A Comprehensive Modeling Framework for Hazmat Network Design, Hazmat Response Team Location, and Equity of Risk

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Abstract

This paper considers a bi-level hazmat transportation network design problem in which hazmat shipments have to be transported over a road network between specified origin-destination points. The bi-level framework involves a regulatory authority and hazmat carriers. The control variables for the regulatory authority are locations of hazmat response teams and which additional links to include for hazmat travel. The regulatory authority (upper level) aims to minimize the maximum transport risk incurred by a transportation zone, which is related to risk equity. Our measure of risk incorporates the average response time to the hazmat incidents. Hazmat carriers (lower level) seek to minimize their travel cost. Using optimality conditions, we reformulate the non-linear bi-level model as a single-level mixed integer linear program, which is computationally tractable for medium size problems using a commercial solver. For large size problems, we propose a greedy heuristic approach, which we empirically demonstrate to find good solutions with reasonable computational effort. We also seek a robust solution to capture stochastic characteristics of the model. Experimental results are based on popular test networks from the Sioux Falls and Albany areas.

Keywords: Hazmat emergency response team; Bi-level network design; Greedy heuristic algorithm; Equity of risk; Robust solution

1 Introduction

Although the majority of hazardous materials (hazmat) are essential for industry and human life, they can be potentially harmful to the public and the environment. Based on the US Department of Transportation's statistics [1], a 10-year (2004–2013) hazmat incident report indicates that highways,

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[1]http://www.phmsa.dot.gov/hazmat/library/data-stats/incidents

with 140,742 number of incidents out of 163,469 in total, have the largest portion of fatalities, injuries, and damage among all modes of transportation. This report indicates the importance of risk on hazmat road networks and the reason why these problems have received so much attention from OR/MS researchers.

Hazmat shipments are controlled under the Federal Hazardous Materials Transportation Act in the United States. There are two main policies available to regulatory agencies to mitigate hazmat transport risk, proactive and reactive (Marcotte et al., 2009). Proactive policies, such as closure of road segments or container design, reduce the probabilities and consequences of incidents involving hazmat release before the incidents happen. Reactive policies, such as deployment of emergency response teams, confine the level of consequences of a hazmat incident, after it occurred.

We focus on a combination of proactive and reactive policies in order to reduce the transport risk; regulating the use of road segments that can be used by hazmat carriers (proactive) and locating the Hazmat Response Teams (HRTs) (reactive). Many existing papers consider toll-setting or road closure policies to deter hazmat carriers from moving along certain road segments. Consideration of network design is a relatively well-studied concept in hazmat transportation. The closest piece of work is that by Kara and Verter (2004), who propose a network design problem to minimize the total hazmat transportation risk while hazmat carriers choose their minimum-cost routes.

In contrast, we propose another similar policy to control the use of roads for hazmat shipments which we refer to as *adding roads*. In addition, we optimize placement of HRTs, recognizing that hazmat transport risk may be significantly reduced by judiciously locating emergency response teams so that they can respond to an incident in a timely manner.

This work presents three main contributions that differentiate our paper from the current literature. First, we consider simultaneous decisions on designing a road network and locating HRTs to mitigate hazmat transport risk. Second, we define a risk measure that includes the average response time to the hazmat incidents. The regulator's objective function incorporates this definition of risk and allows us to capture the interactions between network design decisions and HRT location decisions. Regional jurisdiction of HRT guides us to consider risk equity over network zones, where each geographic zone is assigned to exactly one HRT. Third, we propose a robust solution to deal with the stochastic characteristics of hazmat accident probability and hazmat release consequences.

The remainder of this paper is organized as follows. A review of the relevant literature is provided in Section 2. Section 3 presents our mathematical formulation. Solution methodologies are proposed in Section 4, which is followed by our computational experiments in Section 5. Section 6 contains a summary and future research directions.

2 Literature Review

We present the following three areas of literature that are related to this paper: hazmat network design, equity of risk, and emergency response team locating for hazmat shipments. Each area is reviewed separately. Table 1 shows where our model sits among the available models in the literature.

2.1 Hazmat network design

There are a number of papers in HND that discuss partial or entire road closure to hazmat shipments in an existing network—see a review of network design problems proposed by Yang and Bell (1998). Kara and Verter (2004) provide a bi-level integer programming HND considering the leader-follower relationship between the government and carriers. Their model designs a network for each hazmat group based on risk impact with no interaction between these groups. The government objective is to design a minimum-total risk network, considering the minimum-travel cost route choice behavior of carriers. They applied KKT conditions to replace the lower level model and reformulate their model as a linear mixed integer problem. They solve the linearized form of the problem with an optimization solver. Erkut and Alp (2007) formulate a bi-level tree HND as an integer programming problem to minimize the total transport risk. Later Erkut and Gzara (2008) generalize the model of Kara and Verter (2004) to the undirected road network where the solution stability of single level mixed integer linear model was guaranteed by proposing a heuristic solution method. They add cost in the upper level objective to impose a trade-off between cost and risk. Verter and Kara (2008) define a path-based hazmat transport network design problem, where the construction of a set of alternative paths makes the incorporation of carriers' cost concern in regulator's risk-mitigation decisions easier. The proposed model can be applied for making road-closure decisions that are acceptable for the both regulator and the carrier. They formulate the problem as a single level integer programming assuming that the shortest path is chosen by each carrier. Bianco et al. (2009) provide a bi-level hazmat network design model where both regional and local governments aim to regulate hazmat traffic flow by imposing restrictions on the hazmat traffic volume over the network. They provide a heuristic approach to overcome the non-stability of the single-level mixed integer linearized form of the problem. Amaldi et al. (2011) propose a generalized HND where a subset of roads can be banned by the government. They propose a bi-level integer programming, and solve the compact single-level integer programming that guarantees stability of solution. All these mathematical models seek risk minimization considering minimum-cost route choices of hazmat carriers. Recently, Sun et al. (2016) present a robust HND to model the risk uncertainty on each link in two cases; across all shipments, and for each shipment. They develop a heuristic approach to solve their robust HND.

As a more flexible alternative to network design policies, toll setting approaches have been proposed. See Marcotte et al. (2009), Wang et al. (2012), Esfandeh et al. (2016b), and Bianco et al. (2016) for example. In time-varying networks, Esfandeh et al. (2016a) propose time-dependent network design policies for regulating hazmat traffic.

2.2 Equity of risk

Based on the definition of Keeney (1980), risk equity is the magnitude of the largest difference in the level of risk among a certain set of individuals. There are a number of papers that apply risk equity to the area of hazmat transportation. Gopalan et al. (1990) develop an integer programming formulation to find an equitable set of routes for hazmat shipments where a high degree of equity can be achieved by nominal increase in the total risk and by imposing an average equity over each route to evenly spread the risk among zones. Current and Ratick (1995) present a multi-objective model to analyze location-routing decisions involving hazmat. The model's objective includes risk, equity and cost. They impose risk equity by defining the maximum allowable risk exposure for each individual at a facility site in the form of a constraint in the problem formulation, and minimize this maximum threshold as one of the objectives. Two multi-objective mixed integer programming methods are applied to generate noninferior solutions. Carotenuto et al. (2007) provide a model to generate minimal risk paths for transporting hazmat shipments on a given regional area. The contribution is to select minimum-risk paths which impose the risk on the population in an equitable way. They define an upper limit on the total hazmat transportation risk over congested roads by imposing a risk threshold. They apply a Lagrangian relaxation to find a lower bound on the optimal solution value. The only one paper in HND that discusses risk equity is that by Bianco et al. (2009). In their HND model, the regional regulator tries to minimize total transport risk assuring risk equity, while the local regulator seeks to minimize the risk over the local jurisdiction. They define a maximum link total risk threshold to set an upper limit over the total risk value on each link on the network as a leader (upper level problem). They suggest a heuristic approach to find a solution which ensures stability and feasibility. Kang et al. (2014) proposed a hazmat routing problem based on value-at-risk model. They apply their model to a multi-trip multi-hazmat type problem, which finds the routes with minimum value-at-risk value and risk equity.

2.3 Location of hazmat response teams

The HRT location problem is valued as a prominent logistic problem to mitigate hazmat transportation risk. List and Turnquist (1998) define a multi-objective problem combining a route-siting model with three main elements. They solve the problem in three separate sub-problems, i.e., routing to find the proper routes, flow assignment to the obtained routes, and locating the HRTs based on assigned flows. The objective of the latter sub-problem describes the importance of locating HRTs near links with high flow volume or large exposed population (List, 1993). Hamouda (2004) proposes a risk-based decision support model to find the optimal location of HRTs among candidate nodes to minimize the total network risk. Their model ensures that response time to any demand point does not exceed a specified threshold. Berman et al. (2007) present a model to optimally design a specialized team network for better responding to hazmat accidents. The problem was introduced as a maximal arc-covering model to incorporate the emergency response capability. Later, Zografos and Androutsopoulos (2008) present a decision support system to find distribution routes with respect to minimization of travel time, minimization of risk, while integrating the HRT location decisions with hazmat route decisions. Jiahong and Bin (2010) present an HRT location-routing problem as a maximal arc-covering model that aims to maximize the service level of responding to hazmat incidents, while taking time and cost into account. Recently, Xu et al.

Table 1: Our paper with respect to the HND, hazmat equity modeling, and HRT location literature

Research Area	Existing Papers In Hazmat Transportation Literature		
Network Design	Kara and Verter (2004) Erkut and Alp (2007) Erkut and Gzara (2008) Verter and Kara (2008) Amaldi et al. (2011) Sun et al. (2016)	Bianco et al. (2009)	This paper
Equity Of Risk	Gopalan et al. (1990) Current and Ratick (1995) Carotenuto et al. (2007) Kang et al. (2014)		
Emergency Response Team Location	List (1993) List and Turnquist (1998) Hamouda (2004) Berman et al. (2007) Zografos and Androutsopoulos (2008) Jiahong and Bin (2010) Xu et al. (2013)		

(2013) propose a bi-level optimization model for the HND problem that considers the location of HRTs.

3 Problem Definition

In our bi-level problem, the regulatory authority influences the carriers' decisions by making additional roads available to the hazmat carriers and locating HRTs, whereas carriers can influence the leaders' decisions by their route selections. Figure 1 shows the conceptual representation of our model. To counteract the high consequences of hazmat incidents on the network, the regulatory authority's policy tools encourage the carriers to choose their paths such that, the selected minimum cost routes be closer to the HRTs.

3.1 Risk measurement

A popular way to estimate risk is to multiply the accident probability with estimated incident consequences to evaluate expected damage. The incidet consequence is commonly defined as potential fatalities or dollar value of damage to property (Gopalan et al., 1990). In the case of hazmat accident, the radius of spread depends on the physical and chemical characteristics of the hazmat. Gopalan et al. (1990) showed that if λ represents the radius of spread, people and properties within the λ-neighborhood (boundary of a circle with a radius λ and a center at the incident location) of the incident location can possibly be affected. In this paper, risk measurement is not only a multiplication of accident probability by estimated consequences, but also a function of response time to the incident. Based on a standard of emergency response coverage document[2] prepared by Portland Fire and Rescue (PFR), survival rate will be decreased by 10% for every one minute

[2] www.portlandoregon.gov/fire/article/101052

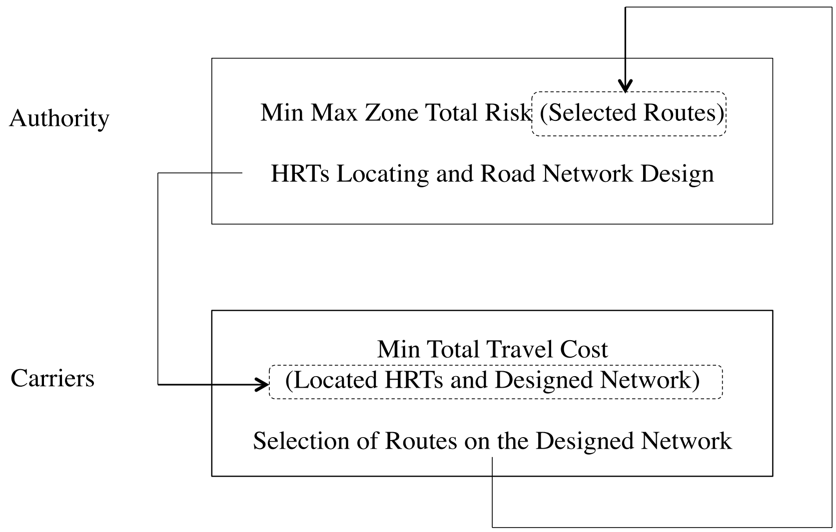

Figure 1: The proposed bi-level model

Table 2: Response time factor [Source: Hazardous Materials Cooperative Research Program (2011)]

Response Time Factor	Description
1	Meets or exceeds desired response time
2	Response time is within 125 percent of desired response time
3	Response time is within 150 percent of desired response time
4	Response time is within 200 percent of the desired response time
5	Response time is more than double the desired response time

delay in defibrillation. PFR's report indicates that the HRTs can reach 90% of hazmat accidents within 18 minutes in urban zones while maintaining an acceptable response level. Delay in response longer than a specified time threshold can highly increase the hazmat incident consequence. The aforementioned document and a guide for evaluating the emergency response needs for hazardous material incidents (Hazardous Materials Cooperative Research Program, 2011) reveal a proportional relation between the response time and the undesirable incident consequence. The risk measure for hazmat accident in this guide is defined as the multiplication of vulnerability (or accident likelihood), consequence, emergency response capability, and response time factor. The desired response time for four target outcomes; i) assess, ii) manage, iii) rescue, and iv) control, is proposed in minutes based on different jurisdiction classes. Five jurisdiction classes are suggested considering the population density in the area that hazmat incident happens. Table 2 shows the response time factor knowing the desired response time in case of a hazmat incident. According to the statistics of Table 2 proposed in the aforementioned guide, it is fair enough to consider that the hazmat consequence increases in proportion to the emergency response time with a linear function.

Let $\mathcal{A}_z$ be the set of all current and potential-to-add links on zone z. Let $\eta^c_{ij}(m)$ be the risk associated with one hazmat shipment of type c on link (i,j) given that hazmat response unit m responds to an incident that occurs on zone z. Definition of $\eta^c_{ij}(m)$ includes the accident probability

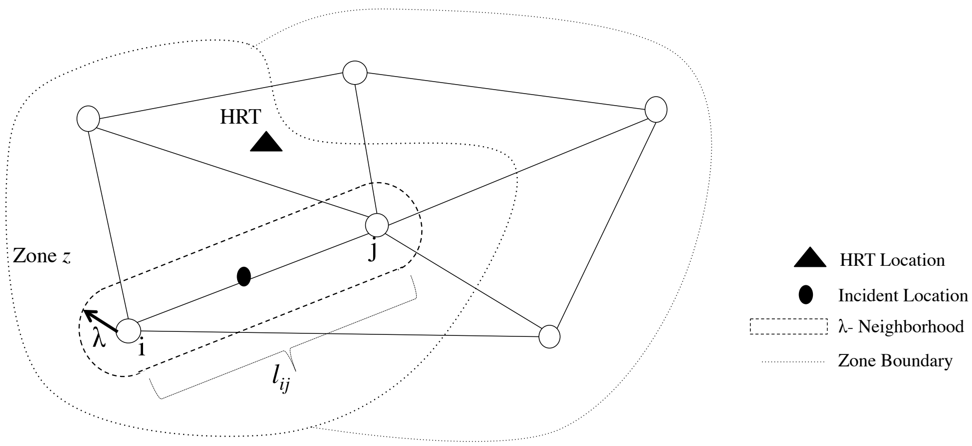

Figure 2: Attributes of risk measurement

and the accident consequence. Accident probability is denoted by $\rho_c l_{ij}$, where ρ_c is the average per-mile accident probability of hazmat type c and l_{ij} is the length of link (i, j), denoted by l_{ij}. We assume that the accident consequence is a non-decreasing function of the response time, denoted by $\xi^c_{ij}(\cdot)$ for each link (i, j) and hazmat type c. Then, we have

$$\eta^c_{ij}(m) = \rho_c l_{ij} \xi^c_{ij}(f^m_{ij}) \qquad \forall (i,j) \in \mathcal{A}_z, c \in \mathcal{C} \tag{1}$$

where f^m_{ij} denotes the average response time (distance) from hazmat response team m to the incident location happening on link (i, j).

To specify the accident consequence function $\xi^c_{ij}(\cdot)$, we adopt the population exposure measure q^c_{ij} on the λ-neighborhood of link (i, j) for hazmat type c as shown in Figure 2. To the best of our knowledge, no certain consequence function is proposed in the literature. Therefore, we suppose that the consequence increases in proportion to the response time with a linear consequence function for simplicity of the following form:

$$\xi^c_{ij}(f^m_{ij}) = q^c_{ij} \frac{f^m_{ij}}{F^c_{ij}} \tag{2}$$

where F^c_{ij} is a positive scaling constant specific to link (i, j) and hazmat type c. The consequence function (2) implicitly assumes that the impact of a hazmat incident extends beyond the λ-neighborhood, if the response time takes longer than F^c_{ij}. It is vital to consider how immediate response units can reach to the incident location.

3.2 Bi-level model formulation

We let $\mathcal{N}$ be the set of nodes, and $\mathcal{A}$ be the set of current road links available for hazmat shipments. We define $\mathcal{A}'$ be the set of all (current and potential-to-add) road links and $\mathcal{A}' \setminus \mathcal{A}$ be the set of potential links to be available for hazmat shipments. Let $\mathcal{C}$ be a set of hazmat shipments, and for each hazmat shipment $c \in \mathcal{C}$, let $o(c)$ and $d(c)$ be, respectively, the origin and destination nodes. Let n_c be the number of shipments from $o(c)$ to $d(c)$ for each $c \in \mathcal{C}$. We assume the existence of at least a path between each origin-destination pair in $G(\mathcal{N}, \mathcal{A})$. Let the binary variable x_{ij}^c be equal to one if hazmat shipment c traverses link (i, j), and zero otherwise. Let the binary variable y_{ij} be equal to one if link $(i, j) \in \mathcal{A}' \setminus \mathcal{A}$ becomes available for hazmat transport, and zero otherwise. Let the binary variable z_m be equal to one if candidate site $m \in \mathcal{M}$ is opened for locating an HRT, and zero otherwise. Binary variable v_z^m denotes the assignment of a HRT located at site $m \in \mathcal{M}$ to zone $z \in \mathcal{Z}$. Let h_{ij} be the cost of adding link $(i, j) \in \mathcal{A}' \setminus \mathcal{A}$ and B be the total available budget for adding links. Constant p denotes the total number of HRT locations to be chosen. The total risk over zone $z \in \mathcal{Z}$ becomes

$$\sum_{(i,j)\in\mathcal{A}_z} \sum_{m\in\mathcal{M}} \sum_{c\in\mathcal{C}} \rho_c l_{ij} n_c q_{ij}^c \frac{f_{ij}^m}{F_{ij}^c} v_z^m x_{ij}^c$$

and we let θ be the maximum zone total risk.

Our bi-level model considers two related optimization problems. The optimal solution of the upper level model (P_1) is affected by the solution of the lower level model (P_2). In real applications, there are two different emergency planning committees; local, and regional. Regional HRTs help the local HRTs to respond the incidents that require a higher level of response capabilities. Thus, local units are the primary HRTs, and regional units are the secondary HRTs. Each local HRT can be supported by one or multiple regional HRTs in case of the need for a higher level response. Our model captures the primary assignment of the local HRTs to the smallest managerial divisions (zones) such that, each zone is assigned to only one local HRT. The added potential links $Y = \{y_{ij}\}$, HRT assignments $V = \{v_z^m\}$ and HRT locations $Z = \{z_m\}$ are the variables determined by the leader to assure minimization of the maximum zone total risk. The followers aim to minimize the network total travel cost by controlling the variables $X = \{x_{ij}^c\}$.

(P_1) :

$$\min_{y_{ij}, z_m, v_z^m} \theta \tag{3}$$

$$\sum_{(i,j)\in\mathcal{A}_z} \sum_{m\in\mathcal{M}} \sum_{c\in\mathcal{C}} \rho_c l_{ij} n_c q_{ij}^c \frac{f_{ij}^m}{F_{ij}^c} v_z^m x_{ij}^c \leq \theta \qquad \forall z \in \mathcal{Z} \tag{4}$$

$$\sum_{(i,j)\in\mathcal{A}'\setminus\mathcal{A}} h_{ij} y_{ij} \leq B \tag{5}$$

$$\sum_{m\in\mathcal{M}} z_m = p \tag{6}$$

$$\sum_{m\in\mathcal{M}} v_z^m = 1 \qquad \forall z \in \mathcal{Z} \tag{7}$$

$$v_z^m \leq z_m \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M} \tag{8}$$

$$y_{ij} \in \{0, 1\} \qquad \forall (i, j) \in \mathcal{A}' \setminus \mathcal{A} \tag{9}$$

$$z_m \in \{0,1\} \qquad \forall m \in \mathcal{M} \tag{10}$$

$$v_z^m \in \{0,1\} \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M} \tag{11}$$

where x_{ij}^c solves

(P_2) :

$$\min \sum_{c \in \mathcal{C}} \sum_{(i,j) \in \mathcal{A}'} n_c l_{ij} x_{ij}^c \tag{12}$$

$$\sum_{i \in \mathcal{N}:(i,j) \in \mathcal{A}'} x_{ij}^c - \sum_{l \in \mathcal{N}:(j,l) \in \mathcal{A}'} x_{jl}^c = \begin{cases} 1 & \text{if } j = o(c) \\ -1 & \text{if } j = d(c) \\ 0 & \text{otherwise} \end{cases} \qquad \forall j \in \mathcal{N}, c \in \mathcal{C} \tag{13}$$

$$x_{ij}^c \leq y_{ij} \qquad \forall (i,j) \in \mathcal{A}' \setminus \mathcal{A}, c \in \mathcal{C} \tag{14}$$

$$x_{ij}^c \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}', c \in \mathcal{C} \tag{15}$$

The first problem P_1 is the *upper level formulation* in which the leader seeks the equity of the hazmat transportation risk among zones when the followers choose their minimum cost routes. The model minimizes the maximum zone total risk (θ) among all the zone total risk values of the existing network G by adding the appropriate available links, finding the best candidate locations for HRTs and, assigning each zone to an HRT. Constraints (4) assure equity of risk on zones, while (5) limits the total cost of links additions to the available budget. Constraint (6) indicates the total number of available HRTs and constraints (7) assign each zone on the network to exactly one hazmat response team. Constraints (8) permit assignments of opened candidate locations to zones. Problem P_2 is the *lower level formulation* that models the follower's behavior of minimizing total travel cost influenced by a feasible flow assignment $X = \{x_{ij}^c\}$, given the added links $Y = \{y_{ij}\}$ by the leader. Constraints (13) are the flow conservation requirements, whereas (14) ensures that the carriers can use only the road links made available by the leader. The proposed bi-level formulation has a non-linear objective in the upper level model. Since the non-linear term is a quadratic function of two binary variables, it can be linearized using common techniques.

4 Solution Methodologies

Due to the computational difficulty of a non-linear bi-level integer programing problem (Jeroslow, 1985), we develop a single level representation of the model to small instances that are solvable using an optimization solver like CPLEX. A greedy heuristic is developed for large size problems.

4.1 Single level representation

As noted earlier, problem P_2 can be solved given a set of available added links determined by the upper problem. While the y_{ij} values are given, the constraints of problem P_2 constitute a totally unimodular matrix and its integrality requirements can be replaced by $x_{ij}^c \geq 0$ without loss of optimality (Kara and Verter, 2004); hence equivalent duality conditions can replace P_2. To obtain this we define:

- π_i^c : dual variables for constraints (13), $i \in \mathcal{N}, c \in \mathcal{C}$
- π_j^c : dual variables for constraints (13), $j \in \mathcal{N}, c \in \mathcal{C}$

We follow the procedure in Amaldi et al. (2011) and replace the lower level model P_2 with constraints (13), (14) and the following constraints in the upper level model P_1.

$$\pi_i^c - \pi_j^c \leq n_c l_{ij} \qquad \forall (i,j) \in \mathcal{A}, c \in \mathcal{C} \tag{16}$$

$$\pi_i^c - \pi_j^c \leq n_c l_{ij} + M(1 - y_{ij}) \qquad \forall (i,j) \in \mathcal{A}' \setminus \mathcal{A}, c \in \mathcal{C} \tag{17}$$

$$\pi_{o(c)}^c - \pi_{d(c)}^c \geq \sum_{(i,j)\in\mathcal{A}'} n_c l_{ij} x_{ij}^c \qquad \forall c \in \mathcal{C} \tag{18}$$

This establishes a new single-level mixed-integer non-linear programming (MINLP) problem P_3:

(P_3) :

$$\min_{y_{ij}, z_m, v_z^m, x_{ij}^c} \theta \tag{19}$$

$$\sum_{(i,j)\in\mathcal{A}_z} \sum_{m\in\mathcal{M}} \sum_{c\in\mathcal{C}} \rho_c l_{ij} n_c q_{ij}^c \frac{f_{ij}^m}{F_{ij}^c} v_z^m x_{ij}^c \leq \theta \qquad \forall z \in \mathcal{Z} \tag{20}$$

$$\sum_{(i,j)\in\mathcal{A}'\setminus\mathcal{A}} h_{ij} y_{ij} \leq B \tag{21}$$

$$\sum_{m\in\mathcal{M}} z_m = p \tag{22}$$

$$\sum_{m\in\mathcal{M}} v_z^m = 1 \qquad \forall z \in \mathcal{Z} \tag{23}$$

$$v_z^m \leq z_m \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M} \tag{24}$$

$$\sum_{i\in\mathcal{N}:(i,j)\in\mathcal{A}'} x_{ij}^c - \sum_{l\in\mathcal{N}:(j,l)\in\mathcal{A}'} x_{jl}^c = \begin{cases} 1 & \text{if } j = o(c) \\ -1 & \text{if } j = d(c) \\ 0 & \text{otherwise} \end{cases} \qquad \forall j \in \mathcal{N}, c \in \mathcal{C} \tag{25}$$

$$x_{ij}^c \leq y_{ij} \qquad \forall (i,j) \in \mathcal{A}' \setminus \mathcal{A}, c \in \mathcal{C} \tag{26}$$

$$\pi_i^c - \pi_j^c \leq n_c l_{ij} \qquad \forall (i,j) \in \mathcal{A}, c \in \mathcal{C} \tag{27}$$

$$\pi_i^c - \pi_j^c \leq n_c l_{ij} + M(1 - y_{ij}) \qquad \forall (i,j) \in \mathcal{A}' \setminus \mathcal{A}, c \in \mathcal{C} \tag{28}$$

$$\pi_{o(c)}^c - \pi_{d(c)}^c \geq \sum_{(i,j)\in\mathcal{A}'} n_c l_{ij} x_{ij}^c \qquad \forall c \in \mathcal{C} \tag{29}$$

$$x_{ij}^c \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}', c \in \mathcal{C} \tag{30}$$

$$y_{ij} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}' \setminus \mathcal{A} \tag{31}$$

$$z_m \in \{0,1\} \qquad \forall m \in \mathcal{M} \tag{32}$$

Table 3: Notation for greedy algorithm

Notation	Description
θ	Objective value (minimum of maximum risk values on zones)
η_{ij}^c	Risk of hazmat type c traveling on link (i,j)
Ω_c	Vector of risk values for the links on path P_c
Ω'_c	Vector of risk values for the links on path P'_c

$$v_z^m \in \{0,1\} \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M} \tag{33}$$

$$\pi_i^c \quad \text{free} \quad \forall i \in \mathcal{N}, c \in \mathcal{C} \tag{34}$$

In the single-level representation, x_{ij}^c and y_{ij} are determined simultaneously. This change results in the loss of the total unimodularity. Therefore, it is necessary to reimpose integrality on the x_{ij}^c variables by replacing $x_{ij}^c \geq 0$ by $x_{ij}^c \in \{0,1\}$. We can linearize the quadratic term in constraints (20) by introducing the variables $w_{ij}^{cmz} = v_z^m x_{ij}^c$ and adding the following constraints:

$$v_z^m + x_{ij}^c - w_{ij}^{cmz} \leq 1 \qquad \forall (i,j) \in \mathcal{A}', c \in \mathcal{C}, m \in \mathcal{M}, z \in \mathcal{Z} \tag{35}$$

$$v_z^m + x_{ij}^c \geq 2w_{ij}^{cmz} \qquad \forall (i,j) \in \mathcal{A}', c \in \mathcal{C}, m \in \mathcal{M}, z \in \mathcal{Z} \tag{36}$$

$$w_{ij}^{cmz} \geq 0 \qquad \forall (i,j) \in \mathcal{A}', c \in \mathcal{C}, m \in \mathcal{M}, z \in \mathcal{Z} \tag{37}$$

This results in a mixed integer linear programming (MILP) problem, for which we had success solving small size problems by CPLEX.

4.2 A Greedy heuristic approach

We present a greedy iterative construction algorithm for cases that CPLEX is unable to solve the model. Let $G(\mathcal{N}, \mathcal{A})$ denote the current hazmat network and $G'(\mathcal{N}, \mathcal{A}')$ denote the hazmat network including all the potential-to-add links and the current links. Table 3 describes the notation used for the greedy algorithm. The main concept behind our greedy heuristic algorithm is that there are two options to transport a hazmat shipment; shortest path on network G, and shortest path on network G'. The first requirement to iteratively implement the greedy algorithm is to find initial locations for HRTs. To effectively determine the HRTs initial locations, we need to find the common links between any two shortest paths corresponding to OD pair $c \in \mathcal{C}$ on networks G and G'. These common links will be used by hazmat carriers no matter which shortest path (Shortest path on network G or shortest path on network G') will be selected for OD pair $c \in \mathcal{C}$ by our greedy algorithm. If there exist multiple shortest paths for OD pair $c \in \mathcal{C}$, we choose any shortest path arbitrarily. Step 0 of the greedy algorithm mathematically describes this procedure. Then, we apply the p-center problem presented in Step 1 to find the initial location of HRTs for the common links found in Step 0.

Knowing the initial location of HRTs, in Step 2, we calculate the links' risk values associated with transporting hazmat shipments $c \in \mathcal{C}$ on either of these two shortest paths, the shortest path with less maximum link risk value (ζ_c) will be selected. Subsequently, we select a hazmat shipment ($\bar{c}$) with its corresponding chosen shortest path, and we assume that hazmat carriers travel on this selected shortest path to transport hazmat shipment $\bar{c}$.

In Step 3, if the selected shortest path belongs to network G', its corresponding potential-to-add links will be added to network G. Then, we update the set of common links in Step 4. Afterwards, we apply p-center problem (presented in Step 1) to update the location of HRTs considering the hazmat shipments related to the selected OD pairs. This procedure continues iteratively until there is i) no more OD pair to select or ii) not enough budget to open any of the potential links. In this case, p-center problem presented in Step 5 will be solved to find the final location of HRTs. We describe our greedy algorithm in detail as follow.

Step 0. Given the two networks G and G', the algorithm, in each iteration, first finds the shortest paths P_c and P'_c, respectively, for each OD pair $c \in \mathcal{C}$. When there exist multiple shortest paths, choose any shortest path arbitrarily. We consider paths P_c and P'_c as sets of links so that set operations are meaningful. Construct the following subset of OD pairs:

$$\mathcal{W} = \{c : c \in \mathcal{C}, \text{and } P_c \neq P'_c\}$$

If $\mathcal{W}$ is an empty set, go to Step 5. Otherwise, construct the following set of links:

$$\mathcal{I} = \bigcup_{c \in \mathcal{C}} \{P_c \cap P'_c\}$$

which is the set of all common links.

Step 1. Given the set $\mathcal{I}$, the greedy algorithm solves the following p-center problem to find the location of HRTs:

$$\min_{z_m, v_z^m} \theta$$

subject to:

$$\sum_{(i,j) \in \mathcal{A}_z \cap \mathcal{I}} \sum_{m \in \mathcal{M}} \sum_{c \in \mathcal{C}} \rho_c l_{ij} n_c q_{ij}^c \frac{f_{ij}^m}{F_{ij}^c} v_z^m \delta_{ij}^c \leq \theta \qquad \forall z \in \mathcal{Z}$$

$$\sum_{m \in \mathcal{M}} z_m = p$$

$$\sum_{m \in \mathcal{M}} v_z^m = 1 \qquad \forall z \in \mathcal{Z}$$

$$v_z^m \leq z_m \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M}$$

$$z_m \in \{0, 1\} \qquad \forall m \in \mathcal{M}$$

$$v_z^m \in \{0, 1\} \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M}$$

where δ_{ij}^c is a constant that equals to 1 if $(i,j) \in P_c \cap P'_c$ for each OD pair c and 0 otherwise. Let the solutions be $\bar{z}_m$ and $\bar{v}_z^m$.

Step 2. From the solutions $\bar{v}_z^m$ to the p-center problem in Step 1, we compute

$$\eta_{ij}^c = \rho_c l_{ij} n_c q_{ij}^c \frac{f_{ij}^m}{F_{ij}^c} \qquad \text{when } \bar{v}_z^m = 1 \text{ and } (i,j) \in \mathcal{A}_z$$

for all $(i,j) \in (P_c \setminus P'_c) \cup (P'_c \setminus P_c)$ and $c \in \mathcal{W}$. Then, we find the following values:

$$\begin{aligned} \eta_c &= \max_{(i,j) \in P_c \setminus P'_c} \{\eta_{ij}^c\} \quad \forall c \in \mathcal{W} \\ \eta'_c &= \max_{(i,j) \in P'_c \setminus P_c} \{\eta_{ij}^c\} \quad \forall c \in \mathcal{W} \\ \zeta_c &= \min\{\eta_c, \eta'_c\} \qquad \forall c \in \mathcal{W} \end{aligned}$$

Then we find the minimum of $\{\zeta_c : c \in \mathcal{W}\}$ and the corresponding OD pair $\bar{c}$.

Step 3. For the chosen OD pair $\bar{c}$, we consider the following three cases:

- Case 1: If $\eta_{\bar{c}} < \eta'_{\bar{c}}$, then update

$$\begin{aligned} \mathcal{I} &\leftarrow \mathcal{I} \cup (P_{\bar{c}} \setminus P'_{\bar{c}}) \\ \mathcal{W} &\leftarrow \mathcal{W} \setminus \{\bar{c}\} \end{aligned}$$

  and go to Step 4.

- Case 2: If $\eta_{\bar{c}} > \eta'_{\bar{c}}$, then we check the remaining budget B.
  - If $\sum_{(i,j) \in P'_{\bar{c}} \setminus P_{\bar{c}}} h_{ij} \leq B$, then update

$$\begin{aligned} \mathcal{I} &\leftarrow \mathcal{I} \cup (P'_{\bar{c}} \setminus P_{\bar{c}}) \\ \mathcal{W} &\leftarrow \mathcal{W} \setminus \{\bar{c}\} \\ B &\leftarrow B - \sum_{(i,j) \in P'_{\bar{c}} \setminus P_{\bar{c}}} h_{ij} \end{aligned}$$

    and go to Step 4.
  - If $\min_{(i,j) \in \mathcal{A}' \setminus \mathcal{A}} \{h_{ij}\} \leq B < \sum_{(i,j) \in P'_{\bar{c}} \setminus P_{\bar{c}}} h_{ij}$ update

$$\begin{aligned} \mathcal{I} &\leftarrow \mathcal{I} \cup (P_{\bar{c}} \setminus P'_{\bar{c}}) \\ \mathcal{W} &\leftarrow \mathcal{W} \setminus \{\bar{c}\} \end{aligned}$$

    and go to Step 4.

– If $B < \min_{(i,j)\in\mathcal{A}'\setminus\mathcal{A}}\{h_{ij}\}$, then update

$$\mathcal{A} \leftarrow \mathcal{I} \cup \left(\bigcup_{c\in\mathcal{W}} \{P_c \setminus P'_c\} \right)$$

and go to Step 5.

- Case 3: If $\eta_{\bar{c}} = \eta'_{\bar{c}}$, consider the vectors

$$\Omega_c = \{\eta^c_{ij} : (i,j) \in P_c \setminus P'_c\}$$
$$\Omega'_c = \{\eta^c_{ij} : (i,j) \in P'_c \setminus P_c\}$$

Since the largest elements in the two sets Ω_c and Ω'_c were the same, we compare the second largest elements. If Ω_c has the greater second largest element, then we follow the steps in Case 1; if Ω'_c does, follow Case 2. If the second largest elements are the same again, we consider the third largest elements. We repeat this process until we can break the tie. When either of the two sets has no element to compare, assume zero (This implies that if either of two sets has less number of elements, we add zeros to that set until $|\Omega_c|$=$|\Omega'_c|$).

Step 4. If $\mathcal{W}$ is an empty set, update

$$\mathcal{A} \leftarrow \mathcal{I}$$

and go to Step 5; otherwise go to Step 1.

Step 5. We have an updated set of links $\mathcal{A}$ that contains new links to build. Given this set $\mathcal{A}$, we solve the following p-center problem to find the final solution.

$$\min_{z_m, v^m_z} \theta$$

subject to:

$$\sum_{(i,j)\in\mathcal{A}_z\cap\mathcal{A}} \sum_{m\in\mathcal{M}} \sum_{c\in\mathcal{C}} \rho_c l_{ij} n_c q^c_{ij} \frac{f^m_{ij}}{F^c_{ij}} v^m_z \delta^c_{ij} \le \theta \qquad \forall z \in \mathcal{Z}$$
$$\sum_{m\in\mathcal{M}} z_m = p$$
$$\sum_{m\in\mathcal{M}} v^m_z = 1 \qquad \forall z \in \mathcal{Z}$$
$$v^m_z \le z_m \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M}$$
$$z_m \in \{0,1\} \qquad \forall m \in \mathcal{M}$$
$$v^m_z \in \{0,1\} \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M}$$

where δ^c_{ij} is a constant that equals to 1 if link $(i,j) \in \mathcal{A}$ for each OD pair $c \in \mathcal{C}$ is used and 0 otherwise. Let the final solutions be θ^*, z^*_m, and v^{*m}_z.

In our heuristic approach we apply an exact algorithm proposed by Özsoy and Pınar (2006) to solve the p-center problem in Step 5.

4.3 Special case with the same unique shortest path on networks G and G' for each OD pair

If there is same unique shortest path between each origin-destination on networks G and G', then the upper-level problem, called as P_1 in section 3, reduces to a p-center problem. Let $\mathcal{C}$ be the set of all OD pairs. We suppose that for all $c \in \mathcal{C}$, OD c has a unique shortest path on both networks G and G', and these two paths are the same. In this case, we can reduce network G' to G with the lower level problem's optimal solution $(\bar{x}^c_{ij})$. All the constraints related to the set of potential-to-add links become redundant, and the original upper level problem reduces to the p-center optimization problem called as P_5:

$$(P_5):$$

$$\min_{z_m, v_z^m} \theta \tag{38}$$

subject to:

$$\sum_{(i,j)\in\mathcal{A}_z} \sum_{m\in\mathcal{M}} \sum_{c\in\mathcal{C}} \rho_c l_{ij} n_c q^c_{ij} \frac{f^m_{ij}}{F^c_{ij}} v^m_z \bar{x}^c_{ij} \leq \theta \qquad \forall z \in \mathcal{Z} \tag{39}$$

$$\sum_{m\in\mathcal{M}} z_m = p \tag{40}$$

$$\sum_{m\in\mathcal{M}} v^m_z = 1 \qquad \forall z \in \mathcal{Z} \tag{41}$$

$$v^m_z \leq z_m \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M} \tag{42}$$

$$z_m \in \{0,1\} \qquad \forall m \in \mathcal{M} \tag{43}$$

$$v^m_z \in \{0,1\} \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M} \tag{44}$$

$$\bar{x}^c_{ij} \text{is an optimal solution}$$

Here, $\bar{x}^c_{ij}$ is the optimal solution given by the lower level problem. P_5 represents the general form of p-center problem presented by Owen and Daskin (1998). Thus for this special situation, the greedy algorithm is able to solve P_5 by solving only a p-center problem.

5 Experimental Results

We organize our experimental results into four parts. The first part analyzes the benefits of jointly deciding on network design and locations of the HRTs. The second shows sensitivity analysis with respect to the number of available HRTs and the available budget. For parts one and two, the single-level form of the proposed model is optimally solved by CPLEX 12.6 solver. The third part examines the efficiency of the greedy algorithm for large-size network instances using a network derived from Albany, New York. The last part seeks a robust solution to the problem using the greedy algorithm. The heuristic algorithm is implemented in Java. All algorithms ran on a PC with 2 GB of RAM. We report the main features of the model as follows:

- $|\mathcal{C}|$: number of OD pairs
- #HRT: the total number of available hazmat response teams
- Inst: the test problem's ID
- θ^*: the optimal objective value of the bi-level model in terms of maximum zone total risk
- Opt. Obj.: the optimal objective value of the bi-level model obtained by solving the single-level MILP formulation with CPLEX
- Heu. Obj.: the objective value of the bi-level model obtained by heuristic algorithm

5.1 Benefits of joint network design and location of HRTs

The data set we used for the material in Sections 5.1 and 5.2 corresponds to the Sioux Falls Road Network (Figure 3) with 24 nodes and 76 links. We consider 8 zones, 10 candidate sites for locating HRTs, and 14 potential-to-add links for this network. We define zones in such a way that each links belongs to only one zone. If a link belongs to two or more zones, that link can be broken into sub-links such that each sub-link belongs to only one zone and represents as a link itself. We indicate the candidate sites with black rectangles and the potential-to-add links with bolded arrows.

The results in Table 4 are presented for two different ways of calculating risk measurement. First, designing the road network for hazmat carriers based on a modified version of the network design model proposed by Kara and Verter (2004), and then, locating hazmat response teams on the designated network by solving a p-center problem with the objective of minimization of maximum zone total risk. We modify the Kara and Verter (2004)'s model such that it conforms to our network design concept. We change the policy of road closure to the policy of making additional road segments available to hazmat carriers. We also change the objective of Kara and Verter (2004)'s model to minimization of maximum zone total risk. To be able to define this objective we incorporate zone's concept in the aforementioned modified network design model. The first approach is denoted as 'Network Design + Location Problem' in Table 4 and the exact network design and location models are described in Appendices A and B, respectively. Second, a joint decision making process of designing a hazmat transportation network and locating HRTs by solving our proposed bi-level model in section 3. All problem instances in Table 4 are solved for the fixed values of parameters B=1000 and p=4.

As it is evident in Table 4, our proposed model gives a smaller maximum zone total risk value, θ^*, for instances $|\mathcal{C}|$=5, 10, and 15. While, the objective value θ^* remains the same for both methods in Table 4 for instance problems $|\mathcal{C}|$=20, and 25 and this happens when the opened links obtained by network design model includes all the opened links found by our proposed model. We emphasize

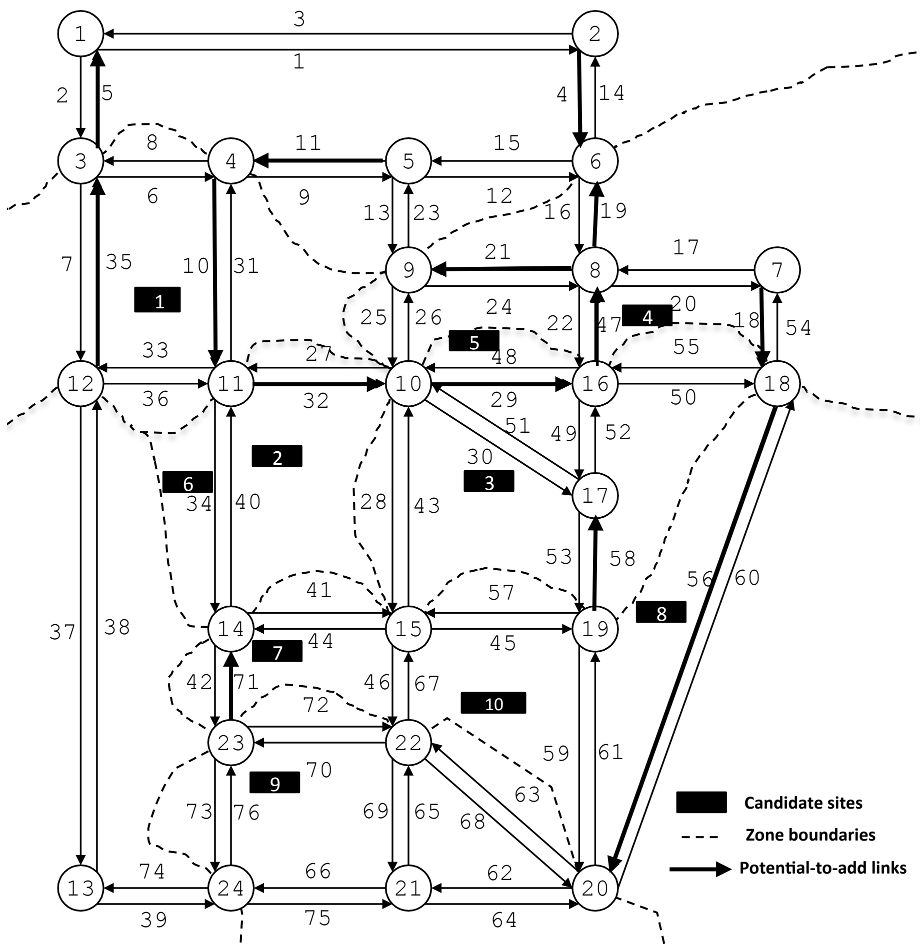

Figure 3: Sioux Falls Road Network

Table 4: Comparison between the joint and separate decision making to locate HRTs in terms of θ.

NS	Network Design + Location Problem (given the chosen routes)				Our Problem			
	Travel Cost	θ^*	Opened Sites	Opened Links	Travel Cost	θ^*	Opened Sites	Opened Links
5	6529	183.7	1, 5	4	8127	173.8	1, 3	10, 32
10	13139	307.3	1, 2	4	15853	290.2	1, 2	10, 32
15	20059	472.1	1, 2	4, 10	20466	461.2	1, 2	4, 7
20	29213	565.2	1, 10	4, 10, 29, 32	29254	565.2	1, 10	4, 10, 32
25	34564	612.3	1, 10	4, 10, 29, 32	34605	612.3	1, 10	4, 10, 32

that for instance problems $|C| = 20$, and 25, the separate decision making approach gives better results in terms of the Travel Cost. In test instances $|\mathcal{C}|$=5, 10, and 15, we have respectively 5.7%, 5.8%, and 2.4% reduction in maximum zone total risk (θ^*) by a nominal increase in the travel cost. The results in Table 4 imply that whenever link addition helps to reduce the risk associated with the response time to a hazmat incident by conducting the hazmat carriers into less risky routes, our joint decision making approach may provides a better solution.

5.2 Sensitivity analysis

There are two types of cost: cost of link addition and cost of HRT deployment. In order to evaluate the effect of cost on risk of hazmat shipments, a sensitivity analysis is performed based on the number of available HRTs and the total available budget for link addition.

Tables 5 and 6 show the results for different sizes of test problems on the Sioux Falls network. The optimal risk values (θ^*) in Table 5 indicate that by increasing the number of available HRTs, a larger number of candidate sites can be opened. Consequently, the maximum zone total risk decreases, but increasing the number of HRTs to more than a specific level may not help to reduce the risk. That level introduces the optimal number of needed HRTs. As an example, for instances 5 and 6 in Table 5, increasing in number of HRTs from 3 to 4 does not show any reduction in θ^*. Moreover, Table 6 represents the impact of budget level from 0 to 1000 on θ^*. It is clearly shown that by increasing the budget from 0 to 400, we achieve a remarkable reduction in risk values. There is no more change in θ^* for a budget level larger than 400. Run time for finding the optimal solutions ranges from 3 seconds to 48 hours in tables 5 and 6, implying that run time for different test problems is highly dependent on the network's topology, number of hazmat shipments, and the model's parameters.

5.3 Efficiency of the greedy heuristic

We consider Albany's road network with 149 links, 90 nodes, 6 zones, 7 potential-to-add links, and 21 candidate sites as a larger network to investigate efficiency of the greedy algorithm. Table 7 shows the results for 50 test problems (10 instances for each $|\mathcal{C}|$=5, 10, 20, 30, 40). The budget B for all test instances in Table 7 is considered as 1000. Number of available HRTs for test instances with $|\mathcal{C}|$=5 is specified as 4. For the rest of test instances with $|\mathcal{C}|$=10, 20, 30, and 40, number of available HRTs is considered as 6. The optimal objective values found by CPLEX (Opt. Obj.) are very close to the heuristic objective values (Heu. Obj.) for all instances, even for some of them the Opt. Obj. values are equal to their corresponding Heu. Obj. values. By comparing run times in both approaches, we notice that greedy heuristic is computationally efficient and able to solve all sizes of test problems presented in Table 7 in less than 3 minutes. On the other hand, run time for the exact optimal solutions by CPLEX ranges from 4 seconds to 10 hours and 30 minutes. The average gap for every 10 instances of the same size $|\mathcal{C}|$, ranges from 0.06% to 1.54%. We tried to find the optimal solution for a test problem with $|\mathcal{C}|$=60, but CPLEX was unable to solve the single-level model. For such cases, the greedy algorithm appears to provide a viable alternative.

Table 5: Results for the single-level MIP formulation of the bi-level model for B=1000, # of HRT=1, 2, 3, and 4.

Inst.	$\lvert\mathcal{C}\rvert$	# HRT	θ^*	Travel Cost	Opened Links	Opened Sites	Run Time
1	5	4	176.1	6529	4	1, 2, 3, 4	3 sec
2	5	3	176.1	6529	4	1, 2, 4	9 sec
3	5	2	183.7	6529	4	1, 4	3 sec
4	5	1	210.5	6529	4	1	3 sec
5	10	4	294.9	13139	4	1, 2, 4, 10	18 sec
6	10	3	294.9	13139	4	1, 2, 4	20 sec
7	10	2	307.3	13139	4	1, 2	17 sec
8	10	1	358.0	13139	4	1	7 sec
9	15	4	443.7	20059	4, 10	1, 2, 4, 10	40 sec
10	15	3	447.8	20059	4, 10	1, 2, 4	6 min, 18 sec
11	15	2	472.1	20059	4, 10	1, 2	58 sec
12	15	1	552.8	20059	4, 10	1	12 sec
13	20	4	531.1	29213	4, 10, 29, 32	1, 2, 4, 10	2 min, 30 sec
14	20	3	537.8	29213	4, 10, 29, 32	1, 2, 10	10 min, 11sec
15	20	2	565.2	29213	4, 10, 29, 32	1, 10	7 min, 34 sec
16	20	1	653.6	29213	4, 10, 29, 32	1	41 sec
17	25	4	577.0	34564	4, 10, 29, 32	1, 3, 4, 10	5 min, 39 sec
18	25	3	584.9	34564	4, 10, 29, 32	1, 2, 10	1 hr, 32 min
19	25	2	612.3	34564	4, 10, 29, 32	1, 10	17 min, 35 sec
20	25	1	705.8	34564	4, 10, 29, 32	1	77 sec
21	35	4	684.1	45295	4, 10, 29, 32	1, 2, 4, 10	74 min, 20 sec
22	35	3	692.1	45295	4, 10, 29, 32	1, 3, 10	56 min, 12 sec
23	35	2	719.4	45295	4, 10, 29, 32	1, 10	41 min, 47 sec
24	35	1	812.9	45295	4, 10, 21, 29, 32	1	1 min, 55 sec
25	45	4	684.1	55168	4, 10, 21, 29, 32	1, 2, 4, 10	36 min, 40 sec
26	45	3	692.1	55168	4, 10, 21, 29, 32	1, 2, 10	23 min, 5 sec
27	45	2	719.4	55168	4, 10, 21, 29, 32	1, 10	38 min, 30 sec
28	45	1	812.9	55168	4, 10, 21, 29, 32	1	2 min

Table 6: Results for the single-level MIP formulation of the bi-level model for # of HRT=4, B=0, 200, 400, 600, 800, and 1000.

Inst	$\|\mathcal{C}\|$	B	θ^*	Travel Cost	Opened Links	Opened Sites	Run Time
1	25	1000	577.0	34564	4, 10, 29, 32	1, 3, 4, 10	5 min, 39 sec
2	25	800	577.0	34564	4, 10, 29, 32	1, 3, 4, 10	8 min, 42 sec
3	25	600	577.0	34564	4, 10, 29, 32	1, 3, 4, 10	43 min, 32 sec
4	25	400	577.0	34605	4, 10, 32	1, 3, 4, 10	59 min, 40 sec
5	25	200	686.0	42760	10	1, 3, 4, 10	22 min, 35 sec
6	25	0	842.2	46204	_	1, 2, 4, 10	42 sec
7	30	1000	684.1	39739	4, 10, 32	1, 2, 4, 10	8 min, 48 sec
8	30	800	684.1	39739	4, 10, 32	1, 2, 4, 10	12 min, 42 sec
9	30	600	684.1	39739	4, 10, 32	1, 2, 4, 10	16 min, 18 sec
10	30	400	684.1	39739	4, 10, 32	1, 2, 4, 10	1 hr, 6 min
11	30	200	793.1	47894	10	1, 2, 4, 10	21 min, 18 sec
12	30	0	949.4	51338	_	1, 2, 4, 10	12 sec
13	35	1000	684.1	45295	4, 10, 21, 29, 32	1, 2, 4, 10	74 min, 20 sec
14	35	800	684.1	45295	4, 10, 21, 29, 32	1, 2, 4, 10	2 hr, 54 min
15	35	600	684.1	45366	4,10, 32	1, 2, 4, 10	3 hr, 37 min
16	35	400	684.1	45366	4,10, 32	1, 2, 4, 10	6 hr, 1 min
17	35	200	793.1	53521	10	1, 2, 4, 10	3 hr, 5 min
18	35	0	949.4	57452	_	1, 3, 4, 10	7 sec
19	40	1000	684.1	50827	4, 10, 32	1, 2, 4, 10	5 hr, 37 min
20	40	800	684.1	50827	4, 10, 32	1, 2, 4, 10	44 min, 50 sec
21	40	600	684.1	50827	4, 10, 32	1, 2, 4, 10	2 hr, 2 min
22	40	400	684.1	50827	4, 10, 32	1, 2, 4, 10	3 hr, 32 min
23	40	200	793.1	58982	10	1, 3, 4, 10	47 min, 30 sec
24	40	0	949.4	62290	_	1, 3, 4, 10	20 sec
25	45	1000	684.1	55168	4, 10, 21, 29, 32	1, 2, 4, 10	36 min, 40 sec
26	45	800	684.1	55168	4, 10, 21, 29, 32	1, 2, 4, 10	8 hr, 45 min
27	45	600	684.1	55168	4, 10, 29, 32	1, 2, 4, 10	1 day, 20 hrs
28	45	400	684.1	55315	4, 10, 32	1, 2, 4, 10	13 hrs, 1 min
29	45	200	793.1	63470	10	1, 2, 4, 10	10 hr, 52 min
30	45	0	949.4	66914	_	1, 2, 4, 10	25 sec

Table 7: Comparison between the optimal and the heuristic solutions of the bi-level model

$\|\mathcal{C}\|$	Inst.	Opt. Obj.	Opt. Travel Cost	Opt. Run Time	Heu. Obj.	Heu. Travel Cost	Heu. Run time	Gap (%)	Average Gap (%)
5	1	9192.65	108327	7 sec	9192.65	108327	2 sec	0.00	0.06
	2	4566.74	98397	82 sec	4566.74	98397	2 sec	0.00	
	3	11035.03	94106	4 sec	11035.03	94106	6 sec	0.00	
	4	3021.64	76136	6 sec	3021.64	76136	1 sec	0.00	
	5	6152.82	52436	3 sec	6152.82	52436	1 sec	0.00	
	6	10445.29	90415	42 sec	10445.29	90415	1 sec	0.00	
	7	11490.04	89950	3 sec	11561.51	89869	1 sec	0.62	
	8	9110.10	82519	11 sec	9110.10	82519	1 sec	0.00	
	9	2744.67	78286	201 sec	2745.21	75343	2 sec	0.02	
	10	7781.97	86752	52 sec	7781.97	86752	1 sec	0.00	
10	1	19348.44	192460	726 sec	19455.20	188922	1 sec	0.55	0.55
	2	25836.16	220153	231 sec	25836.16	220153	1 sec	0.00	
	3	16097.55	204195	575 sec	16676.38	204903	1 sec	3.47	
	4	19848.14	218100	218 sec	19848.14	218100	1 sec	0.00	
	5	17398.41	176012	496 sec	17398.41	176012	1 sec	0.00	
	6	15313.00	191194	282 sec	15313.00	191194	1 sec	0.00	
	7	26624.96	242320	436 sec	26636.80	239341	1 sec	0.04	
	8	14594.46	172583	91 sec	14792.51	167693	1 sec	1.34	
	9	20467.79	160751	75 sec	20467.79	160751	1 sec	0.00	
	10	16773.34	155129	27 sec	16788.15	157028	1 sec	0.09	
20	1	27346.37	387139	16 min, 20 sec	27706.68	385324	1 sec	1.30	1.19
	2	32294.22	376751	1 hr, 4 min	32940.13	364241	3 sec	1.96	
	3	38186.24	372706	12 min, 18 sec	38207.35	371989	1 sec	0.06	
	4	27409.57	336513	1 hr, 17 min	27467.17	332913	2 sec	0.21	
	5	35961.42	371198	21 min	37602.66	373613	4 sec	4.36	
	6	27239.16	363770	10 min, 8 sec	27584.23	354223	3 sec	1.25	
	7	42934.09	377022	13 min	43567.31	377250	8 sec	1.45	
	8	27944.35	319351	6 min, 12 sec	27992.28	317319	4 sec	0.17	
	9	21091.19	335927	32 min	21213.42	328630	19 sec	0.58	
	10	38405.14	408498	1 hr, 35 min	38610.09	399860	2 sec	0.53	
30	1	49277.33	553239	45 min, 6 sec	49368.92	544546	2 sec	0.19	0.76
	2	46337.34	615978	31 min 12 sec	46692.60	610451	2 sec	0.76	
	3	42641.84	527140	36 min, 30 sec	42971.42	522854	35 sec	0.77	
	4	61520.17	570554	2 hr, 21 min	61593.09	564512	8 sec	0.12	
	5	45523.25	507762	1 hr, 19 min	46177.68	510211	10 sec	1.42	
	6	40199.45	562969	1 hr, 5 min	40835.47	554571	1 sec	1.56	
	7	46383.95	510302	1 hr, 11 min	46881.51	510684	9 sec	1.06	
	8	51941.59	530246	1 hr, 22 min	52016.76	529967	17 sec	0.14	
	9	40975.60	468967	20 min, 40 sec	41575.09	472019	29 sec	1.44	
	10	61520.17	570554	2 hr, 11 min	61593.09	564512	8 sec	0.12	
40	1	69870.07	759049	10 hr, 30 min	71374.22	754049	7 sec	2.11	1.54
	2	60786.12	750448	1 hr, 17 min	61587.14	748992	21 sec	1.30	
	3	62094.19	642510	3 hr, 5 min	62919.89	642216	72 sec	1.31	
	4	66901.61	653331	4 hr, 5 min	67768.18	653101	33 sec	1.28	
	5	57690.60	684856	2 hr, 49 min	58207.74	675634	37 sec	0.89	
	6	45927.99	635732	6 hr, 46 min	46768.04	632065	32 sec	1.80	
	7	73126.41	739666	1 hr, 58 min	74560.76	743012	12 sec	1.92	
	8	67976.26	671582	2 hr, 19 min	68610.82	665225	13 sec	0.92	
	9	53221.16	709176	6 hr, 41 min	54105.63	703872	4 sec	1.63	
	10	69123.77	629537	1 hr, 2 min	70721.21	802324	158 sec	2.26	

Table 8: Hazmat accident rate per mile

Hazmat Category	$\rho_c(\times 10^{-7})$
Class 1 – Explosives	6.58170
Class 2 – Gases	2.37209
Class 3 – Flammable liquids	4.96414
Class 4 – Flammable solids	6.85756
Class 5 – Oxidizers and organic peroxides	3.03833
Class 6 – Toxic (poison) materials and infectious substances	2.29576
Class 7 – Radioactive materials	3.94605
Class 8 – Corrosive materials	1.32109
Class 9 – Miscellaneous dangerous goods	7.16646

5.4 Robust solution development

For hazmat problems, precise estimates of the required parameters and their probability distributions are almost impossible. In addition, the consequences of hazmat accidents depend on the nature of accidents, hazmat types, population and properties surrounding the release points. As per a report prepared for Federal Motor Carrier Safety Administration in 2001, which is available on the US Department of Transportation Pipeline and Hazardous Materials Safety Administration website (`http://phmsa.dot.gov`), the accident probabilities are estimated for all of the hazmat categories based on the historical record. All hazmats are separated into nine classes according to the Code of Federal Regulations. Table 8 shows these nine hazmat classes and their corresponding accident probabilities (ρ_c). All statistics and information presented in this section are taken from this report.

There are two kinds of enroute hazmat accidents: release and non-release. Since we consider HRT's response time to enroute hazmat incidents in our model, we confine the data presented for the impact of hazmat accidents to the release type of accidents. The estimated occurence rates are presented in Table 9 for three release types: release only, fire but no explosion, and explosion. We calculate the hazmat accident rates by dividing the number of hazmat release type by the total number of release incidents (for each type of accident release).

Table 10 shows the breakdown of estimated annual release accident impact costs into the three mentioned types. The total impact cost includes all costs of clean up, product loss, carrier damage, property damage, environmental damage, injury, fatality, evacuation and delay. Having estimated annual total cost and estimated annual total number of incidents, estimated total impact cost per incident can be easily found. Although we obtain the estimated annual impact costs or consequences for each incident release type r (q_r), estimated annual release rates for hazmat type c (ϕ_c^r), and accident probabilities for hazmat type c (ρ_c), we still need to consider data uncertainty to deal with the insufficiency of available data. To this end, we consider the interval data for q^r as shown in Table 10 with the minimum possible and maximum possible values. In fact, the accident probability for each hazmat incident release type (π_c^r) can be calculated as $\pi_c^r = \rho_c \times \phi_c^r$.

Table 9: Estimated annual rate for all release accident types

Hazmat Category	Estimated annual number of release accidents			Estimated annual release accident rate ($\phi_c^r\%$)		
	Fire	Explosion	Release only	Fire	Explosion	Release only
1	0.20	0.10	11	0.0030674	0.0041356	0.0162237
2	9	2	64.02	0.1380368	0.0818933	0.0944219
3	50	22.02	418	0.7668711	0.9016665	0.6165009
4	0	0.00	8	0	0	0.0117990
5	2	0.00	27	0.0306748	0	0.0398218
6	1	0.00	14	0.0153374	0	0.0206483
7	0	0.00	6	0	0.0000204	0.0088492
8	2	0.00	71	0.0306748	0	0.1047166
9	1	0.30	59	0.0153374	0.0122840	0.0870180
All Categories	65.20	24.42	678.02	1	1	1

We consider the interval $[\pi_c^r - 0.5 \times 10^{-5}, \pi_c^r + 0.5 \times 10^{-5}]$ as the range in which each hazmat accident probability can change. Then, we randomly generate the model's parameters in their interval data for 9 hazmat types and 3 en-route release incident types (3×9 scenarios). All scenarios are defined for a test problem with 60 number of OD pairs on Albany's road network. In order to find the best policy for all these 27 scenarios, we can take advantage of the computational efficiency of our greedy heuristic algorithm and solve the proposed model for all scenarios. We seek the worst-case optimal policies that depend on all the 27 scenarios under consideration and provide guaranteed performances for designed networks within the specified interval of the accident probability data. In such cases, finding the robust solution is crucial in risk assessment and the main tool used is minimax method, which suggests robust policies with guaranteed optimal performance (Rustem and Howe, 2009). Figure 4 presents the heuristic results for all scenarios, where the numbers on z-axis indicate the objective values (θ_τ^ψ) associated with policy τ, found by heuristic approach for scenario $\psi = \tau$, and applied for scenarios $\psi \neq \tau$, ψ =1 to 27. Clearly, the θ_τ^ψ values where $\psi = \tau$ have to be the minimum values for $\psi = 1$ to 27. It is noted that the heuristic algorithm does not necessarily find the optimal objective values. We obtained 6 different policies (solutions) for all 27 scenarios. As it is evident from Figure 4, policies 2 and 4 have smaller risk values in their worst-case scenarios (scenario 22). To choose the best policy, the minmax method has been applied as follow:

$$\min_{\text{Policy}_\tau} \max_{\text{Scenario}_\psi} \{\theta_\tau^\psi\} \tag{45}$$

Using data in shown Figure 4, we have: $\min\{\theta_2^{22}, \theta_4^{22}\} = \theta_4^{22} = 534.3$. Therefore, policy 4 is selected as the robust solution for all the 27 scenarios under consideration.

Table 10: Probability distributions for estimated annual total impact costs for all incident types

Incident Type	Total cost of damage	Total number of accidents	Total cost per incident (estimated impact cost q^r)	Distribution of estimated impact cost (consequences)
Enroute accident release only	276392494	678.02	407646.5	Uniform[400000, 500000]
Enroute accident fire	77160758	65.02	1186723.4	Uniform[1150000, 1250000]
Enroute accident explosion	62208606	24.42	2547236.3	Uniform[2500000, 2600000]

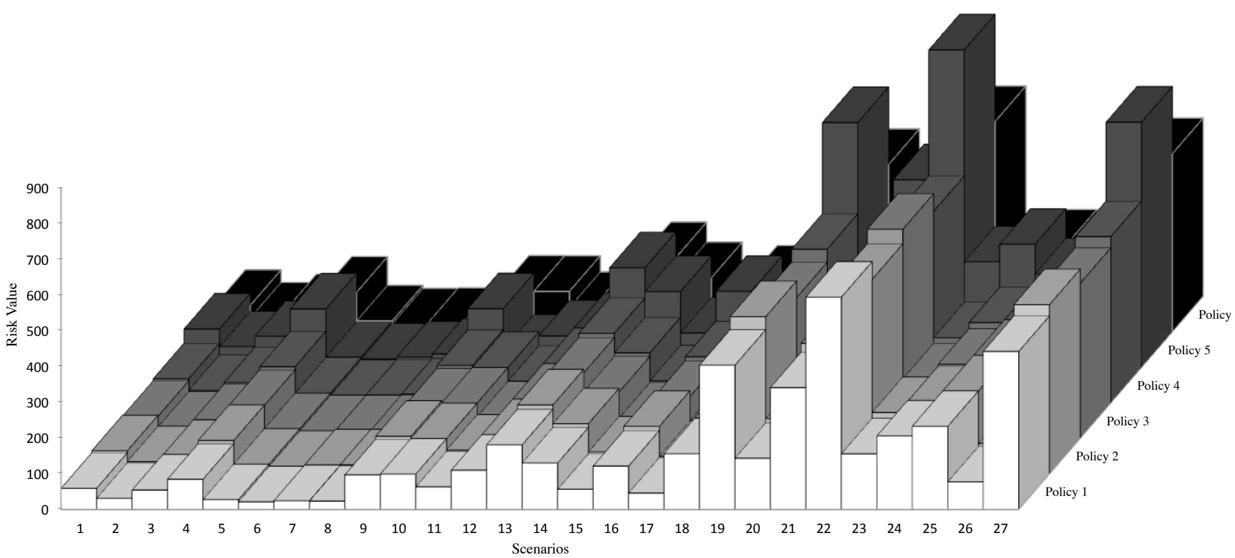

Figure 4: Scenario-based risk values shown for 6 policies obtained by Greedy Algorithm

6 Summary and Future Research

In this paper, we have proposed a bi-level network design model for hazmat transportation. The model aims to minimize the maximum zone total risk and guarantees risk equity. There is a leader-follower relationship between the regulatory authorities and hazmat carriers. The authority tries to find the best locations among all candidate sites to deploy HRTs and to make additional road segments available to hazmat carriers. On the other hand, the hazmat carriers select their minimum-cost routes on the designated network. Authority's decisions about opening road segments and locating HRTs expand the possible route choices for hazmat carriers and help to reduce the average response time to a hazmat incident. The presented non-linear bi-level model is reformulated into a single-level mixed integer linear problem. The single-level model is solved using CPLEX 12.6 for a small size network. The greedy heuristic approach is able to find very good (near optimal or sometimes optimal) solutions in a short time period for large size test problems. Experimental results show that joint decision of network design and deployment of emergency response team may result in better risk reduction. Increasing the total number of available HRTs for deployment and the total available budget for link addition has a remarkable impact on risk mitigation.

We also conclude that the greedy algorithm is computationally efficient and delivered high quality solutions. Finally, a robust solution is obtained for 27 scenarios under consideration by applying the proposed heuristic approach for a large size test problem ($|\mathcal{C}|$=60).

In practice, other emergency response units may be dispatched to the incident site, such as, Emergency Medical Service, Fire, and Police. This suggests a future work dedicated to joint deployment of all emergency units. Furthermore, the average response time is highly dependent on traffic congestion and incident location, thus, a robust solution should be investigated considering uncertainty in determining the response time. Another research opportunity is to consider the problem of adding additional HRTs to a situation where a certain number of HRTs already exist.

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Appendices

In Table 4, the results under the title 'Network Design + Location Problem' are obtained by solving the models in Appendices A and B sequentially.

A Network Design Model Applied in Table 4

The following bi-level network design model is a developed version of the network design model proposed by Kara and Verter (2004). Set $\mathcal{A}$ denotes the set of all the links (potential to add links and current links) on the hazmat network. In the following model, for simplicity, we assume that the cost of opening a link (h_{ij}) is 0, if it is a current (existing) link, and greater than 0 if it is a potential-to-add link. This assumption makes all of the current links available at no price. Therefore, the model decides on which of the potential links to open.

$$\min_{y_{ij}} R \tag{46}$$

$$\sum_{(i,j)\in\mathcal{A}_z} \sum_{c\in\mathcal{C}} \rho_c l_{ij} n_c q_{ij}^c x_{ij}^c \leq R \qquad \forall z \in \mathcal{Z} \tag{47}$$

$$\sum_{(i,j)\in\mathcal{A}} h_{ij} y_{ij} \leq B \tag{48}$$

$$y_{ij} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A} \tag{49}$$

where x_{ij}^c solves

$$\min \sum_{c\in\mathcal{C}} \sum_{(i,j)\in\mathcal{A}} n_c l_{ij} x_{ij}^c \tag{50}$$

$$\sum_{i\in\mathcal{N}:(i,j)\in\mathcal{A}} x_{ij}^c - \sum_{l\in\mathcal{N}:(j,l)\in\mathcal{A}} x_{jl}^c = \begin{cases} 1 & \text{if } j = o(c) \\ -1 & \text{if } j = d(c) \\ 0 & \text{otherwise} \end{cases} \qquad \forall j \in \mathcal{N}, c \in \mathcal{C} \tag{51}$$

$$x_{ij}^c \leq y_{ij} \qquad \forall (i,j) \in \mathcal{A}, c \in \mathcal{C} \tag{52}$$

$$x_{ij}^c \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}, c \in \mathcal{C} \tag{53}$$

B Location Model Applied in Table 4

The bi-level location model presented in this appendix seeks to find the optimal location of HRTs on a designated network.

$$\min_{z_m, v_z^m} \theta \tag{54}$$

$$\sum_{(i,j)\in\mathcal{A}_z^*} \sum_{m\in\mathcal{M}} \sum_{c\in\mathcal{C}} \rho_c l_{ij} n_c q_{ij}^c \frac{f_{ij}^m}{F_{ij}^c} v_z^m x_{ij}^c \leq \theta \qquad \forall z \in \mathcal{Z} \tag{55}$$

$$\sum_{m\in\mathcal{M}} z_m = p \tag{56}$$

$$\sum_{m\in\mathcal{M}} v_z^m = 1 \qquad \forall z \in \mathcal{Z} \tag{57}$$

$$v_z^m \leq z_m \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M} \tag{58}$$

$$z_m \in \{0,1\} \qquad \forall m \in \mathcal{M} \tag{59}$$

$$v_z^m \in \{0,1\} \qquad \forall z \in \mathcal{Z}, m \in \mathcal{M} \tag{60}$$

where x_{ij}^c solves

$$\min \sum_{c\in\mathcal{C}} \sum_{(i,j)\in\mathcal{A}^*} n_c l_{ij} x_{ij}^c \tag{61}$$

$$\sum_{i\in\mathcal{N}:(i,j)\in\mathcal{A}^*} x_{ij}^c - \sum_{l\in\mathcal{N}:(j,l)\in\mathcal{A}^*} x_{jl}^c = \begin{cases} 1 & \text{if } j = o(c) \\ -1 & \text{if } j = d(c) \\ 0 & \text{otherwise} \end{cases} \qquad \forall j \in \mathcal{N}, c \in \mathcal{C} \tag{62}$$

$$x_{ij}^c \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}^*, c \in \mathcal{C} \tag{63}$$

where set $\mathcal{A}^*$ includes all the links on the designated network obtained by solving the model presented in Appendix A.

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