary variable indicating on/off status of the adjustable load j of i^{th} LCE during time interval $t \in \mathcal{T}$, r_{ijt} be the energy consumption level, and σ_{ij} be the total required consumption. Then,

$$\underline{R_{ij}}\, y_{ijt} \leq r_{ijt} \leq \overline{R_{ij}} y_{ijt}, \quad \forall i \in \mathcal{C},\ \forall j \in \mathcal{A}_i,\ \forall t \in \mathcal{T}, \tag{2}$$

$$\sum_{t=\underline{R_j}}^{\overline{R_j}} r_{ijt} = \sigma_{ij}, \qquad \forall i \in \mathcal{C},\ \forall j \in \mathcal{A}_i. \tag{3}$$

Let $\mathcal{F}_i$ be the set of fixed loads and $f_{ijt} \in \mathcal{F}_i$ be the j^{th} fixed load of LCE i at time interval t. Then, the total energy that LCE i consumes at a given time interval t is:

$$d_{it} = \sum_{j \in \mathcal{F}_i} f_{ijt} + \sum_{j \in \mathcal{S}_i} s_{ij} x_{ijt} + \sum_{j \in \mathcal{A}_i} r_{ijt},\ \forall t \in \mathcal{T},\ i \in \mathcal{C}. \tag{4}$$

Thus, the energy that the ALCE must buy from the grid at a give time period is:

$$d_t = \begin{cases} \sum_{i\in\mathcal{C}} d_{it} - q_t, & t_s \le t \le t_e, \\ \sum_{i\in\mathcal{C}} d_{it}, & \text{Otherwise}, \end{cases} \quad (5)$$

where q_t is the energy bought form the AEV at time interval t within the option window defined by time intervals t_s and t_e. Recall that a plain call option can only be exercised once, and the ALCE has the right, but not the obligation, to exercise. Hence, we need to add the following constraints. Let $z_t = 1$, if energy is purchased by the ALCE from the AEV during the time interval t and 0 otherwise, and Q is the option quantity. Then we can write that

$$\sum_{t=t_s}^{t_e} z_t \le 1, \text{ and } q_t = Qz_t, \qquad \forall t_s \le t \le t_e. \quad (6)$$

Let Π_t, K and V be the market price of electricity, the option strike price, and the option value (paid once a day) respectively. The ALCE aims to minimize the total cost of its LCEs using the model below.

$$u^{\mathsf{ALCE}}(K, V, Q, \mathbf{\Pi}) =$$

$$\min \sum_{t\in\mathcal{T}} \Pi_t d_t + \sum_{t=t_s}^{t_e} K q_t + V, \quad (7)$$

s.t., (1)–(6),

$$d_t, d_{it}, q_t, r_{ijt} \ge 0, \quad \forall t \in \mathcal{T},\ \forall i \in \mathcal{C}, \forall j \in \mathcal{A}_i, \quad (8)$$

$$x_{ijt} \in \{0, 1\}, \quad \forall t \in \mathcal{T},\ \forall i \in \mathcal{C}, \forall j \in \mathcal{S}_i, \quad (9)$$

$$y_{ijt} \in \{0, 1\}, \quad \forall t \in \mathcal{T},\ \forall i \in \mathcal{C}, \forall j \in \mathcal{A}_i, \quad (10)$$

$$z_t \in \{0, 1\}, \quad \forall t \in \mathcal{T}. \quad (11)$$

2) AEV's model for plain option: Let $\mathcal{B}$ denote the set of EV batteries in the AEV. For a given time interval $t \in \mathcal{T}$, energy balance of the battery $b \in \mathcal{B}$ can be written as:

$$\phi_b s_{bt} = \phi_b s_{b,t-1} + p^+_{bt} - p^-_{bt}, \qquad \forall t \in \mathcal{T},\ \forall b \in \mathcal{B}, \quad (12)$$

where ϕ_b is the maximum capacity of the battery b, $s_{bt} \in (0, 1)$ is the state of charge of battery b at the end of time interval t, p^+_{bt} is the amount of energy that the b^{th} battery draws from the grid at time interval t, and p^-_{bt} is the amount of energy that is extracted from battery. We assume that, the state of charge of EV batteries are not allowed to be 0 nor 1, and hence the following constraint is added to the model.

$$\underline{S_{bt}} \le s_{bt} \le \overline{S_{bt}}, \forall t \in \mathcal{T},\ \forall b \in \mathcal{B}. \quad (13)$$

Note that, both $\underline{S}_{bt}$ and $\overline{S}_{bt}$ are input parameters that satisfy $0 < \underline{S}_{bt} \le \overline{S}_{bt} < 1$. Furthermore, the charging (discharging) rate of a battery have a technical upper bound, which in general is a convex and monotonically decreasing (increasing) function of the current state of charge. For simplicity, we assume the bounds to be constant. Hence, we can write that

$$0 \le p^+_{bt} \le P^+ w^+_{bt} \qquad \forall t \in \mathcal{T},\ \forall b \in \mathcal{B}, \text{ and} \quad (14)$$

$$0 \le p^-_{bt} \le P^- w^-_{bt}, \qquad \forall t \in \mathcal{T},\ \forall b \in \mathcal{B}, \quad (15)$$

where P^+ (P^-) is the charging (discharging) upper bound, and w^+_{bt} (w^-_{bt}) is 1 if battery b is charging (discharging) at time interval t, and 0 otherwise. The next constraint guarantees that the battery b is not in charging and discharging simultaneously during time interval t:

$$w^+_{bt} + w^-_{bt} \le \omega_{bt}, \qquad \forall t \in \mathcal{T},\ \forall b \in \mathcal{B}, \quad (16)$$

where ω_{bt} is a binary parameter with the value of 1 if the b^{th} battery is connected, i.e., the EV is in the parking lot, and 0 otherwise.

We assume that the EV owners are assessed a flat price g (¢$/kWh$) for charging, even though the aggregation (AEV) pays to the system operator based on time varying prices. The flat price assumption is considered to relieve EV owners of price anxiety. Note that, the value of the flat price (g (¢$/kWh$)) can always be adjusted by the aggregation to meet its objective (profit or non profit). Hence, AEV receives a revenue from each EV equal to $g(s_{bT_b} - s_{b0})\phi_b$, where s_{b0} is the initial state of charge and $T_b \in \mathcal{T}$ is the departure time of the b^{th} EV. We denote the minimum required state of charge at the time of departure of the b^{th} EV as δ_{bT_b}. Similarly, we denote the desired state of charge at the time of departure as ρ_{bT_b}. If the state of charge at the time of departure is above ρ_{bT_b}, the revenue for the surplus energy is assessed by AEV at a lower rate of $g - \mu_1$, where μ_1 is the overcharge penalty. The AEV also pays the EV owner an undercharge penalty μ_2 (¢$/kWh$) for each unit of energy below at the time of departure. To calculate the total amount of undercharge and overcharge penalties, we introduce two continuous variables as follows:

$$s_{bT_b} - \rho_{bT_b} = s^1_b - s^2_b \qquad \forall b \in \mathcal{B}, \quad (17)$$

where $s^1_b, s^2_b \ge 0$. Then the total overcharge and undercharge penalty (revenue losses) are computed as $\mu_1 \sum_{b\in\mathcal{B}} \phi_b s^1_b$ and $\mu_2 \sum_{b\in\mathcal{B}} \phi_b s^2_b$, respectively. Note that, δ_{bT_b} is not required to compute the undercharge penalty. This parameter is the minimum required state of charge at the time of departure, which is a constraint that is always met by the optimization model ($\underline{S}_{bT_b} = \delta_{bT_b}$). The undercharge penalty is calculated using ρ_{bT_b} (desired state of charge). Each EV owner discloses both ρ_{bT_b} and δ_{bT_b}. The optimization model guarantees that the state of charge at departure is at least δ_{bT_b} and it considers an undercharge penalty if the state of charge at departure is below ρ_{bT_b}. The option related constraints are discussed next.

The following constraint is introduced to account for the power that AEV commits to ALCE in the option window:

$$\sum_{b\in\mathcal{B}} p^-_{bt} = \tilde{q}_t(\mathbf{\Pi}, K), \qquad t_s \le t \le t_e. \quad (18)$$

The option quantity must be supplied using the stored power if the option is exercised. However, the AEV does not know the decision making process of the ALCE, therefore it must estimate q_t. We denote as estimate of the vector $\boldsymbol{q}$ given the random price of electricity $\mathbf{\Pi}$ as $\tilde{\boldsymbol{q}}(\mathbf{\Pi}, K)$, or in component-wise form, $\tilde{q}_t(\mathbf{\Pi}, K)$. Hence the AEV's model for the plain option is formulated as follows.

$$u^{\mathsf{AEV}}(K, V, Q, \mathbf{\Pi}) =$$

$$\min \sum_{t\in\mathcal{T}}\sum_{b\in\mathcal{B}}\Pi_t p^+_{bt} + \mu_1\sum_{b\in\mathcal{B}}\phi_b s^1_b + \mu_2\sum_{b\in\mathcal{B}}\phi_b s^2_b$$
$$- g(s_{bT_b} - s_{b0})\phi_b - K\sum_{t=t_s}^{t_e}\tilde{q}_t(\mathbf{\Pi},K) - V, \quad (19)$$

s.t., (12)–(18),

$$p^+_{bt}, p^-_{bt}, s_{bt}, s^1_b, s^2_b \geq 0, \qquad \forall t\in\mathcal{T},\ \forall b\in\mathcal{B}, \quad (20)$$
$$w^+_{bt}, w^-_{bt} \in \{0,1\}, \qquad \forall t\in\mathcal{T},\ \forall b\in\mathcal{B}, \quad (21)$$

where

$$\tilde{q}_t(\mathbf{\Pi},K) = Q\tilde{z}_t(\mathbf{\Pi},K), \quad t_s \leq t \leq t_e, \quad (22)$$

$$\tilde{\boldsymbol{z}}(\mathbf{\Pi},K) = \arg\max_{\tilde{\boldsymbol{z}}} \sum_{t=t_s}^{t_e}(\Pi_t - K)\tilde{z}_t, \quad (23)$$

$$\text{s.t.}, \sum_{t=t_s}^{t_e}\tilde{z}_t \leq 1, \quad (24)$$

$$\tilde{z}_t \in \{0,1\}, \quad t_s \leq t \leq t_e. \quad (25)$$

For simplicity of notation, we define $\boldsymbol{C} = (K,V,Q)$ for the plain option, and write the disutility functions as $u^{\mathsf{ALCE}}(\boldsymbol{C},\mathbf{\Pi})$ and $u^{\mathsf{AEV}}(\boldsymbol{C},\mathbf{\Pi})$. Note that these disutilities are random variables.

C. Operational Models with swing option

Swing call option for electricity, has the following key characteristics: 1) purchase of contract quantity can be divided among one or more time intervals within the window, 2) purchase quantities may have time dependent bounds, 3) the strike price may either be fixed or vary for different time intervals, and 4) the ramp up/down rates of quantity purchased may also be bounded. In the swing option model considered here, we only consider characteristics 1 and 2.

1) ALCE's model for swing option: In addition to constraints (1)–(5) and (8)–(11) in the ALCE's model for plain option, we need a few other constraints as described below. In a swing contract, if ALCE exercises the option, the energy bought at each interval as well as the total quantity bought over the contract window must satisfy

$$\underline{Q_t}z_t \leq q_t \leq \overline{Q_t}z_t, \quad t_s \leq t \leq t_e, \text{ and} \quad (26)$$

$$\underline{Q}\,z \leq \sum_{t=t_s}^{t_e} q_t \leq \overline{Q}\,z, \quad (27)$$

where $\underline{Q_t}(\overline{Q_t})$ and $\underline{Q}(\overline{Q})$ are the lower (upper) bounds for energy purchase during a time interval t and over the total contract window, respectively. Also, $z_t = 1$ if the option is exercised at time interval t and 0 otherwise, and $z = 1$ if the option is exercised at least once within the window. Therefore the relationship between z_t and z is given as

$$\sum_{t=t_s}^{t_e} z_t \leq (t_e - t_s + 1)z. \quad (28)$$

Then the ALCE model for a swing call option can be given as

$u^{\mathsf{ALCE}}(K,V,\underline{Q_t},\overline{Q_t},\underline{Q},\overline{Q},\mathbf{\Pi}) =$

$$\min \sum_{t\in\mathcal{T}}\Pi_t d_t + \sum_{t=t_s}^{t_e} Kq_t + V, \quad (29)$$

$$\text{s.t.}, (1)-(5), (8)-(11), (26)-(28), z\in\{0,1\}. \quad (30)$$

2) AEV's model for swing option: Since the AEV is subjected to the value of q_t chosen by the ALCE, the same general model proposed for plain option in (19) applies for the second stage problem in a swing contract. However, the first stage must consider the additional contract parameters. Then we have that

$u^{\mathsf{AEV}}(K,V,\underline{Q_t},\overline{Q_t},\underline{Q},\overline{Q},\mathbf{\Pi}) =$

$$\min \sum_{t\in\mathcal{T}}\sum_{b\in\mathcal{B}}\Pi_t p^+_{bt} + \mu_1\sum_{b\in\mathcal{B}}\phi_b s^1_b + \mu_2\sum_{b\in\mathcal{B}}\phi_b s^2_b$$
$$- g(s_{bT_b} - s_{b0})\phi_b - K\sum_{t=t_s}^{t_e}\tilde{q}_t(\mathbf{\Pi},K) - V, \quad (31)$$

s.t., (12)–(18), (20), (21),

where

$\tilde{\boldsymbol{q}}(\mathbf{\Pi},K) =$

$$\arg\max_{\tilde{\boldsymbol{q}}} \sum_{t=t_s}^{t_e}(\Pi_t - K)\tilde{q}_t, \quad (32)$$

$$\text{s.t.}, \underline{Q_t}\tilde{z}_t \leq \tilde{q}_t \leq \overline{Q_t}\tilde{z}_t, \quad t_s \leq t \leq t_e, \quad (33)$$

$$\underline{Q}\,\tilde{z} \leq \sum_{t=t_s}^{t_e}\tilde{q}_t \leq \overline{Q}\,\tilde{z}, \quad (34)$$

$$\sum_{t=t_s}^{t_e}\tilde{z}_t \leq (t_e - t_s + 1)\tilde{z}, \quad (35)$$

$$\tilde{z}_t \in \{0,1\} \quad t_s \leq t \leq t_e, \quad (36)$$

$$\tilde{z} \in \{0,1\}. \quad (37)$$

For simplicity of notation, we define $\boldsymbol{C'} = (K,V,\underline{Q_t},\overline{Q_t},\underline{Q},\overline{Q})$ for the swing option, and write the disutility functions as $u^{\mathsf{ALCE}}(\boldsymbol{C'},\mathbf{\Pi})$ and $u^{\mathsf{AEV}}(\boldsymbol{C'},\mathbf{\Pi})$. This two-stage formulation was adapted from [13].

III. Call option contract design

In this section, we present the model to obtain the optimal strike price and option value when all the other option parameters are given for the plain and swing option contracts. We use the Nash's approach to the bargaining problem to obtain a fair option contract for both ALCE and AEV while considering their relative market power.

The objectives of ALCE and AEV are to minimize their disutility by establishing an optimal option contract. However, since the objectives are in conflict, a contract that simultaneously minimizes their costs does not exist. In such a scenario, the aggregators may cooperatively bargain with each other to find the most appropriate contract. The bargaining problem can be formalized as follows [41]. Let $n = 1,2,...,N$ be the set of players, and S be a closed and convex subset of $\mathbb{R}^N$ that represents the set of feasible payoff (cost) allocations that the players can get if they cooperate. Let u^k_0 denote the minimal (maximal) payoff (cost) that the k^{th} player would expect without cooperation. The vector $(\mathrm{S}, u_0^1, ..., u_0^N)$ is called a N-person bargaining problem. We chose the Nash bargaining solution (NBS) to address the two person (ALCE and AEV) bargaining problem. NBS is known to be invariant, Pareto optimal, independent of irrelevant alternatives, and symmetrical. In a bilateral negotiation, it is reasonable to expect that the player with higher market power will have a larger share of the benefits than the weaker player. To incorporate the market power, we use the generalized Nash bargaining solution (GNBS) approach [18]. The GNBS for the plain option contract can be formulated as:

$$\begin{aligned} \max\ & \Big(\mathbb{E}[u^{\mathsf{ALCE}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{ALCE}}(\boldsymbol{C},\mathbf{\Pi})]\Big)^{\alpha} \\ & \Big(\mathbb{E}[u^{\mathsf{AEV}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{AEV}}(\boldsymbol{C},\mathbf{\Pi})]\Big)^{1-\alpha} \\ \text{s.t., }& (1) - (25), \end{aligned} \tag{38}$$

where $\alpha \in (0,1)$ is an indicator of ALCE's relative market power, and $\mathbb{E}[u^{\mathsf{ALCE}}(\mathbf{0},\mathbf{\Pi})]$ is ALCE's expected payoff at the disagreement point; $\mathbb{E}[u^{\mathsf{AEV}}(\mathbf{0},\mathbf{\Pi})]$ denotes the same for AEV.

The GNBS formulation for the swing option is similar to that of plain option, the only difference being in the set of constraints that define the feasible set. It can be written as

$$\begin{aligned} \max\ & \Big(\mathbb{E}[u^{\mathsf{ALCE}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{ALCE}}(\boldsymbol{C'},\mathbf{\Pi})]\Big)^{\alpha} \\ & \Big(\mathbb{E}[u^{\mathsf{AEV}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{AEV}}(\boldsymbol{C'},\mathbf{\Pi})]\Big)^{1-\alpha} \\ \text{s.t., }& (1) - (5), (8) - (11), (12)\text{–}(18), (20), (21), \\ & (26) - (28), (30), (32) - (37). \end{aligned} \tag{39}$$

Note that, $u^{\mathsf{ALCE}}(\boldsymbol{C},\mathbf{\Pi})$ and $u^{\mathsf{AEV}}(\boldsymbol{C},\mathbf{\Pi})$ can be written as

$$u^{\mathsf{ALCE}}(\boldsymbol{C},\mathbf{\Pi}) = u^{\mathsf{ALCE}}(K,0,Q,\mathbf{\Pi}) + V, \tag{40}$$

$$u^{\mathsf{AEV}}(\boldsymbol{C},\mathbf{\Pi}) = u^{\mathsf{AEV}}(K,0,Q,\mathbf{\Pi}) - V. \tag{41}$$

Similar expressions can be written for the swing option.

In the rest of this section, we present an approach for obtaining the optimal values of the option parameters. An expression for the option value V can be found using the first and second order conditions for a given strike price K. Let, for the plain option, we denote the objective function of the NBS problem as $\overline{N}$, where

$$\begin{aligned} \overline{N} =& \Big(\mathbb{E}[u^{\mathsf{ALCE}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{ALCE}}(\boldsymbol{C},\mathbf{\Pi})]\Big)^{\alpha} \\ & \Big(\mathbb{E}[u^{\mathsf{AEV}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{AEV}}(\boldsymbol{C},\mathbf{\Pi})]\Big)^{1-\alpha}. \end{aligned} \tag{42}$$

For swing option, the expression for $\overline{N}$ is same as above with $\boldsymbol{C}$ replaced by $\boldsymbol{C'}$.

Proposition 1. *For any given K and Q, the optimal option value V is given as*

$$\begin{aligned} V =& (1-\alpha)\Big(\mathbb{E}[u^{\mathsf{ALCE}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{ALCE}}(\tilde{\boldsymbol{C}},\mathbf{\Pi})]\Big) \\ & - \alpha\Big(\mathbb{E}[u^{\mathsf{AEV}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{AEV}}(\tilde{\boldsymbol{C}},\mathbf{\Pi})]\Big), \end{aligned} \tag{43}$$

where $\tilde{\boldsymbol{C}} = [K,0,Q]$ for the plain option and $\tilde{\boldsymbol{C}} = [K,0,\underline{Q}_t,\overline{Q}_t,\underline{Q},\overline{Q}]$ for the swing option.

Proof. The value of V in (43) maximizes $\overline{N}$. Note that, since V does not appear in any of the constraints of the GNBS model (38) and (39), we can use the first and the second order conditions to obtain its value. Solving for V directly is not straightforward due to the product and the powers in $\overline{N}$. Taking the logarithm of $\overline{N}$ removes both the products and the powers:

$$\begin{aligned} \log(\overline{N}) =& \alpha \log\Big(\mathbb{E}[u^{\mathsf{ALCE}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{ALCE}}(\boldsymbol{C},\mathbf{\Pi})]\Big) + \\ & (1-\alpha)\log\Big(\mathbb{E}[u^{\mathsf{AEV}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{AEV}}(\boldsymbol{C},\mathbf{\Pi})]\Big). \end{aligned} \tag{44}$$

Note that, we can write from (40) and (41) that

$$\frac{\partial}{\partial V}\mathbb{E}[u^{\mathsf{ALCE}}(\boldsymbol{C},\mathbf{\Pi})] = 1, \text{ and}$$

$$\frac{\partial}{\partial V}\mathbb{E}[u^{\mathsf{AEV}}(\boldsymbol{C},\mathbf{\Pi})] = -1.$$

Then, we have that $\frac{\partial \log(\overline{N})}{\partial V}$ can be obtained as

$$\begin{aligned} \frac{\partial \log(\overline{N})}{\partial V} =& \frac{-\alpha}{\mathbb{E}[u^{\mathsf{ALCE}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{ALCE}}(\boldsymbol{C},\mathbf{\Pi})]} + \\ & \frac{1-\alpha}{\mathbb{E}[u^{\mathsf{AEV}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{AEV}}(\boldsymbol{C},\mathbf{\Pi})]}. \end{aligned} \tag{45}$$

By making $\frac{\partial \log(\overline{N})}{\partial V} = 0$, we have that

$$\begin{aligned} & \alpha(\mathbb{E}[u^{\mathsf{AEV}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{AEV}}(\boldsymbol{C},\mathbf{\Pi})]) = \\ & (1-\alpha)(\mathbb{E}[u^{\mathsf{ALCE}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{ALCE}}(\boldsymbol{C},\mathbf{\Pi})]). \end{aligned} \tag{46}$$

Finally, by solving for V, we obtain the optimal V in (43). To show that it maximizes the GNBS, we must check the second-order condition, i.e., $\frac{\partial^2 \log(\overline{N})}{\partial V^2} < 0$. Then, by taking the second derivative, we have that

$$\begin{aligned} \frac{\partial^2 \log(\overline{N})}{\partial V^2} =& -\frac{\alpha}{(\mathbb{E}[u^{\mathsf{ALCE}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{ALCE}}(\boldsymbol{C},\mathbf{\Pi})])^2} - \\ & \frac{1-\alpha}{(\mathbb{E}[u^{\mathsf{AEV}}(\mathbf{0},\mathbf{\Pi})] - \mathbb{E}[u^{\mathsf{AEV}}(\boldsymbol{C},\mathbf{\Pi})])^2} < 0, \end{aligned} \tag{47}$$

for all $\alpha \in (0,1)$. Therefore, (43) returns the value of V that maximizes (42). Given that all values of V are feasible to the ALCE and AEV's problems (since V does not appears in the constraints), the unconstrained solution given by (43) also solves problems in (38) and (39). Note that the option value V can be obtained by independently solving the models of ALCE and AEV.

Note that, if both K and V are given, since there are no other common variables between the ALCE's and the AEV's models, the optimal solution of the GNBS formulation in (38) and (39) can be found by solving the models of the ALCE and AEV individually. However, if only V is given, the optimal solution of the problem can be found by effectively exploring the possible values of K. Also, to obtain optimal values for the option parameters (K and V) as well as the GNBS solution, we need to asses the value of $\mathbb{E}[u^{\mathsf{ALCE}}(\tilde{\boldsymbol{C}},\mathbf{\Pi})]$ and $\mathbb{E}[u^{\mathsf{AEV}}(\tilde{\boldsymbol{C}},\mathbf{\Pi})]$, for which we use the sequential Montecarlo simulation approach as implemented in [30] and [29].

So far we have discussed how to obtain the optimal strike price and option value considering that ALCE and AEV are interested in bargaining for these two quantities. Other contract parameters, i.e., option quantity and time window must also be establish to fully characterize the option contract. Regarding the time window, it is in the interest both ALCE and AEV that the time window encompasses the daily on-peak time periods at which the electricity prices are normally higher [22]. These time periods are important to the ALCE because it can hedge against higher prices and also to AEV because it can negotiate higher values for strike price and option value. Regrading the option quantity, the cost reductions of the participating aggregations increase with increasing option quantity. Hence, both ALCE and AEV should be interested in having a high value for the option quantity. This value, however, is bounded by either AEV's capacity to deliver or ALCE's needs. In our numerical study section, we show the impact of the choice of option quantity in the total cost reduction and on the option value.

IV. Numerical study

The numerical study objectives are: 1) to evaluate the cost and benefit of ALCE and AEV by entering into a bilateral trading contract, and 2) to examine the optimal choices of the contract parameter values. For this purpose we construct a sample numerical problem as follows. It is considered, for simplicity, that the ALCE and the AEV are connected to the same node of a network. The ALCE is comprised of five load consuming entities (LCEs), which are identical except that each has a separate time window (3–11, 5–14, 7–14, 12–21, and 10–17 hours) to operate its shiftable and adjustable loads. The total load of the ALCE is obtained by scaling down load data (by a factor of 240) from the DAY node of the PJM network in the U.S. The scaling factor was chosen to make the scope of ALCE's operation comparable to that of the AEV, described later. Of the total ALCE load, 40% is considered fixed and the remaining 60% is divided equally between shiftable and adjustable loads. The AEV is comprised of 200 EVs, each with battery capacity rating of 30 kWh. We assume that all EVs arrive at the parking facility at 8 AM and depart at 6 PM; random arrival and departure of EVs have been modeled in [31]. We also assume that EVs arrive to the parking facility with an average of 50% state of charge (SOC) and have an average desired SOC of 70% at the time of departure. The minimum and maximum SOC at the time of departure are 60% and 90%, respectively. It is considered that the EV owners pay a flat price to AEV for charging @ 8¢/kWh. The AEV incurs an undercharge/overcharge penalty (paid to the EV owners @ 5¢/kWh) if the SOC of an EV at the time of departure is below or above the desired SOC of 70%. The hourly locational marginal prices (LMPs) of electricity at the network node, where ALCE and AEV are connected, are obtained as follows. We consider the LMP data from DAY node of the PJM network during July 15, 2017 to July 30, 2017. From this LMP data, we calculate the mean and variance for each hour, and use those as parameters of the normal distributions that we assume to describe the hourly LMP variations. Random samples from these hourly distributions are drawn to generate a number of *daily price scenarios*, which are used in the Montecarlo simulation approach to solve the GNBS model.

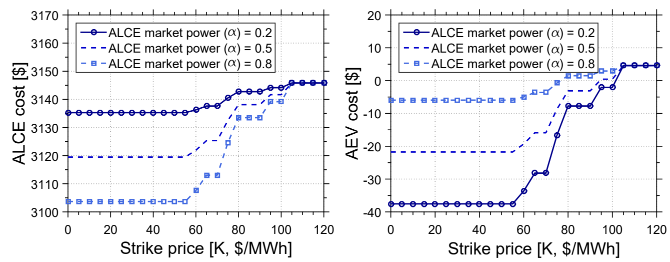

Fig. 2: ALCE and AEV cost for different optimal strike prices in plain call option

Our GNBS model considers option quantity and market power as input. The decision variables are strike price and option value. However, since simultaneous optimization of both decision variables is computationally challenging, we optimize one parameter given the other. For the sample numerical problem, the time window for the option contract is considered to be from 3 PM to 6 PM. For the swing call option, the upper limit for quantity in any given time interval within the window is 250 kWh. Our model is implemented using Julia-0.6.2 and GUROBI 7.5.2. The results are summarized in Figures 2 through 4.

We first addressed our objective of assessing benefits of ALCE and AEV in entering into a bilateral contract. For this, we obtained the optimal values of their costs with and without contract, for various combinations of contract parameters K, V, and α, and a constant option quantity of 1000kWh. Figure 2 presents the results obtained from a plain option contract. For each value of the ALCE market power (α) and a wide range of strike prices (K), our solution approach obtained the corresponding optimal option values (V) as well as the expected costs for both ALCE and AEV (denoted as $\mathbb{E}[u^{\text{ALCE}}]$ and $\mathbb{E}[u^{\text{AEV}}]$). As observed from the figure, the cost reduces with increasing market power, for both parties. The cost curves have three distinct regions: for K values from \$0/MWh to \$55/MWh, \$55/MWh to \$105/MWh, and over \$105/MWh. The first region presents a number of alternative optimal solutions (i.e., various optimal combinations of K and V). In the second region, the cost increases with K for both ALCE and AEV. This is due to the fact that for $K >$\$55/MWh, the contract is often not exercised as some of the daily price scenarios do not exceed the strike price within the contract time window, thus reducing the benefits derived from the contract. In the third region, for $K >$\$105/MWh, none of the daily price scenarios generated for our sample numerical problem exceed the strike price. This yields an option value of zero, and the corresponding costs represent the case with no contract (disagreement point). The disagreement cost for ALEA and AEV are \$3145.8 and \$4, respectively. It is evident from the results that, for the chosen numerical problem and the price scenarios, the ALCE and AEV should select any strike price that is below the threshold of \$55/MWh to maximize their benefits from a bilateral contract. The maximum total benefit resulting from such a bilateral contract is \$52.65 per day, which is the sum of the differences between the disagreement cost and cost for K below \$55/MWh for ALCE and AEV. Their respective shares of the benefit are $\alpha \times \$52.65$ and $(1-\alpha) \times \$52.65$, respectively. Similar cost/benefit patterns have also been observed for swing option, and hence not presented here. We also examined benefits as a function of the option quantity (Q). Increasing trends for benefits vs. option quantity were observed for both option types, for the range Q between 0-1000 kWh. This is depicted in Figure 3 for ALCE. This increasing pattern should hold as long as ALCE has the capacity to fully consume the option quantity. If Q grows too large beyond ALCE's capacity, then the plain option benefit will sharply drop to zero. In our numerical problem, the AEV did not have the capacity to offer Q larger than what ALCE can accommodate. Hence, we could not generate a scenario where the plain option benefit would drop to zero.

Hereafter, per objective 2 of our numerical study, we explored the relationships between the optimal values of the contract parameters for plain option. Figure 4 (left) shows the optimal option values (V) corresponding to a range of strike prices (K), for various levels of ALCE market power (α). The optimal option values were obtained solving (43). It can be observed that V decreases monotonically (up to a certain point) with increases in K. Interestingly, the value of V drops below zero in some cases, which indicates that, beyond a certain value of K (e.g., approximately \$36/MWh for α=0.8), the GNBS makes the option value negative, where AEV pays the fee to ALCE. For lower values of ALCE's relative market power, V becomes negative at relatively higher strike prices. Beyond a certain strike price (\$70/MWh), the option value paid by AEV starts to decrease (move towards zero). This is due to the fact that, at such high values of K, an increasing number of the price scenarios remain below the strike price, thus not triggering the option purchase and reducing AEV's revenue. Further increases in the value of K gradually pushes the V to zero. It is observed that the turning point for V is independent of market power, as it depends only on the strike price and the considered set of price scenarios. A similar trend is observed (not presented here) for the swing call option, where the turning point for V is lower and approximately at K = \$55/MWh. This reduction is as expected since AEV's revenue is higher in plain call option in this numerical example. Figure 4 (right) depicts the impact of option quantity Q on parameters V and K, for α=0.2. We observe the following. V increases with Q, albeit at a slower pace as K increases. Beyond a certain strike price (e.g., K =\$70/MWh), V decreases with increasing Q. Finally, if K is increased further (e.g., K ≥\$75/MWh), V remains constant at zero.

Based on the observations made from the numerical study, we have developed the following step-by-step procedure for implementing the GNBS approach for obtaining an optimal option contract:

1) Option window is chosen by ALCE such that it encompasses the on-peak time periods of the day.
2) Option quantity can be selected as the minimum between ALCE's need and AEV's capacity to deliver.
3) A threshold for the strike price is obtained as follows. This can be done by either ALCE or AEV since in either case the threshold will be the same.
   a) Take random samples of daily electricity prices (Π).
   b) Set the option value V to be a small arbitrary number.
   c) Initialize the search for the threshold with two seeds K_0 and K_1 ($K_0 < K_1$) of the strike price.
   d) Solve the operational model for the chosen option type for both K_0 and K_1.
   e) Compute the rate of change of the cost with respect to the strike price.
   f) Set $K_0 \leftarrow K_1$ and increase the current value of K_1.
   g) Recompute the rate of change of the cost with respect to the strike price using the current values of K_0 and K_1.
   h) If the rate of change differs from the one in the previous iteration, STOP, the threshold is equal to K_0; otherwise return to Step 3f.
4) Choose any strike price below the threshold obtained from Step 3 and use equation (43) to obtain the optimal option value V. We note that this step will have to be solved in a centralized manner as operational models of both ALCE and AEV are required to solve (43).

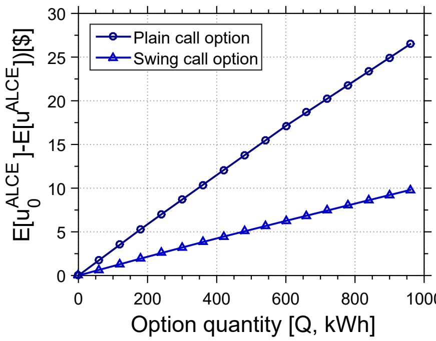

Fig. 3: ALCE cost saving comparison for different option quantities (for $\alpha = 0.5$)

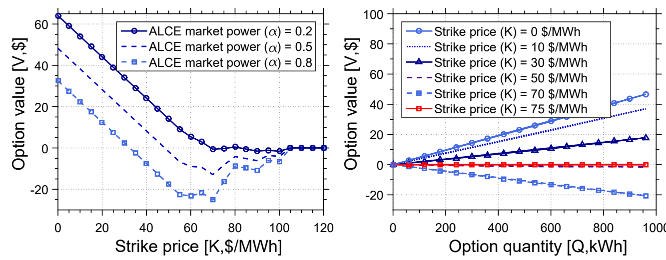

Fig. 4: Optimal option values for given strike prices and quantities (plain option)

V. Concluding Remarks

Although energy trading in power markets is expanding among prosumers at the peer-to-peer level, trading among aggregations of end-use consumers has not yet been adequately explored. In this paper, our objective has been to promote local energy trading among aggregations of empowered end-use consumers. We have developed a bilateral option contract framework between two types of such aggregations , namely, aggregation of load consuming entities only (ALCE) and aggregation of electric vehicles (AEV). The framework uses a generalized Nash bargaining solution approach to find the optimal contract parameters. Using a sample numerical problem, we have examined the properties of two different kinds of option contracts (plain and swing) and assessed their benefits to the participating aggregations. We have demonstrated via numerical results that the aggregations of end-use consumers can benefit from bilateral contracts for trading electricity.

In what follows, we discuss some of the desired properties of obtaining an optimal option contract using the GNBS approach and how it relates to the general multi-objective optimization framework. Reaching an agreement between two aggregations is a multi-objective optimization problem (MOOP) in which each aggregation has its own objective function to optimize. In general, when solving a MOOP all (or some) of the Pareto-optimal solutions are identified to construct the so called Pareto frontier or non-dominated frontier. Computing each Pareto-optimal solution might be computationally expensive [5]. Obtaining the non-dominated frontier might be infeasible in practice. Hence, the aim could be to find a single Pareto-optimal solution that considers the objectives of both aggregations while guaranteeing fairness. A possible approach is to use a weighted sum of the objectives with the hope to find a good solution by exploring different values of the weights. However, for non-convex MOOPs, there may exist many (and possibly infinite) Pareto-optimal points that cannot be obtained by optimizing a weighted sum of the objective functions. Such points are called unsupported Pareto-optimal points [7]. Hence, a major disadvantage of this method is that it completely ignores the existence of unsupported Pareto-optimal points that can possibly attain a better balance (more fair) between different objectives.

Approaches to obtain fair solution can be found in the literature, e.g., the max-min approach which maximizes the objective of the least satisfied aggregation. However, this category of approaches, while being fair, may disregard efficiency (total cost minimization) of the solution. The GNBS approach provides a natural compromise between fairness and efficiency. This is so, since maximizing the geometric mean leads to a more balanced valuation without neglecting efficiency [6]. Some of the desirable properties of the GNBS include 1) it returns a Pareto-optimal solution, 2) It can decrease the computational time significantly when compared to complete multi-objective optimization approach, 3) it generally avoids the endpoints of the non-dominated frontier and thus it generates a good balance between objectives, 4) it does not ignore unsupported Pareto-optimal points and it might yield such points as solution and, 5) it attempts to ensure that the benefits are distributed fairly with respect to the relative market power of aggregations.

EV owners receive payment from selling energy to the ALCE. However, concerns regarding degradation of batteries due to discharge for trading may naturally arise, prompting the EV owners to hesitate to participate in P2P trading. This concern can be addressed as follows. It is standard practice that when a battery capacity reduces bellow certain level, it is recommended for replacement. Some authors consider such replacement when battery capacity reduces to 80%, while others consider replacement at 50% [26]. For replacement at 80%, the battery goes through approximately 3650 cycles, and the corresponding numbers for 65% and 50% replacement rule are approximately 6400 and 9150, respectively [26]; this numbers consider a full depth of discharge (DOD) cycle.

For the problem considered in this paper, there is only one contract window per day. Even when the contract is exercised for a given option quantity, only a subset of the batteries may need to be discharged, that too perhaps partially. It is well known that DOD significantly affects the battery longevity [39] and the equivalent number of cycles is proportional to the DOD [10]. If we consider 1.5 cycles per day, 0.5 due to trading and one for traveling, the EV batteries may need to be replaced approximately on 7, 12, and 17 years as per 80%, 65%, and 50% replacement rule, respectively. Even with two full DOD per day, with 65% replacement rule, an EV battery should last approximately 9 years.

The cost of battery degradation is not included in our model. Based on the nature of option contract, the daily expected discharge and the corresponding degradation cost are constant [21]. Consideration of this fixed cost does not fundamentally alter our model, though it will alter the value of contact parameters in equilibrium. For example, the degradation cost, if considered, will be added to the term $\mathbb{E}[u^{\mathsf{AEV}}(\tilde{\boldsymbol{C}}, \boldsymbol{\Pi})]$ in Equation (43). This will yield a higher magnitude of the option value for any given strike price in an optimal option contract. This bias can added at the end of the optimization process.

Our methodology has a few limitations that may be addressed in future work. First, we have assumed that both ALCE and AEV are loads on the same bus of the network. In practice, energy trading can occur between aggregations connected to different buses. In that case, congestion costs and differences in hourly LMPs must be considered for option contract design. This will require incorporation of an optimal power flow model in our methodology. Second, we have assumed in our model implementation that the EVs arrive to and depart from the parking facility at set times. A more generalized model implementation will consider the parking lots as smart hubs in which the EVs come and go throughout the day depending on their trip plans and charging needs. Finally, although we incorporate relative market power in bilateral contract design and examine its impact, it is not clear how to estimate its numerical value. The traditional approach to study market power in electricity markets uses concentration measures such as the Hirschman-Herfindahl Index (HHI) or the quantity modulated price index (QMPI) [23]. These measures are mostly used to estimate generators' market power and are used by FERC as fundamental screening tools for merger analysis in this sector. Research has shown that concentration based measures can be misleading indicators of the market power in electricity markets [4]. From a demand-side perspective, a body of research agrees that market power is a function of demand elasticity and the ability to respond to time-varying price signals. However, how to estimate the relative market power of peers engaged in energy trading remains an open research question.

References

[1] M. Alipour, B. Mohammadi-Ivatloo, M. Moradi-Dalvand, and K. Zare. Stochastic scheduling of aggregators of plug-in electric vehicles for participation in energy and ancillary service markets. *Energy*, 118:1168–1179, 2017.

[2] R. Alvaro-Hermana, J. Fraile-Ardanuy, P. J. Zufiria, L. Knapen, and D. Janssens. Peer to peer energy trading with electric vehicles. *IEEE Intelligent Transportation Systems Magazine*, 8(3):33–44, 2016.

[3] E. J. Anderson and H. Xu. Optimal supply functions in electricity markets with option contracts and non-smooth costs. *Mathematical Methods of Operations Research*, 63(3):387–411, 2006.

[4] S. Borenstein and J. Bushnell. An empirical analysis of the potential for market power in california's electricity industry. *The Journal of Industrial Economics*, 47(3):285–323, 1999.

[5] H. Charkhgard, K. Keshanian, R. Esmaeilbeigi, and P. Charkhgard. The magic of nash social welfare in optimization: Do not sum, just multiply! *Preprint. Available at https://www.researchgate.net/publication/340022329_The_Magic_of_Nash_Social_Welfare_in_Optimization_Do_Not_Sum_Just_Multiply*, 2020.

[6] R. Cole and V. Gkatzelis. Approximating the nash social welfare with indivisible items. *SIAM Journal on Computing*, 47(3):1211–1236, 2018.

[7] M. Ehrgott. *Multicriteria optimization*, volume 491. Springer Science & Business Media, 2005.

[8] W. El-Baz, P. Tzscheutschler, and U. Wagner. Integration of energy markets in microgrids: A double-sided auction with device-oriented bidding strategies. *Applied energy*, 241:625–639, 2019.

[9] A. Fayaz-Heidari, M. Fotuhi-Firuzabad, and R. Ghorani. Economic valuation of demand response programs using real option valuation method. In *2019 27th Iranian Conference on Electrical Engineering (ICEE)*, pages 685–691. IEEE, 2019.

[10] T. Guena and P. Leblanc. How depth of discharge affects the cycle life of lithium-metal-polymer batteries. In *INTELEC 06-Twenty-Eighth International Telecommunications Energy Conference*, pages 1–8. IEEE, 2006.

[11] R. Jing, M. N. Xie, F. X. Wang, and L. X. Chen. Fair p2p energy trading between residential and commercial multi-energy systems enabling integrated demand-side management. *Applied Energy*, 262:114551, 2020.

[12] J. Kang, R. Yu, X. Huang, S. Maharjan, Y. Zhang, and E. Hossain. Enabling localized peer-to-peer electricity trading among plug-in hybrid electric vehicles using consortium blockchains. *IEEE Transactions on Industrial Informatics*, 13(6):3154–3164, 2017.

[13] R. M. Kovacevic and G. C. Pflug. Electricity swing option pricing by stochastic bilevel optimization: a survey and new approaches. *European Journal of Operational Research*, 237(2):389–403, 2014.

[14] H. Le Cadre, P. Jacquot, C. Wan, and C. Alasseur. Peer-to-peer electricity market analysis: From variational to generalized nash equilibrium. *European Journal of Operational Research*, 282(2):753–771, 2020.

[15] C. Long, Y. Zhou, and J. Wu. A game theoretic approach for peer to peer energy trading. *Energy Procedia*, 159:454–459, 2019.

[16] A. Lüth, J. M. Zepter, P. C. del Granado, and R. Egging. Local electricity market designs for peer-to-peer trading: The role of battery flexibility. *Applied energy*, 229:1233–1243, 2018.

[17] K. A. Melendez, V. Subramanian, T. K. Das, and C. Kwon. Empowering end-use consumers of electricity to aggregate for demand-side participation. *Applied Energy*, 248:372 – 382, 2019.

[18] Y. Moon and C. Kwon. Online advertisement service pricing and an option contract. *Electronic Commerce Research and Applications*, 10(1):38–48, 2011.

[19] F. Moret, T. Baroche, E. Sorin, and P. Pinson. Negotiation algorithms for peer-to-peer electricity markets: Computational properties. In *2018 Power Systems Computation Conference (PSCC)*, pages 1–7. IEEE, 2018.

[20] T. Morstyn, A. Teytelboym, and M. D. McCulloch. Bilateral contract networks for peer-to-peer energy trading. *IEEE Transactions on Smart Grid*, 10(2):2026–2035, 2019.

[21] S. Muhammad, J. Moshtagh, M. Shafie-khah, A. Rastgou, and J. P. Catalão. Operational scheduling of a smart distribution system considering electric vehicles parking lot: A bi-level approach. *International Journal of Electrical Power & Energy Systems*, 105:159 – 178, 2019.

[22] P. Muniain and F. Ziel. Probabilistic forecasting in day-ahead electricity markets: Simulating peak and off-peak prices. *International Journal of Forecasting*, 2020.

[23] V. Nanduri and T. K. Das. A reinforcement learning model to assess market power under auction-based energy pricing. *IEEE transactions on Power Systems*, 22(1):85–95, 2007.

[24] H. K. Nunna and S. Doolla. Demand response in smart distribution system with multiple microgrids. *IEEE transactions on smart grid*, 3(4):1641–1649, 2012.

[25] S. S. Oren. Integrating real and financial options in demand-side electricity contracts. *Decision Support Systems*, 30(3):279–288, 2001.

[26] M. A. Ortega-Vazquez. Optimal scheduling of electric vehicle charging and vehicle-to-grid services at household level including battery degradation and price uncertainty. *IET Generation, Transmission & Distribution*, 8(6):1007–1016, 2014.

[27] S. Pineda and A. J. Conejo. Using electricity options to hedge against financial risks of power producers. *Journal of Modern Power Systems and Clean Energy*, 1(2):101–109, 2013.

[28] W. Saad, Z. Han, H. V. Poor, and T. Başar. A noncooperative game for double auction-based energy trading between phevs and distribution grids. In *2011 IEEE international conference on smart grid communications (SmartGridComm)*, pages 267–272. IEEE, 2011.

[29] J. A. Schachter and P. Mancarella. Demand response contracts as real options: a probabilistic evaluation framework under short-term and long-term uncertainties. *IEEE Transactions on Smart Grid*, 7(2):868–878, 2015.

[30] H. R. Sheybani and M. O. Buygi. Put option pricing and its effects on day-ahead electricity markets. *IEEE Systems Journal*, (99):1–11, 2017.

[31] V. Subramanian and T. K. Das. A two-layer model for dynamic pricing of electricity and optimal charging of electric vehicles under price spikes. *Energy*, 167:1266–1277, 2019.

[32] S.-C. Suh and Q. Wen. Multi-agent bilateral bargaining and the nash bargaining solution. *Journal of Mathematical Economics*, 42(1):61–73, 2006.

[33] W. Tushar, T. K. Saha, C. Yuen, M. I. Azim, T. Morstyn, H. V. Poor, D. Niyato, and R. Bean. A coalition formation game framework for peer-to-peer energy trading. *Applied Energy*, 261:114436, 2020.

[34] W. Tushar, T. K. Saha, C. Yuen, D. Smith, and H. V. Poor. Peer-to-peer trading in electricity networks: An overview. *https://arxiv.org/abs/2001.06882 (last accessed on Jan 27, 2020)*, 2020.

[35] W. Tushar, C. Yuen, H. Mohsenian-Rad, T. Saha, H. V. Poor, and K. L. Wood. Transforming energy networks via peer-to-peer energy trading: The potential of game-theoretic approaches. *IEEE Signal Processing Magazine*, 35(4):90–111, 2018.

[36] H. Wang and J. Huang. Bargaining-based energy trading market for interconnected microgrids. In *2015 IEEE International Conference on Communications (ICC)*, pages 776–781. IEEE, 2015.

[37] H. Wang and J. Huang. Incentivizing energy trading for interconnected microgrids. *IEEE Transactions on Smart Grid*, 9(4):2647–2657, 2016.

[38] Website. Global ev outlook 2019. https://www.iea.org/reports/global-ev-outlook-2019, 2019. accessed: January 26, 2020.

[39] S. Xie, X. Hu, Q. Zhang, X. Lin, B. Mu, and H. Ji. Aging-aware co-optimization of battery size, depth of discharge, and energy management for plug-in hybrid electric vehicles. *Journal of Power Sources*, 450:227638, 2020.

[40] B. Zhang, C. Jiang, J.-L. Yu, and Z. Han. A contract game for direct energy trading in smart grid. *IEEE Transactions on Smart Grid*, 9(4):2873–2884, 2018.

[41] Z. Zhang, J. Shi, H.-H. Chen, M. Guizani, and P. Qiu. A cooperation strategy based on nash bargaining solution in cooperative relay networks. *IEEE Transactions on Vehicular Technology*, 57(4):2570–2577, 2008.

[42] S. Zou, Z. Ma, X. Liu, and I. Hiskens. An efficient game for coordinating electric vehicle charging. *IEEE Transactions on Automatic Control*, 62(5):2374–2389, 2016.